

Syllabus:

1. Simple stress, strain

$$\text{Simple stress} = P/A$$

(i) Actual stress

(ii) Normal stress.

2. Complex stress.

3. Principal stress and theories of failure

4. S.F.D and B.M.D.

5. Torsion.

6. Bending stress and shear stress

7. Thin cylinders.

Simple stress and strain:

* stress is a tensor quantity.* Young's modulus of concrete = $5000 \sqrt{f_{ck}}$ (19456 : 2000)Young's modulus of steel = $2 \times 10^5 \text{ N/mm}^2$

$$\text{Young's modulus, } E = \frac{\sigma}{\epsilon}$$

Poisson's ratio $\mu = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$

$$\mu = 0 - 0.5$$

Bottle cork $\mu = 0$.

Creep:

Long term deflection due to sustained loading.

Fatigue:

Failure due to cyclic loading.

$$151348 : 2999$$

$$EC = 5700 \sqrt{f_{ck}}$$

Strength Of Materials:

The internal resistance offered by the body to sustain external loading without any deformation.

Stress:

* It is the resistance offered by the body per unit area

* This resistance is due to cohesion b/w the particles

* Its unit is $= \text{N/mm}^2$ (or) MPa .

where

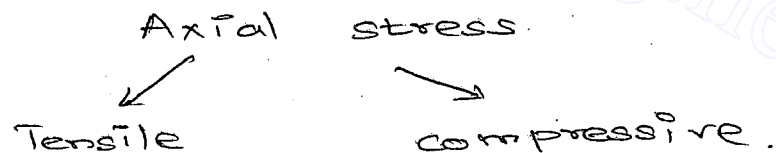
$$1 \text{ Pa} = 1 \text{ N/m}^2$$

* It is a tensor quantity:

Broad Classification:

- * Simple stress
- * complex stress
- * Bending stress.
- * shear stress

Simple stress:



Other classification:

- * Engineering stress.
- * True stress.

Engineering stress:

$$\sigma_{\text{engg}} = \frac{P}{A_0}$$

A_0 - original c/s Area

True stress:

$$\sigma_{\text{true}} = \frac{P}{A_i}$$

A_i - Instantaneous c/s Area

Relationship b/w True & Engg Stress

$$\sigma_{\text{true}} = \frac{P}{A_i} \times \frac{A_0}{A_0}$$

$$\sigma_{\text{true}} = \sigma_{\text{eng}} \times \frac{A_0}{A_i}$$

Strain (ϵ):

It is the ratio b/w change in length and original length.

$$\text{Dimension} = M^0 L^0 T^0$$

Hooke's Law:

It is given by "Robert Hook" and it states that stress is directly proportional to the strain within the elastic limit.

$$f \propto \epsilon$$

$$f = E \epsilon$$

$$E = \frac{f}{\epsilon}$$

$$f = E \epsilon$$

$$\frac{P}{A} = E \times \frac{\delta L}{L}$$

$$\delta L = \frac{PL}{AE}$$

Poisson's Ratio:

It is the ratio b/w lateral strain and longitudinal strain.

It is denoted by μ (or) $\frac{1}{m}$ (or) ν .

Ex:

Material	μ
Concrete	0.2
Steel	0.3
Glass	0.318
Cork	0
Rubber	0.5

μ lies between 0 - 0.5.

Elasticity:

* It is defined as the property of material due to which the material regains its original shape after removal of loads.

* The materials can be classified into Elastic, plastic and Rigid.

Elastic:

When an elastic material is subjected to external loading undergoes deformation such that deformation disappears after removal of loading.

Plastic:

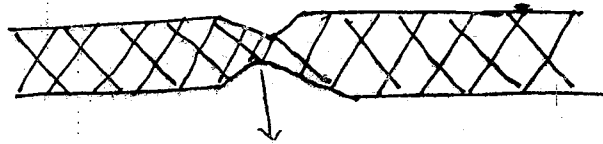
A plastic material undergoes continuous deformation during the period of loading and deformation is permanent and the material does not regain its shape.

Rigid:

* A rigid material does not undergo a deformation when it is subjected to loading.

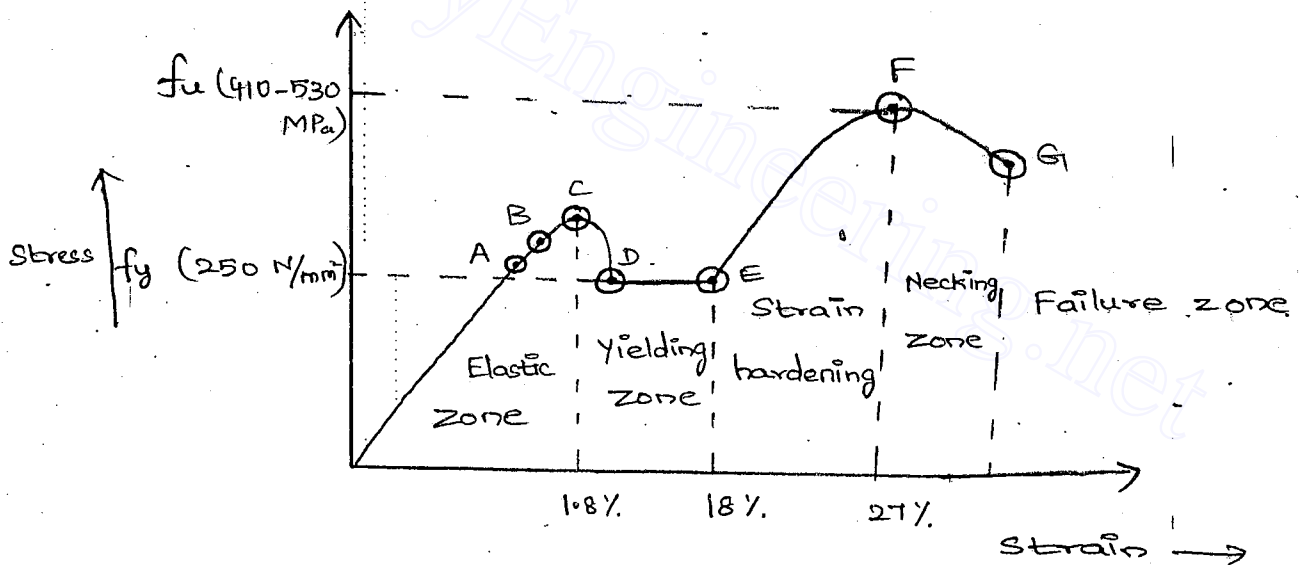
* Practically no material is purely plastic, elastic and rigid.

stress-strain graph of Mild steel:



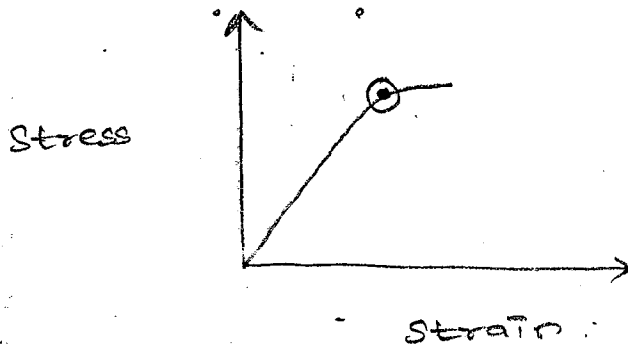
This type of failure is known as cup and cone fracture.

stress-strain graph of Mild steel:



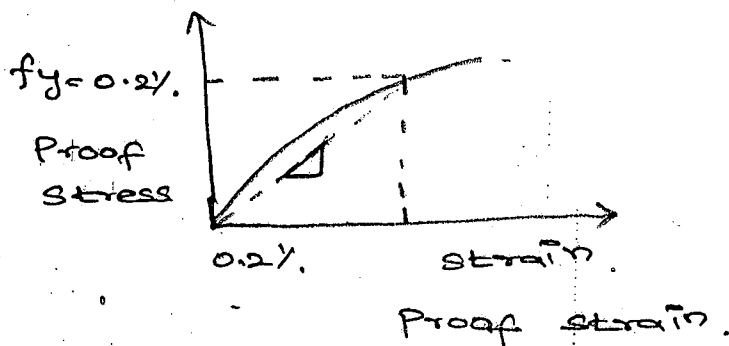
- A - Limit of proportionality.
- B - Elastic Limit
- C - Upper yield point
- D-E - Lower yield point
- F - Ultimate yield point
- G - Failure point.

Stress strain graph of glass.

Proof - stress:

When a material does not show an distinct yield point (Due to its nature) then 0.2% of proof strain is considered (a line drawn parallel to elastic line (limit of proportionality of line) and the corresponding stress considered as the yield stress of a material and it is known as proof stress.

This behaviour is seen in Aluminium, cast iron and high strength steel.

stress - strain graph of HYSD:

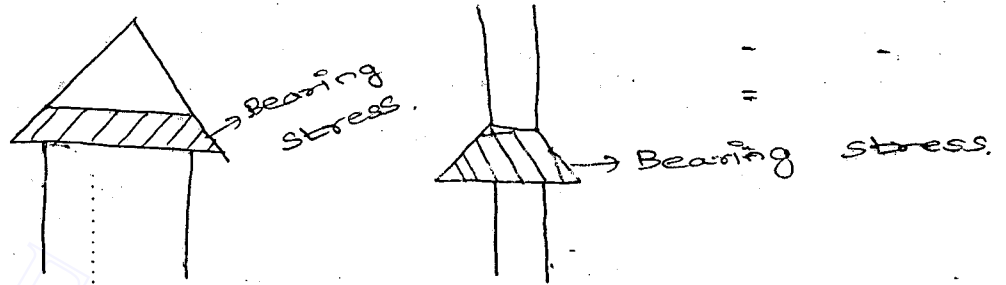
Normal

stresses classified into 3 :

- * Axial stress
- * Bearing stress
- * Bending stress.

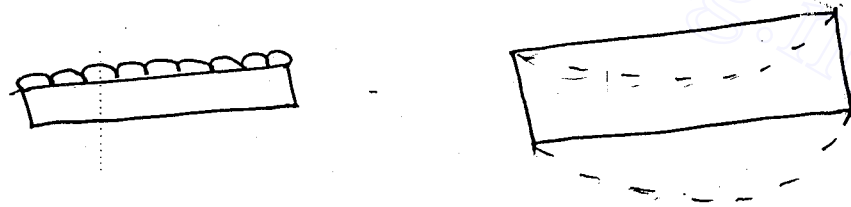
Bearing stress:

compressive stress arising when one body is supported by another is called bearing stress.



Bending stress:

Bending tensile stress and compressive stress is produced when it is subjected to bending.



Shearing stress:

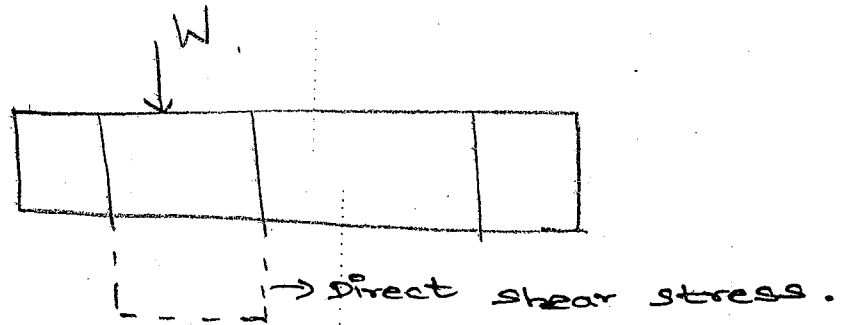
It is the stress acting in the plane of a section.

There are 2 kinds of shearing stress

- * Direct shear stress.
- * Indirect shear stress.

Direct shear stress:

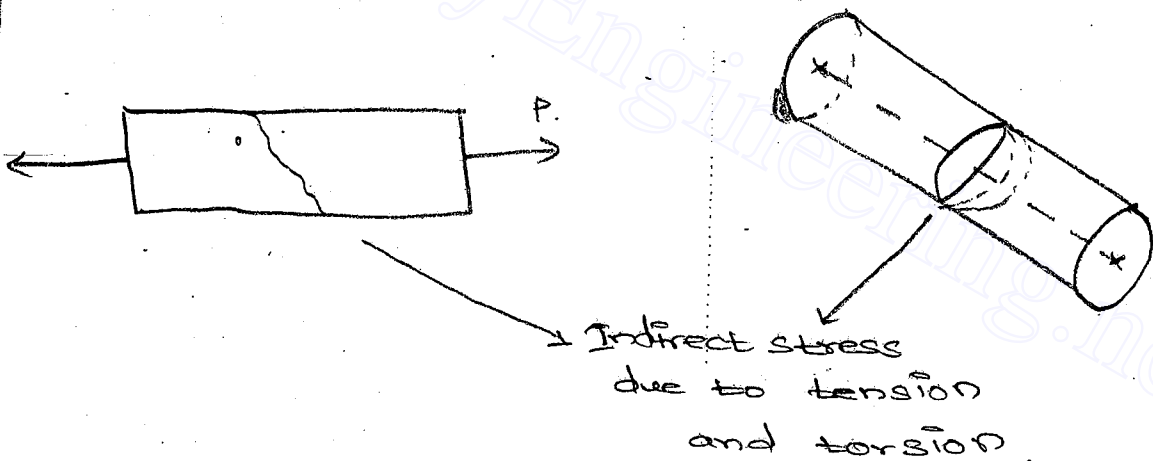
shear stress created due to direct action of forces in trying to cut through the materials.



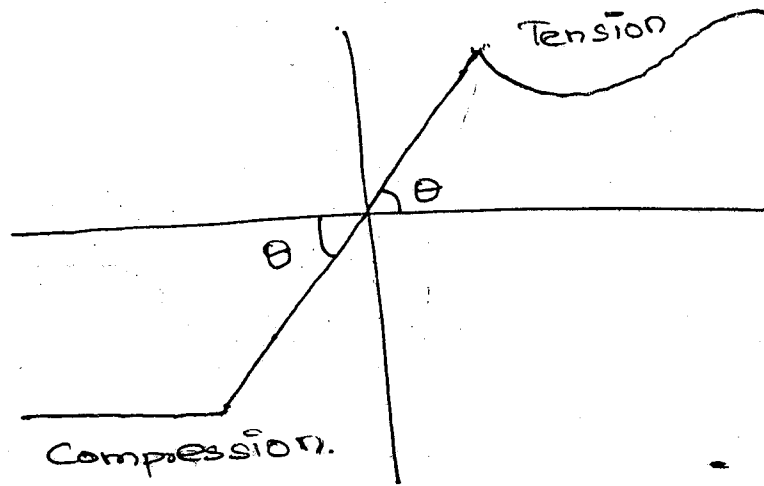
Indirect shear stress:

Indirect shear stress is due to

- (a) Tension (or) Compression.
- (b) Torsion.



under compression :



If structural steel is subjected to compression instead of tension - the stress-strain curve will be same through its straight line portion and through beginning of curve portion corresponding through yield and strain hardening.

For large value of strain, stress strain curve will diverged.

In compression no-necking occurs

Modulus of elasticity } = Modulus of elast
in compression } in tensile

STRENGTH OF MATERIALS

Creep - Time dependent deformation under sustained loading

* Creep is an important parameter that affects or influences.

* Influencing factor $n, E,$
Fatigue -

* Rate of creep decrease with time because as stress increases strain hardening takes place

* It depends on Temperature level, stress level, time, type of loading (static or dynamic)

* Generally effect of creep becomes noticeable at approx. 30% of melting point for metals.

Fatigue:

* Deterioration of material due to repeated cycles of loading of stress or strain resulting in progressive cracking that eventually produces fracture. Is called fatigue.

Plasticity:

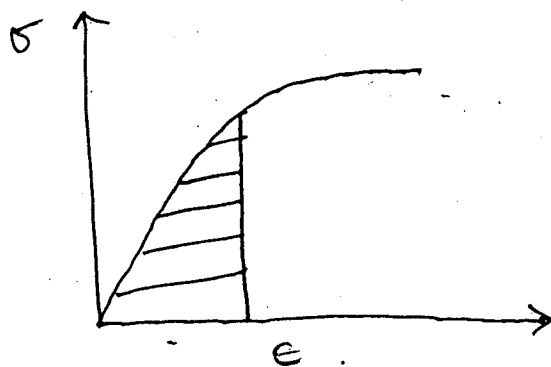
The characteristics of a material by which it undergoes inelastic strain beyond the strain at the elastic limit is known as plasticity.

Resilience:

* It is the property of a material to absorb energy when it deformed elastically and then upon unloading to have this energy recovered.

Hence greater the resilience more is desirable for spring action.

The area under stress strain curve within elastic limit is called as Modulus of resilience.



For a linearly elastic material strain energy stored per unit volume

$$= \frac{\sigma \times E \times \text{Volume}}{2} = \frac{\sigma \times \sigma}{2E}$$

$$= \frac{\sigma^2}{2E} \times V \quad (\text{Modulus of resilience})$$

Area under load - under - deformation within elastic limit is called as resilience.

Endurance limit:

Endurance limit is the stress level below which even large no. of stress cycle cannot produce fatigue failure.

* For structural steel, endurance limit = $\frac{1}{2} \times$ ultimate strength

* For non-ferrous metal stress @ failure continues to decrease. hence we define fatigue limit as stress corresponding to failure after a specific no. of loading cycles.

* Due to corrosion effect, endurance limit is reduced upto 50% of that under normal condition.

Relaxation:

* The decrease ~~in~~ stress in steel as a result of creep within steel under prolonged strain is called relaxation.

Stiffness:

* Stiffness may be defined as a ability of a material to withstand high load without major deformation.

Deformable bodies:

Def: Undergoes deformation when external forces acted upon them.

Volumetric strain: $\left(\frac{dv}{v}\right)$

Ratio of change in volume to original volume.

$$E_v = \frac{dv}{v}$$

Factor of Safety:

Ratio of ultimate tensile stress to the permissible stress is called factor of safety.

$$F.O.S = \frac{\text{ultimate stress}}{\text{Permissible stress}}$$

There are 3 types of Elastic constant

E, G, k .

1.) Modulus of Elasticity (or) Young's Modulus. (E)

2.) Bulk Modulus. (k)

3.) Shear Modulus (or) Rigidity Modulus (or) Modulus of rigidity. (G)

Bulk modulus:

Bulk modulus of a substance measures the substance's resistance to uniform compression.

It is defined as the ratio of direct stress to the resulting relative decrease of the volume.

$$K = \frac{\text{Direct stress}}{\text{Volumetric strain}}$$

Shear Modulus (or) Rigidity Modulus.

[G, C, N]:

Ratio of shear stress to shear strain.

$$C = \frac{\tau}{\phi}$$

Relationship b/w elastic constant.

$$E = 3K(1-2\mu)$$

$$E = 2G(1+\mu)$$

* If $K = \mu$

$$E = 3\mu(1-2\mu)$$

$$= 3\mu - 6\mu^2$$

$$= 3\mu - 6\mu^2$$

* If $K = G$, $E = K$

$$E = 3K(1-2\mu)$$

$$\boxed{\mu = 1/3}$$

* If $E = G$

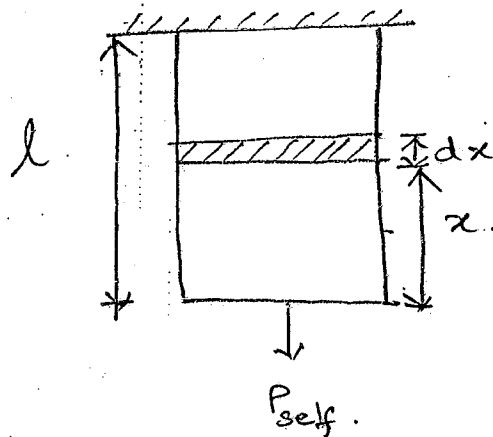
$$G = 2G(1+\mu)$$

$$= 2 + 2\mu$$

$$\boxed{\mu = -1/2}$$

* $E = \frac{9KG}{3K+G}$

STRENGTH OF MATERIAL.



$$P_{self} = \gamma \times A \times l.$$

$$\begin{aligned} \delta l_x &= \frac{P_x l_x}{A_x \times E} \\ &= \frac{\gamma \times A \times x \, dx}{AE} \\ &= \int_0^l \frac{\gamma A x \, dx}{AE} \end{aligned}$$

$$\begin{aligned} \delta l &= \frac{\gamma A}{AE} \left[\frac{x^2}{2} \right]_0^l \\ &= \frac{\gamma A \times l \times l}{2AE} \\ &= \frac{(\gamma A l) l}{2AE} \end{aligned}$$

$$\delta l = \frac{W_{total} \times l}{2AE}$$

$$\delta l = \frac{(0 + W_{\text{total}})}{2} \times l$$

AE.

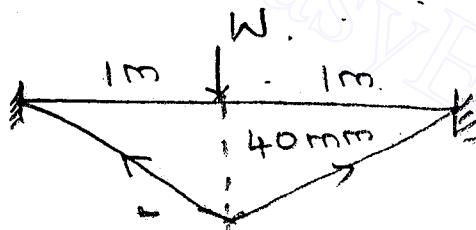
$$\delta l = \frac{W_{\text{total}} \times l}{2AE}$$

Note:

* Elongation of tapered circular dia rod $D_1 =$ small dia $D_2 =$ large dia due to load P .

$$\delta l = \frac{Pl}{\frac{\pi}{4} D_1 D_2 E}$$

1)



ϕ of wire = 1mm

lengths of deformed wire

$$\delta l = \frac{\delta l}{l} = \frac{\sqrt{1000^2 + 40^2} - 1000}{1000}$$

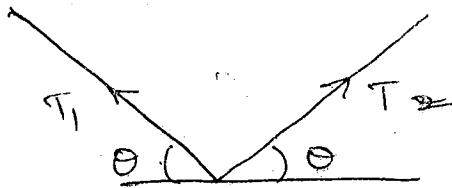
$$\delta l = 1000.799 - 1000$$

$$\delta l = 0.799 \text{ mm}$$

$$\delta l = \frac{Pl}{AE}$$

$$0.799 = \frac{P \times 1000}{\left(\frac{\pi \times 1^2}{4}\right) \times 2 \times 10^5}$$

$$P = 125.61 \text{ N}$$



$$\theta = \tan^{-1}(40/1000)$$

$$\boxed{\theta = 2.29}$$

$$\sum H = 0$$

$$T_1 \cos \theta = T_2 \cos \theta$$

$$\boxed{T_1 = T_2}$$

$$\sum V = 0$$

$$T_1 \sin \theta + T_2 \sin \theta = W$$

$$W = 2(125.51 \sin 2.29)$$

$$\boxed{W = 10.03 \text{ N}}$$

$$\sigma = \frac{8 E \delta}{l}$$

$$= \frac{8 \times 2 \times E}{l}$$

$$\boxed{\sigma = 159.80 \text{ N}}$$

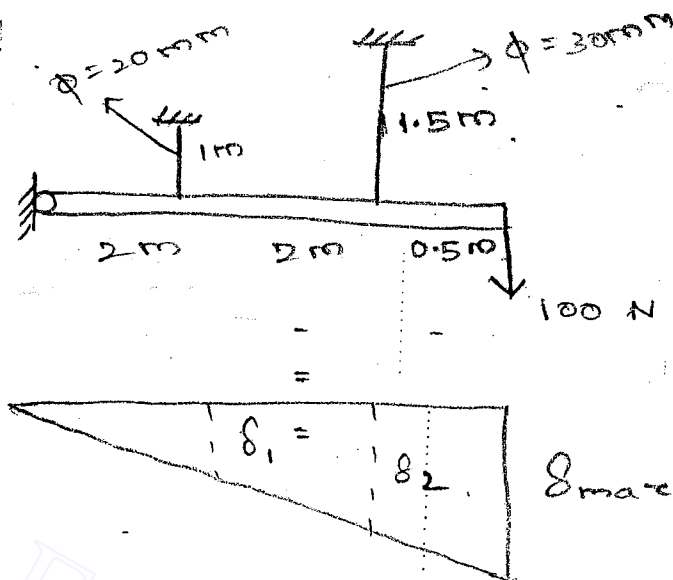
$$T = \sigma \times A$$

$$= 159.808$$

$$\times \frac{\pi \times 1^2}{4}$$

$$\boxed{T = 125.51 \text{ N}}$$

2. A rigid beam shown in figure. Determine stresses developed in the rod supporting the rigid beam.
- ↓
linear variation



$$\sum V = 0$$

$$T_1 + T_2 + R_A = 100 \text{ N} \quad \text{--- (1)}$$

$$\sum M_B = 0$$

$$2T_1 + 4T_2 = 100 \times 4.5$$

$$2T_1 + 4T_2 = 450 \quad \text{--- (2)}$$

By similar triangle property,

$$\frac{\delta_1}{2} = \frac{\delta_2}{4}$$

$$\boxed{\delta_2 = 2\delta_1}$$

$$\delta = \frac{Pl}{AE}$$

$$2 \times \frac{T_1 l_1}{A_1 E} = \frac{T_2 l_2}{A_2 E}$$

$$\frac{2 \times T_1 \times 1000}{\frac{\pi \times 20^2}{4} \times E} = \frac{T_2 \times 1500}{\frac{\pi \times 30^2}{4} \times E}$$

$$T_2 = \frac{2 \times 1000 \times 30^2}{20^2 \times 1500} T_1$$

$$T_2 = 3T_1$$

$$2T_1 + (4 \times 3)T_1 = 450$$

$$14T_1 = 450$$

$$T_1 = 32.14 \text{ N}$$

$$T_2 = 96.43 \text{ N}$$

$$\sigma_1 = \frac{T_1}{A_1} = \frac{32.14}{\left(\frac{\pi \times 20^2}{4}\right)} = 0.102 \text{ N/mm}^2$$

$$\sigma_2 = \frac{T_2}{A_2} = \frac{96.43}{\left(\frac{\pi \times 30^2}{4}\right)} = 0.136 \text{ N/mm}^2$$

$$R_A = 100 - 96.43 - 32.14$$

$$= -28.57$$

$$R_A = 28.57 \text{ N (down)}$$

CONCRETE STRUCTURES

→ Concrete Technology

- * Properties of concrete
- * Basics of mix design.

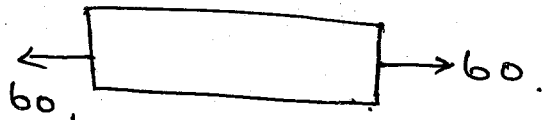
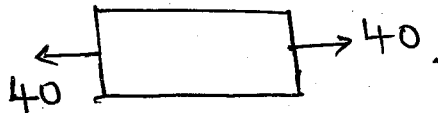
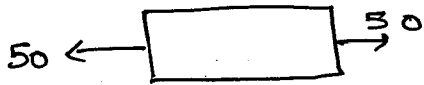
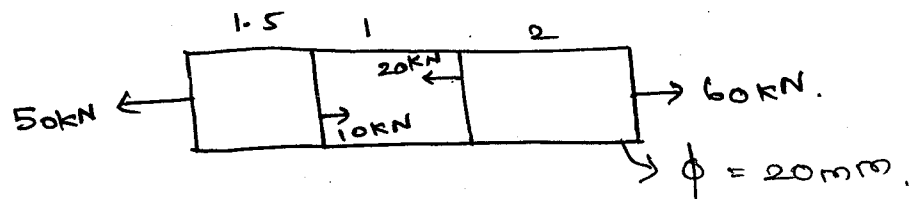
→ Concrete Design :

- * Basic working stresses and limit state design concepts
- * Analysis of ultimate load capacity
- * Design of members subjected to flexure, shear, compression and torsion by limit state method.

→ Prestressed concrete

- * Basic elements of prestressed concrete
- * Analysis of beam sections at transfer and service load.

1. Determine the change in length of a loaded member



$$\delta l = \frac{P_1 l_1}{A_1 E} + \frac{P_2 l_2}{A_2 E} + \frac{P_3 l_3}{A_3 E}$$

$$= \frac{1}{AE} (50 \times 1.5 + 1 \times 40 + 60 \times 2)$$

$$\delta l = \frac{235 \times 10^3 \times 10^3}{AE}$$

$$= \frac{235 \times 10^6}{\left(\frac{\pi \times 20^2}{4}\right) \times 2 \times 10^5}$$

$$\delta l = 3.74 \text{ mm}$$

2. A metallic bar of $l = 200 \text{ mm}$ Axial tensile load = 160 kN . $c/s = 40 \text{ mm} \times 40 \text{ mm}$. Elongation of bar is 0.2 mm . $\delta b = 0.005 \text{ mm}$. Determine $E, \mu =$

$$\delta l = \frac{Pl}{AE}$$

$$E = \frac{Pl}{A\delta l} = \frac{160 \times 10^3 \times 200}{40 \times 40 \times 0.2}$$

$$\boxed{E = 1 \times 10^5}$$

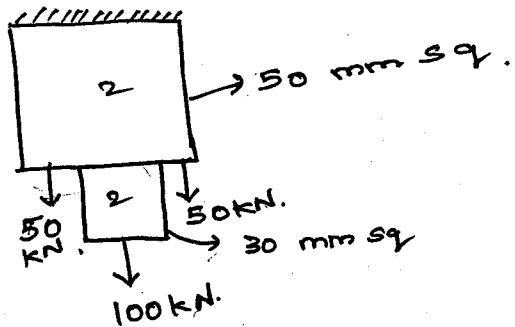
$$\text{lateral strain} = \frac{0.005}{40} = 0.125 \times 10^{-3}$$

$$\text{longitudinal strain} = \frac{0.2}{200} = 1 \times 10^{-3}$$

$$\mu = \frac{0.125 \times 10^{-3}}{1 \times 10^{-3}}$$

$$\boxed{\mu = 0.125}$$

1) Determine maximum stress developed in material as shown in.

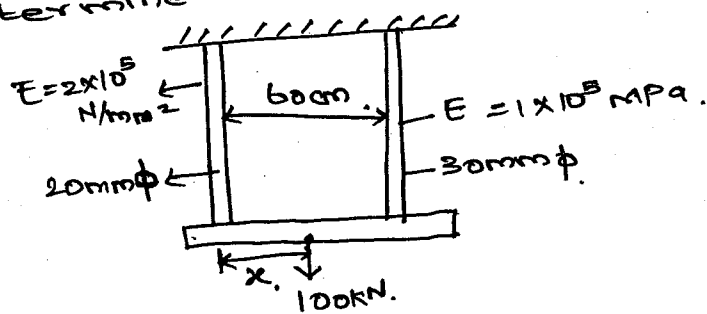


$$f_2 = \frac{P_2}{A_2} = \frac{100 \times 10^3}{30 \times 30} = 111.11 \text{ N/mm}^2$$

$$f_1 = \frac{P_1}{A_1} = \frac{(50 + 50 + 100) \times 10^3}{50 \times 50} = 80 \text{ N/mm}^2$$

∴ Maximum stress acting on the member is 80 N/mm².

2. 2 metallic bars are used to support a load as shown in figure. Determine the position of load such that the bottom supporting member remains horizontal. Also determine the stresses in the bars.



$$P_2 + P_1 = 100 \text{ kN}$$

$$P_2 = 100 - P_1$$

$$\Sigma M = 0$$

$$P_1 \times x = P_2 \times (60 - x)$$

$$(100 - P_2) \times x = 60P_2 - P_2 x$$

By compatibility equation.
 $100x + P_2 x$

$$\delta l_1 = \delta l_2$$

$$\frac{P_1 l_1}{A_1 E_1} = \frac{P_2 l_2}{A_2 E_2}$$

$$\frac{P_1}{\frac{\pi \times 20^2}{4} \times 2 \times 10^3} = \frac{P_2}{\frac{\pi \times 30^2}{4} \times 1 \times 10^3}$$

$$P_1 = P_2 \times \frac{20^2 \times 2}{30^2}$$

$$P_1 = 0.89 P_2$$

$$(0.89 P_2) + P_2 = 100$$

$$P_2 = 52.94 \text{ kN}$$

$$P_1 = 47.06 \text{ kN}$$

$$\Sigma M = 0$$

$$(100 \times x) = (47.06 \times 60)$$

$$60x = 2823.5$$

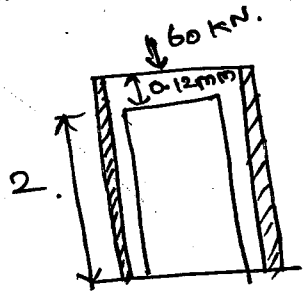
$$x = 31.76 \text{ cm}$$

$$\sigma_1 = \frac{P_1}{A_1} = 149.8 \text{ N/mm}^2$$

$$\sigma_2 = 74.89 \text{ N/mm}^2$$

3.

A steel rod of 20mm dia coaxially placed inside a brass cylinder of inner dia 22mm $D_o = 30\text{mm}$. Brass is longer than steel by 0.12mm. the length of the steel is 2m. External coaxial compressive load is 60 kN. Determine the stresses developed in steel and brass: $E_s = 2 \times 10^5 \text{ N/mm}^2$, $E_b = 1 \times 10^5 \text{ N/mm}^2$



$$\delta l = \frac{P_b l}{AE}$$

$$P_b = \frac{0.12 \times \pi \times (30^2 - 22^2) \times 1 \times 10^5}{4 \times 2000}$$

$$P_b = 10.96 \text{ kN.}$$

$$P_{\text{total}} = 60 - 10.96 = 58.04 \text{ kN.}$$

$$P_b' + P_s = 58.04 \times 10^3$$

Eg

By compatibility condition,

$$\delta l_{P_b'} = \delta l_{P_s}$$

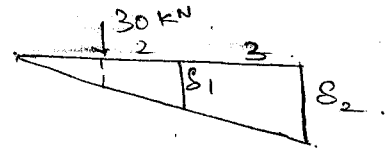
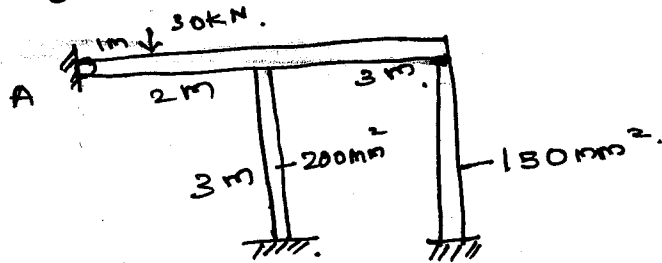
$$\frac{P_b' l}{A_b E_b} = \frac{P_s l}{A_s E_s}$$

$$\frac{P_b'}{\frac{\pi (30^2 - 22^2) \times 1 \times 10^5}{4}} = \frac{P_s}{\frac{\pi \times 20^2 \times 2 \times 10^5}{4}}$$

$$P_b' = \frac{P_s \times (30^2 - 22^2)}{20^2 \times 2}$$

$$P_b' = 0.52 P_s$$

7. A rigid beam as shown in figure.



$$\sum V = 0$$

$$R_A + R_B + R_C = 30 \text{ kN} \quad \text{--- (1)}$$

$$\sum M_A = 0$$

$$30 \times 1 = (R_B \times 2) + (R_C \times 5)$$

$$\frac{\delta_1}{2} = \frac{\delta_2}{5}$$

$$\delta_1 = \frac{2\delta_2}{5}$$

$$\frac{P_1 l_1}{A_1 E} = \frac{2 P_2 l_2}{5 A_2 E}$$

$$\frac{R_B \cancel{E}}{200 \times \cancel{E}} = \frac{2 \times R_C \times 3}{5 \times 150}$$

$$R_B = \frac{2 \times 200}{5 \times 150} \times R_C$$

$$R_B = 0.533 R_C$$

$$(0.533 R_C \times 2) + 5 R_C = 30$$

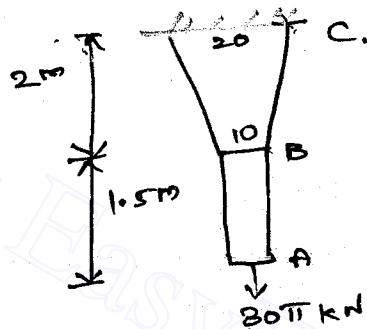
$$R_C = 4.945 \text{ kN}$$

$$R_B = 2.6357 \text{ kN}$$

$$R_C = 30 - 4.945 - 2.6357$$

$$R_A = 22.419 \text{ kN}$$

1. A tapered circular rod dia varying from 20mm to 10mm. is connected to another circular rod 10mm as shown in fig. both the bars are made of same material where $E = 2 \times 10^5 \text{ MPa}$. when subjected to a load of $30\pi \text{ kN}$, the deflection @ point A is _____ mm.



$$\delta l = \delta l_1 + \delta l_2.$$

$$= \frac{Pl}{\frac{\pi}{4} \times D_1 \times D_2 \times E} + \frac{Pl}{\frac{\pi}{4} D^2 \times E}.$$

$$= \frac{30\pi \times 10^3}{2 \times 10^5} \left[\frac{4 \times 2000}{\pi \times 20 \times 10} + \frac{1500 \times 4}{\pi \times 10^2} \right].$$

$$\delta l = 15 \text{ mm.}$$

2. For a beam of width $b = 230\text{mm}$
 $d = 500\text{mm}$. the no. of rebars of 12mm
 ϕ required to satisfy minimum tension
 reinforcement as per IS 456:2000.
 Assuming $F_{yk} = 500$ is —

$$A_{st} = \frac{0.85bd}{f_y}$$

$$= \frac{0.85 \times 230 \times 500}{500}$$

$$= 195.5 \text{ mm}^2$$

$$n = \frac{195.5}{\frac{\pi \times 12^2}{4}} = 1.72$$

$$n \approx 2$$

3. In a R.C.C section the stress @ the
 fiber in compression is 5.8MPa .
 $x_a = 58\text{mm}$. M_{25} Assuming linear elastic
 behaviour of concrete the effective
 curvature of the section is —

$$\sigma_{cbC} = 5.8\text{MPa} \quad x_a = 58\text{mm} \quad f_{ck} = 25$$

Bending equation

$$\frac{M}{I} = \frac{f}{y} = \frac{E}{R}$$

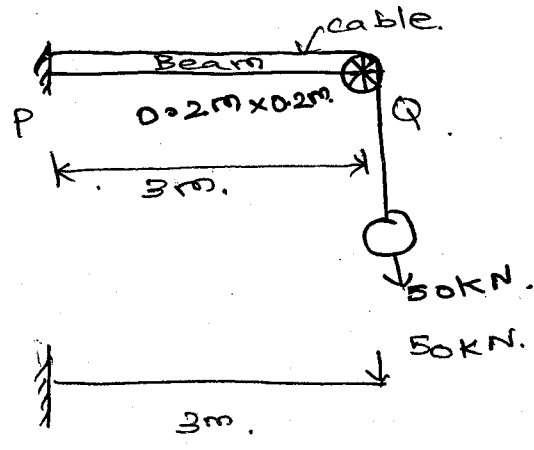
$$\frac{\sigma}{y} = \frac{E}{R}$$

$$\text{Radius of curvature } \frac{1}{R} = \frac{\sigma}{y \times E} = \frac{5.8}{58 \times 5000 \times \sqrt{25}}$$

$$\boxed{\frac{1}{R} = 4 \times 10^{-6} / \text{mm}}$$

4. The values of axial stress in KN/m^2 .

B.M in KN.m . S.F in KN . acting at point P. for the arrangement as shown in figure respectively.

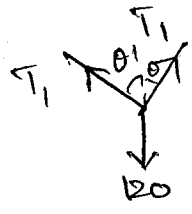
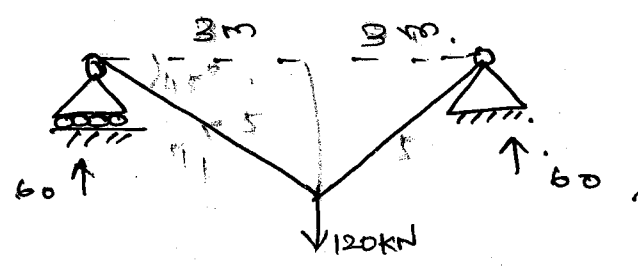


$B.M = 50 \times 3 = 150 \text{ KN.m}$

Axial stress = $\frac{50}{0.2 \times 0.2} = 1250 \text{ KN/m}^2$

Axial force = 50 KN

5. The tension in 10m long cable is shown in figure neglecting its self weight is.



$\sum V = 0$

$2T_1 \cos \theta = 120$

$2T_1 \cos \theta = 120$

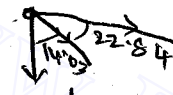
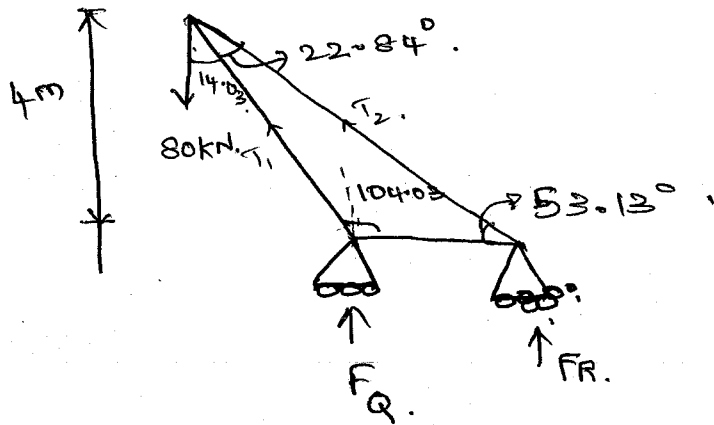
$T_1 = \frac{120 \times 5}{2 \times \frac{4}{5}}$

$T_1 = 75 \text{ KN}$

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6. Mathematical Idealization of crane has.

3 bars with vertices as shown in figure with a load of 80 kN hanging vertically the co-ordinates of the vertices are given in the diagram. The forces in PQ.



$$\sum H = 0$$

$$T_2 \sin 36.87 - T_1 \sin 14.03$$

$$T_2 = -0.404 T_1$$

$$\sum V = 0$$

$$80 = T_2 \cos 36.87 + T_1 \cos 14.03$$

$$= -0.404 T_1 \cos 36.87 + T_1 \cos 14.03$$

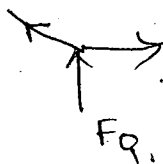
$$= -0.326 T_1 + 0.97 T_1$$

$$80 = T_1 0.644$$

$$T_1 = 124.19 \text{ kN}$$

$$F_{QR} = T_1 \sin 14.03$$

$$F_{QR} = 30.107 \text{ kN}$$



Euler column Load $P_E = \frac{\pi^2 EI}{L^2}$

Thermal stresses:

change in length, $\delta l = l \alpha \Delta T$

Thermal stress $\sigma_{\text{thermal}} = E \epsilon$
 $= \frac{\delta l}{l} \times E$
 $= \frac{l \alpha \Delta T}{l} \times E$

$$\sigma_{\text{Thermal}} = \alpha \Delta T E$$

Compression member

$\delta l = (l_c \times \Delta t) - \text{net elongation}$

Tension member $\delta l = \text{net elongation} - (l_c \times \Delta t)$

Thermal Stress:

1. Two copper plates and one steel plate are rigidly connected to each other and they are heated to a temperature to 315°C . The room temperature is 15°C . Net the elongation of assembly is 2mm. Determine the original length of the material and stress developed in the material $\alpha_s = 12 \times 10^{-6} / ^{\circ}\text{C}$, $\alpha_c = 17.5 \times 10^{-6} / ^{\circ}\text{C}$
 $E_s = 2 \times 10^5 \text{ N/mm}^2$ $E_c = 1 \times 10^5 \text{ N/mm}^2$.

Using Compa
 Solution:

$$\delta l_c = l_c \alpha \Delta T \quad -2 \text{ (Compression)}$$

$$\delta l_s = 2 - l_s \alpha \Delta T \quad \text{(Tension)}$$

Stress in steel

$$f_{\text{steel}} = \left(\frac{\delta l_s}{l} \right) \times E_s$$

$$= \frac{[2 - (l_s \alpha \Delta T)] \times E_s}{l}$$

Stress in copper.

$$f_{\text{copper}} = \frac{\delta l_c}{l} \times E_c$$

$$= \frac{(l_c \alpha \Delta T - 2) \times E_c}{l}$$

$$F_{\text{steel}} = F_{\text{copper}}$$

$$\frac{[2 - (l_s \alpha \Delta T)] \times E_s \times A}{l} = \frac{(l_c \alpha \Delta T - 2) \times E_c \times 2A}{l}$$

$$2 - (l \times 12 \times 10^{-6} \times 300) \times 2 \times 10^5 = (l \times 17.5 \times 10^{-6} \times 300 - 2) \times 1 \times 10^5$$

$$4 = l [(12 \times 10^{-6} \times 300) + (17.5 \times 10^{-6} \times 300)]$$

$$l = 451.98 \text{ mm}$$

$$\delta l = \alpha \Delta T L$$

$$2 = 12 \times 10^{-6} \times 300 \times$$

linear

- * when two different bars are connected rigidly and have two different α value such that $\alpha_1 > \alpha_2$.

→ then material 1 is under Compression
Material 2 is under Tension

$$\delta l_{\pm} = \text{Net elongation} - l \alpha_2 \Delta T$$

- * when there is no yielding of support.

$$\delta l = l \alpha \Delta T$$

- * when there is yielding of support

$$\delta l = (\text{yielding length}) = \left[\frac{l \alpha \Delta T - \text{yielding length}}{l} \right]$$

- * EI - Flexural rigidity
 AI - Axial rigidity
 GA - Shear rigidity
 GJ - Torsional rigidity

- * $\frac{EI}{L}$ - Flexural stiffness.
 $\frac{AI}{L}$ - Axial stiffness
 $\frac{GA}{L}$ - Shear stiffness
 $\frac{GJ}{L}$ - Torsional stiffness.

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$$f_{\text{steel}} = \frac{2 - (451.8 \times 12 \times 10^{-6} \times 300)}{451.8} \times 2 \times 10^5$$

$$= 165.35 \text{ N/mm}^2 \quad (\text{Tensile force})$$

$$f_{\text{copper}} = \frac{(451.8 \times 17.5 \times 10^{-6} \times 300 - 2)}{451.8} \times 10^5$$

$$= + 82.49 \text{ N/mm}^2. \quad (\text{compressive})$$

2.) Determine f_{thermal} of steel rod $l = 6\text{m}$ fixed @ the end:
 T rises from 20°C - 120°C

(i) No yielding of support

(ii) 1mm yielding of support.

Solution:

(i) No yielding of support.

$$\sigma_{\text{thermal}} = \alpha \Delta T E$$

$$= 12 \times 10^{-6} \times 2 \times 10^5 \times 100$$

$$= 240 \text{ N/mm}^2$$

(ii) Yielding of support by 1mm

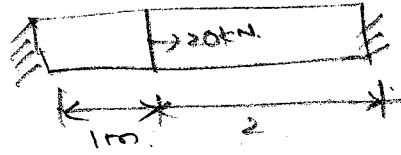
$$\sigma_{\text{thermal}} = \frac{\delta l}{l} \times E$$

$$= \frac{(\alpha \Delta T - 1) \times E}{l}$$

$$= \frac{[(6000 \times 12 \times 10^{-6} \times 100) - 1] \times 2 \times 10^5}{6000}$$

$$= 206.67 \text{ N/mm}^2.$$

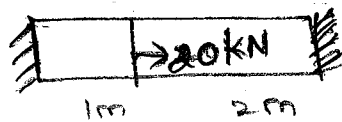
* stress will be more in non yielding
when compared to stress induced due
to yielding.



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3) Determine

support reactions



$$\sum H_A = 0$$

$$R_A + R_B = 20$$

$$\delta l_1 = \delta l_2 = 0 \quad (\text{Due to ends are fixed})$$

$$\delta l_1 = \delta l_2$$

$$\frac{R_A l_1}{AE} = \frac{R_B l_2}{AE}$$

$$R_A \times 1 = 2R_B$$

$$2R_B + R_B = 20$$

$$R_B = \frac{20}{3} = 6.67 \text{ kN}$$

$$R_A = 20 - 6.67$$

$$R_A = 13.33 \text{ kN}$$

4. A steel plate 10mm to be drilled to make a hole of 20mm ϕ ultimate shear stress 300 MPa. Determine Compressive stress required to make the hole.

solution:

$$\text{Load} = \text{Shear stress} \times \text{Area}$$

$$= 300 \times \pi \times \frac{20^2}{4} \times 10 \times 10^{-3}$$

$$\text{Load} = 94.25 \text{ kN}$$

$$\text{Load} = 188.495 \text{ kN}$$

$$\text{Compressive stress} = \frac{188.495 \times 10^3}{\pi \times \frac{20^2}{4} \times 10 \times 10^{-3}}$$

$$\text{Compressive stress} = 300 \text{ N/mm}^2$$

5. A metal bar of $l = 100 \text{ mm}$ is inserted b/w 2 rigid support and its temp $\Delta T = 10^\circ \text{C}$. If $\alpha = 12 \times 10^{-6} / ^\circ \text{C}$. $E = 2 \times 10^5 \text{ N/mm}^2$. What is the stresses in the bar.

$$\begin{aligned}
 f_{\text{thermal}} &= \alpha \Delta T E \\
 &= 12 \times 10^{-6} \times 10 \times 2 \times 10^5 \\
 &= 24 \text{ N/mm}^2
 \end{aligned}$$

6. A mild steel specimen is under uniaxial tensile stress $E = 2 \times 10^5 \text{ N/mm}^2$. $f_y = 250 \text{ N/mm}^2$. Max. amount of strain energy / unit volume that can be stored in the material is.

$$\begin{aligned}
 \frac{\text{Strain energy}}{\text{unit volume}} &= \frac{\sigma^2}{2E} = \frac{250^2}{2 \times 2 \times 10^5} \\
 &= 0.15625 \text{ Nmm/mm}^3
 \end{aligned}$$

7. 2 steel columns P and Q (length l , and $2l$), Fe 250 and 500 respectively have the same d 's and end condition the ratio of buckling load of the P to that column load Q is 4

$$0.5, 1, 2, \quad P_p = \frac{\pi^2 EI}{l_{eff}^2} = \frac{1}{l^2} = \underline{4}$$

$$P_Q = \frac{\pi^2 EI}{(2l)^2}$$

THERMAL STRESS:

α - co-efficient of thermal expansion

for eg: for steel $\alpha_{steel} = 12 \times 10^{-6} \text{ m/m/}^\circ\text{C}$

change in length

for 1mm in length and 1° rise in temperature

$$\delta l = \alpha$$

For 1mm length Δt rise in temperature

$$\delta l = \alpha \Delta t$$

For l mm length Δt rise in temperature

$$\delta l = l \alpha \Delta t$$

Thermal stress:

$$f_{thermal} = \frac{\delta l}{l} \times E = \frac{l \alpha \Delta t}{l} \times E$$

$$f_{thermal} = \alpha \Delta t E$$

Note:

when compound bars are used materials with higher α will try to expand until the free expansion limit, but the material with lower α will not allow the higher α material till free expansion.

It will pull back the higher α material. As a result, material with higher α subjected compression.

Material with lower α will expand more than its free expansion since higher α material will pull it

As a result material with higher lower α subjected to tension.

1. Brass rod 2.4m long is placed b/w 2 rigid walls, 2.43m apart. As shown in figure the temperature of rod is raised untill the rod is fixed b/w the wall and has a compressive stress 210 kg/cm^2 . The rod is restrained from bending. what is the rise in temperature.
 $E = 1.05 \times 10^6 \text{ kg/cm}^2$. $\alpha = 11.8 \times 10^{-6} / ^\circ\text{C}$.

Solution:

$$\Delta l = l \alpha \Delta T$$

$$0.03 = 2.4 \times 11.8 \times 10^{-6} \times \Delta T$$

$$\Delta T = 1059.32 \text{ } ^\circ\text{C}$$

$$\text{Compressive stress} = \frac{\Delta l}{l} E$$

$$21 = \frac{\Delta l}{l} \alpha E \times \Delta T$$

$$\Delta l = 21 \times 2.4 = 50.4 \text{ cm}$$

$$E = \frac{21}{1.05 \times 10^{-5}}$$

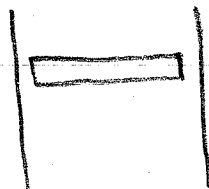
$$E = 2 \times 10^4 \text{ m}$$

$$E = \alpha \Delta T$$

$$\Delta T = \frac{2 \times 10^{-4}}{11.8 \times 10^{-6}} = 16.949 \text{ } ^\circ\text{C}$$

$$\text{Total change in temp} = 1059.32 + 16.949$$

$$= 1076.269 \text{ } ^\circ\text{C}$$



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2. A rod is composed of 3 segments as shown in figure. The rod is held b/w rigid supports. find the stress developed in each material when the temp $\Delta T = 50^\circ\text{C}$. under following 2 different condition
- when the supports are perfectly rigid.
 - when the right hand support yield by 0.2 mm

$$E_s = 2 \times 10^6 \text{ kg/cm}^2$$

$$E_{al} = 0.7 \times 10^6 \text{ kg/cm}^2$$

$$\alpha_c = 1.8 \times 10^{-5} / ^\circ\text{C}$$

$$E_c = 1 \times 10^6 \text{ kg/cm}^2$$

$$\alpha_{\text{steel}} = 1.2 \times 10^{-5} / ^\circ\text{C}$$

$$\alpha_{\text{alum}} = 2.4 \times 10^{-5} / ^\circ\text{C}$$

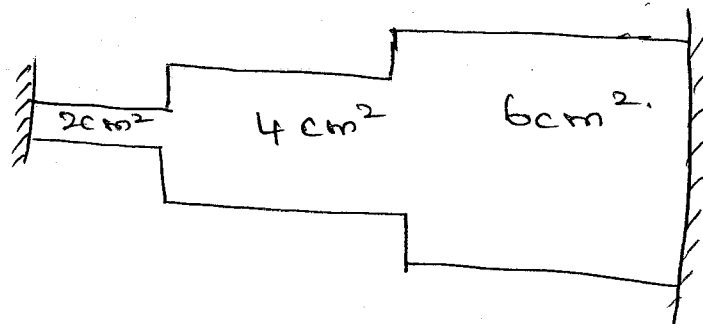
$$\text{a) stress} = f_1 + f_2 + f_3$$

$$= \alpha_1 \Delta T E_1 + \alpha_2 E_2 \Delta T + \alpha_3 E_3 \Delta T$$

$$= \left[\left(1.2 \times 10^{-5} \times 2 \times 10^6 \right) + \left(1.8 \times 10^{-5} \times 1 \times 10^6 \right) + \left(2.4 \times 10^{-5} \times 0.7 \times 10^6 \right) \right] \times 50$$

$$= (2.4 + 1.8 + 1.68) \times 50$$

$$= 294 \text{ N/mm}^2$$



$$\delta l = \delta l_1 + \delta l_2 + \delta l_3$$

$$= \left[(0.15 \times 1.2 \times 10^{-5} \times 2 \times 10^5) + (0.2 \times 1.8 \times 10^{-5} \times 1 \times 10^5) \right. \\ \left. + (0.15 \times 2.4 \times 10^{-5} \times 0.7 \times 10^5) \right] \times 50$$

$$\boxed{\delta l = 0.045}$$

$$f_{st} = f_{cu} = F_{A1}$$

$$\frac{f_{st}}{2} = \frac{f_c}{4} = \frac{f_{A1}}{6}$$

$$\boxed{f_{st} = \frac{f_c}{2}}$$

$$\boxed{f_{A1} = \frac{3}{2} f_c}$$

$$\delta l = \frac{f_s l_s}{E_s} + \frac{f_c l_c}{E_c} + \frac{f_{A1} l_{A1}}{E_{A1}}$$

$$0.045 = \frac{f_c \times 0.15}{2 \times 2 \times 10^5} + \frac{f_c \times 0.2}{1 \times 10^5} + \frac{3 f_c}{0.7 \times 10^5 \times 2}$$

$$= f_c \left\{ \right.$$

$$f_c = 913.1 \text{ kg/cm}^2$$

$$f_{st} = \frac{913.1}{2} \quad f_{st} = 456.55 \text{ kg/cm}^2$$

$$f_{A1} = 608.7 \text{ kg/cm}^2$$

$$\delta l - 2 = \frac{f_s}{E_s} \times l_s + \frac{f_c \times l_c}{E_c} + \frac{f_{AL} + l_{AL}}{E_{AL}}$$

$$f_s = 1014.4 \text{ kg/cm}^2$$

$$f_c = 507.2 \text{ kg/cm}^2$$

$$f_{AL} = 338.1 \text{ kg/cm}^2$$

3. A surveyor steel tape nominally 30m. Is 1.25 wide and 1mm thick. Its length is correct when used @ a temperature of 16°C. and under a pull of 10kg. By how much it will be in error when used at a temperature 50°C under a pull of 5kg. $E = 2 \times 10^6 \text{ kg/cm}^2$

$$\alpha = 11 \times 10^{-6} / ^\circ\text{C}$$

$$\delta l = l \alpha \Delta T$$

$$= 30 \times 11 \times 10^{-6} \times 34$$

$$\delta l = 0.0112 \text{ m}$$

$$= \underline{\underline{1.12 \text{ cm}}}$$

30m - Wd 1.25mm
 t = 1mm
 @ 16°C
 P = 10kg
 E = 2 x 10⁶
 5 / (1.25 x 0.1) = 4E

Load / Area = $\sigma \Rightarrow \frac{P}{A} = \epsilon E$ Strain x Modulus

$$\frac{\delta l}{l} = \frac{P}{A E}$$

$$\delta l = \frac{P \times l}{A \times E}$$

$$= \frac{5 \times 30 \times 100 \text{ cm}}{1.25 \times 0.1 \times 2 \times 10^6 \text{ kg/cm}^2}$$

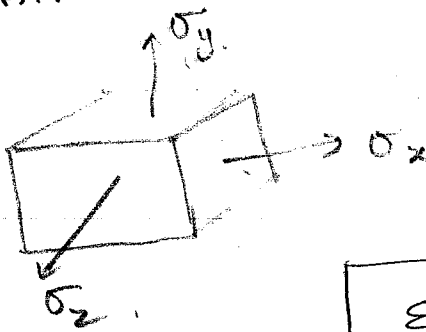
$$= \underline{\underline{0.06 \text{ cm}}}$$

Error in length = 1.06 cm = (1.12 - 0.06)

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ELASTIC

CONSTANT :



$$\epsilon_v = \epsilon_x + \epsilon_y + \epsilon_z$$

$$= \frac{\sigma_x}{E} + \frac{\sigma_y}{E} + \frac{\sigma_z}{E} + 2\mu \left[\frac{\sigma_x + \sigma_y + \sigma_z}{E} \right]$$

$$\epsilon_v = \frac{\sigma_x + \sigma_y + \sigma_z}{E} (1 - 2\mu)$$

$$E = \frac{\sigma_x + \sigma_y + \sigma_z}{\epsilon_v} (1 - 2\mu)$$

$$\text{If } \sigma_x = \sigma_y = \sigma_z = \sigma$$

$$E = \frac{3\sigma}{\epsilon_v} (1 - 2\mu)$$

$$E = 3K (1 - 2\mu)$$

$$E = 2G (1 + \mu)$$

$$E = 3K (1 - 2\mu) \Rightarrow \mu = - \left(\frac{E}{6K} - \frac{1}{2} \right)$$

$$E = 2G \left(1 - \frac{E}{6K} + \frac{1}{2} \right)$$

$$E = \frac{2G}{2} \left(\frac{3}{2} + \frac{E}{6K} \right)$$

$$= 2G \left(\frac{3K + E}{6K} \right)$$

$$6KE = 18KG + 2GE \Rightarrow$$

$$E = \frac{6KG}{3K + G}$$

$$\epsilon_x = \frac{\sigma_x}{E} - \mu \left[\frac{\sigma_y}{E} + \frac{\sigma_z}{E} \right]$$

$$\epsilon_y = \frac{\sigma_y}{E} - \frac{\mu \sigma_x}{E} - \frac{\mu \sigma_z}{E}$$

$$\epsilon_z = \frac{\sigma_z}{E} - \frac{\mu \sigma_x}{E} - \frac{\mu \sigma_y}{E}$$

$$\frac{\sigma}{\epsilon_v} = K$$

EI - flexural rigidity.

$$\frac{N}{mm^2} \times mm^4 = Nm^2$$

GA - shear rigidity.

$$\frac{kg/cm^2}{cm} \times m^2$$

GJ - Torsional rigidity.

$$\frac{kg/cm^2}{cm} \times m^2$$

AE - Axial rigidity.

$\frac{EI}{L}$ - flexural stiffness.

$\frac{GA}{L}$ - shear stiffness.

$\frac{GJ}{L}$ - torsional stiffness.

$\frac{AE}{L}$ - Axial stiffness.

Effective due to impact Load = 2x static load

Shear Force Diagram & Bending Moment Diagram

* Beam is a structural member subjected to load \perp to the axis of the member. and it transmits the load to the support through bending only.

* Beam is a bending member which is designed on the basis of max. BM and max. S.F.

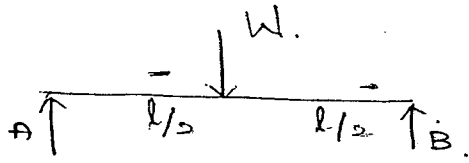
* The algebraic sum of all moments considered from extreme end to any section of beam is called B.M @ that section.

* The graphical representation of B.M along with its nature is called B.M diagram. which is very essential to find out the position of steel bars in R.C sections.

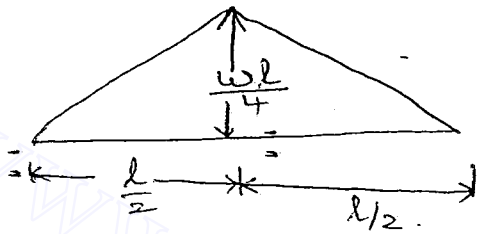
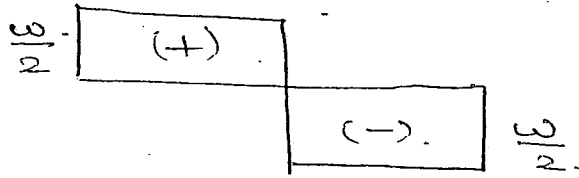
* The algebraic sum of all forces considered from extreme end to any section of beam is called S.F at that section.

Standard Cases :

1.)

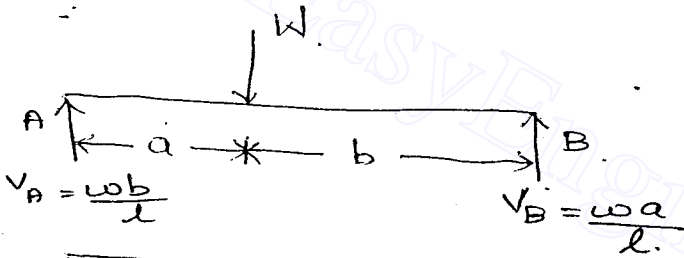


$$V_A = V_B = \frac{W}{2}$$



$$\frac{Wl}{4}$$

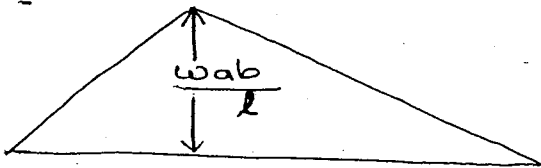
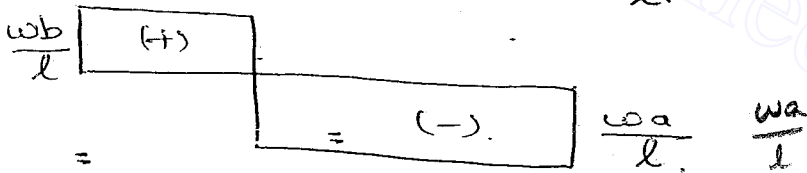
2.)



$$V_A = \frac{Wb}{l}$$

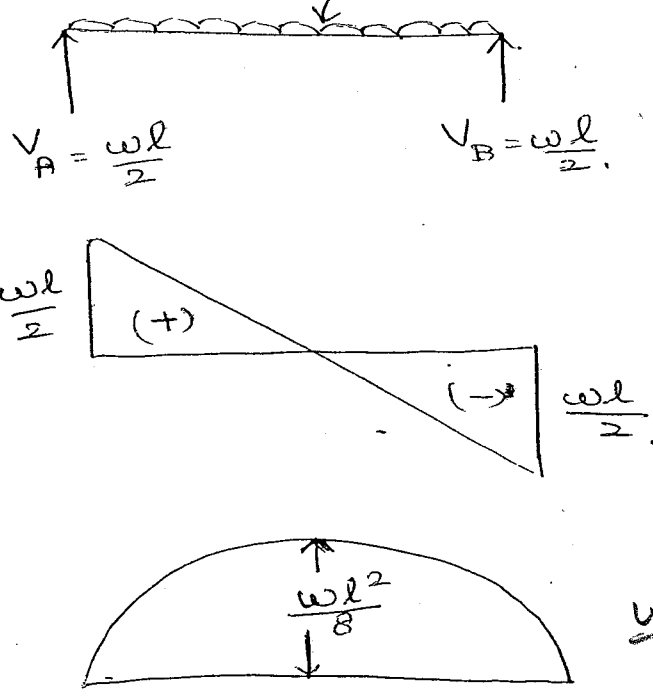
$$V_B = \frac{Wa}{l}$$

$$\frac{Wb}{l}$$



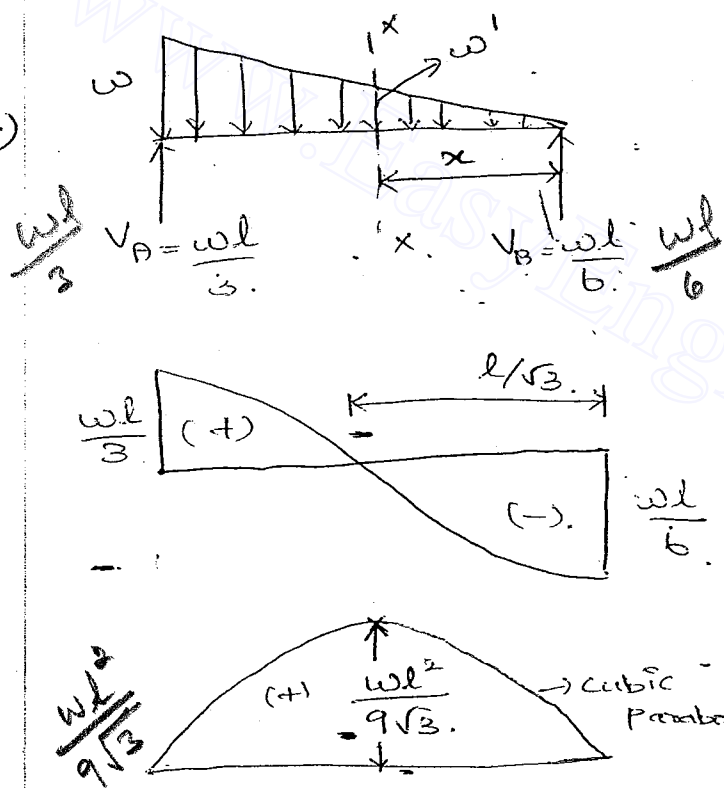
$$\frac{Wab}{l}$$

3.)



Load intensity
= Area of loading Diagram -
 $W = w \times l$

4.)



$M_A = 0$

$R_B \times l = \frac{1}{2} \times w \times l \times \frac{l}{3}$

$R_B = \frac{wl}{6}$

$R_A = \frac{wl}{2} - \frac{wl}{6}$

$R_A = \frac{2wl}{6} = \frac{wl}{3}$

$\Sigma F = 0$

$\Sigma M = 0$

$R_B - \frac{1}{2} \times w' \times x = 0$

By similar triangles property

$\frac{w'}{w} = \frac{x}{l}$

$w' = \frac{wx}{l}$

B.M @ $x = \frac{l}{\sqrt{3}}$

$= R_B \times \frac{l}{\sqrt{3}} - \frac{1}{2} \times w' \times \frac{l}{\sqrt{3}} \times \left(\frac{l}{\sqrt{3}}\right) \times \frac{1}{3}$

$= \left(\frac{wl}{6} \times \frac{l}{\sqrt{3}}\right) - \frac{1}{2} \times \frac{w \times l}{l \sqrt{3}} \times \frac{l}{\sqrt{3}} \times \frac{l}{3\sqrt{3}} \times \frac{2wl^2}{6} = wx^2$

$= \frac{wl^2}{6\sqrt{3}} - \frac{wl^2}{18\sqrt{3}}$

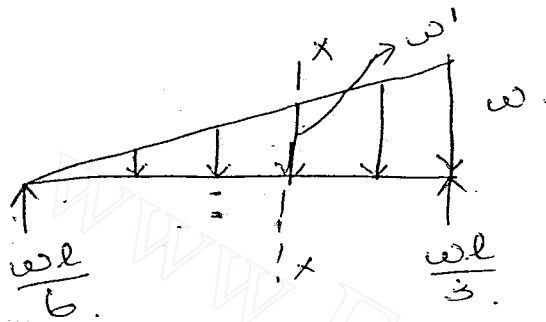
$\frac{wl}{6} = \frac{1}{2} \times \frac{wx}{l} \times x$

$x = \frac{l}{\sqrt{3}}$ S.F = 0

$$= \frac{1}{\sqrt{3}} \left(\frac{\omega l^2}{6} - \frac{\omega l^2}{18\sqrt{3}} \right)$$

$$= \frac{1}{\sqrt{3}} \left(\frac{2\omega l^2}{18} \right)$$

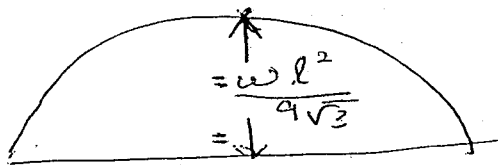
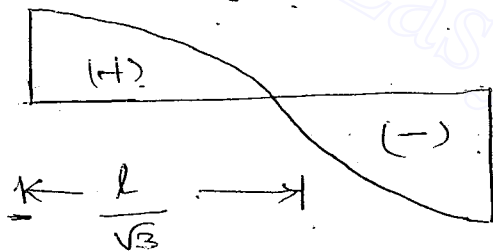
B.M @ $x = \frac{l}{\sqrt{3}}$ } = $\frac{\omega l^2}{9\sqrt{3}}$



$$V_{xx} = 0$$

$$R_A = \frac{1}{2} \times \omega' \times x = 0$$

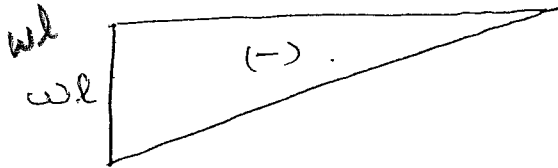
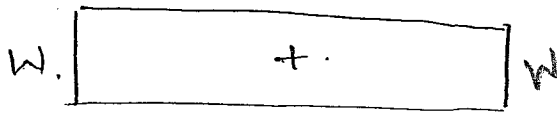
$$x = \frac{l}{\sqrt{3}}$$



5.)



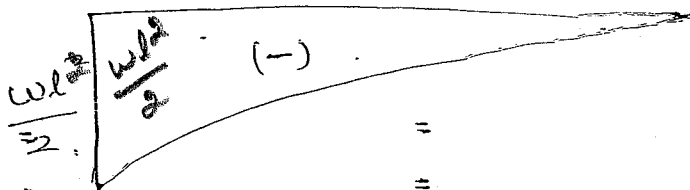
$R_A = W$



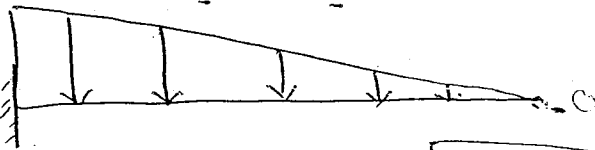
6.)



$R_A = wl$

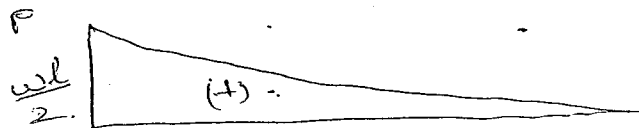


7.)

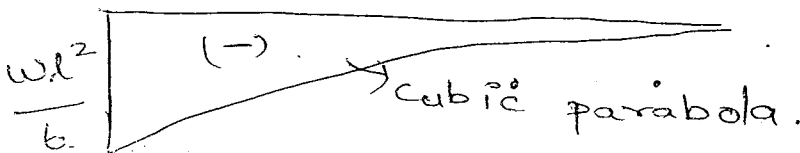


$R_A = \frac{1}{2} \times w \times l$

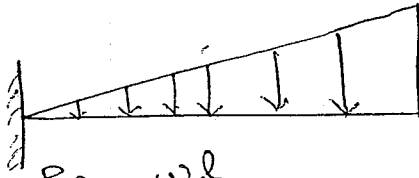
$R_A = \frac{wl}{2}$



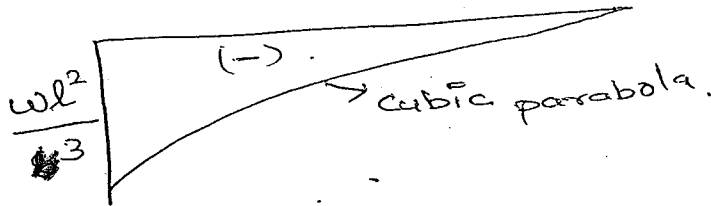
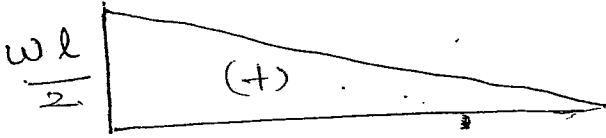
$B.M = \frac{wl}{2} \times \frac{l}{3}$
 $= \frac{wl^2}{6}$



8.)



$$R_A = \frac{wl}{2}$$



B.M @ support

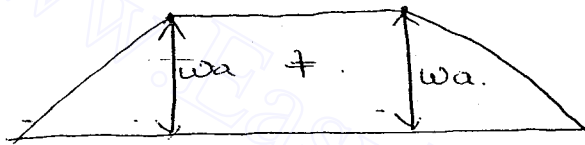
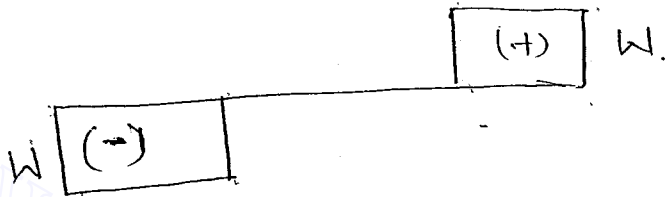
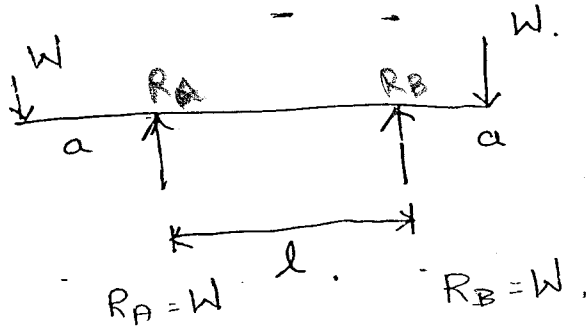
$$= \frac{w \times l}{2} \times \frac{l}{3}$$

$$= \frac{wl^2}{3}$$

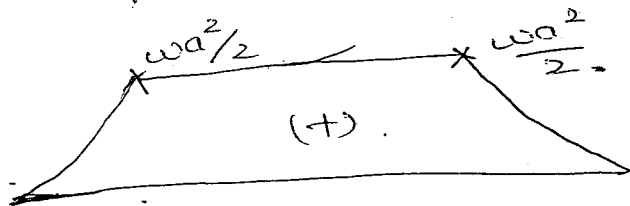
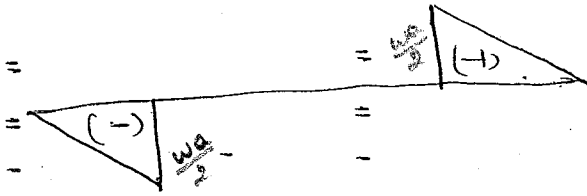
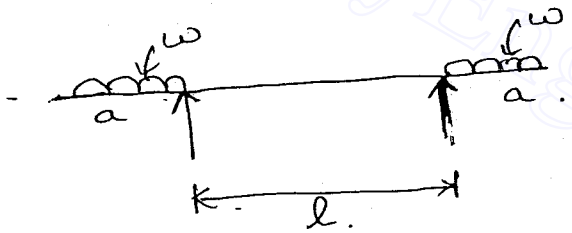
www.EasyEngineering.net

SFD and BMD.

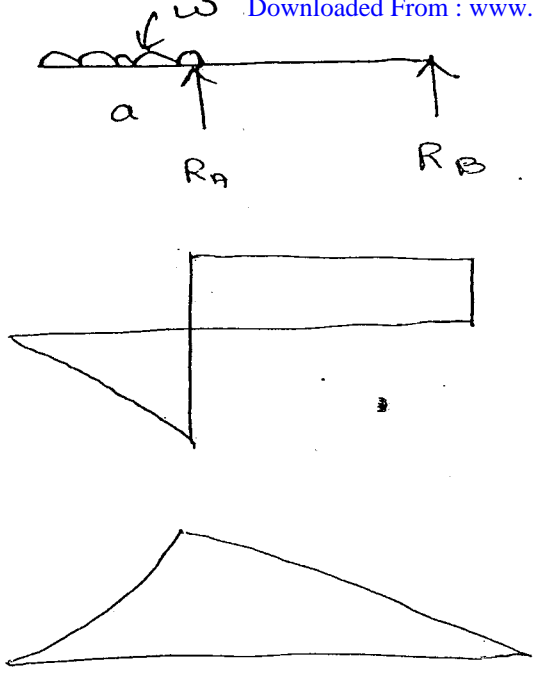
1.)



2.)



3.)



$$R_A + R_B = wa$$

$$\sum M_A = 0$$

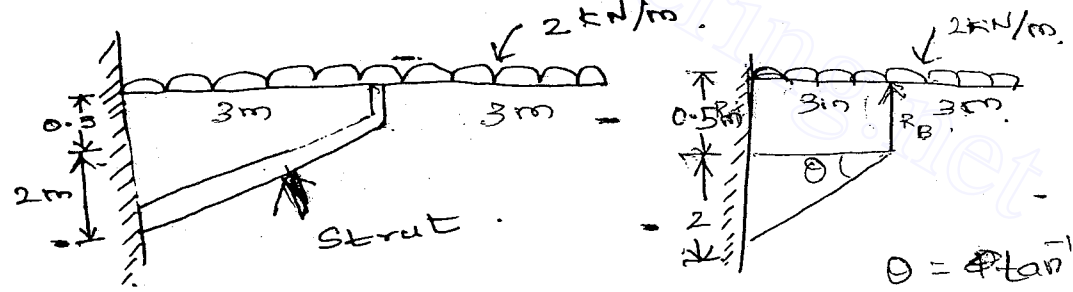
$$R_B l = -\frac{wa^2}{2}$$

$$R_B = -\frac{wa^2}{2l}$$

$$R_A = wa + \frac{wa^2}{2l}$$

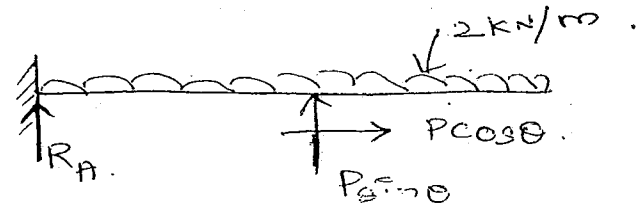
4.)

A beam is supported by a strut. Determine force in the strut, thrust in the beam, Max. B.M in the beam, Max. S.F in the beam and S.F.D and B.M.D.



$$\theta = \tan^{-1} \left(\frac{2}{3} \right)$$

$$\theta = 33.69^\circ$$



$$R_A + P \cos \theta = 2 \times 6$$

$$\sum M_A = 0$$

$$(P \sin \theta) \times 3 + (P \cos \theta \times 0.5) - \frac{2 \times 6^2}{2} = 0$$

$$3P \sin 33.69 + 0.5P \cos 33.69 = 36$$

$$P = 17.31 \text{ kN.}$$

$$R_A = 12 - 17.31 \sin 33.69$$

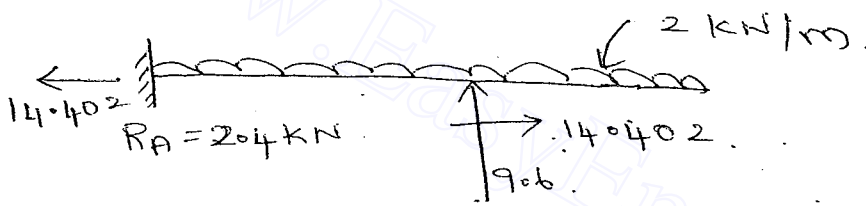
$$R_A = 2.4 \text{ kN.}$$

$$P \sin \theta = 17.31 \sin 33.69$$

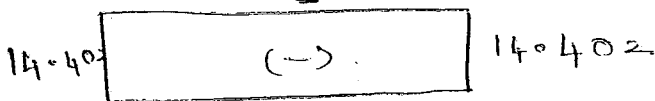
$$= 9.6$$

$$P \cos \theta = 17.31 \cos 33.69$$

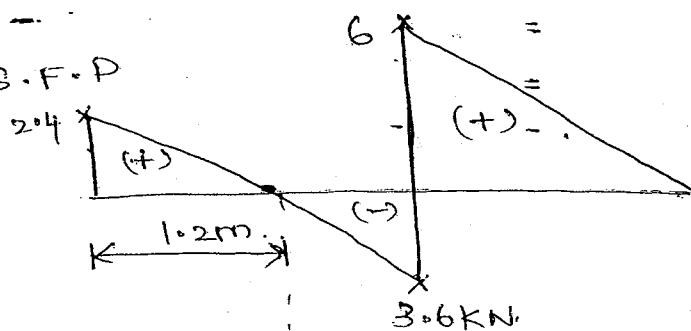
$$= 14.402 \text{ kN.}$$



Thrust -



S.F.P



$$V_{xx} = 0$$

$$R_A - 2x = 0$$

$$2.04 = 2x$$

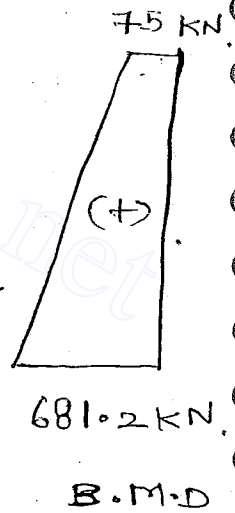
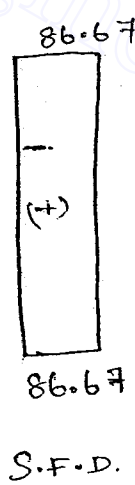
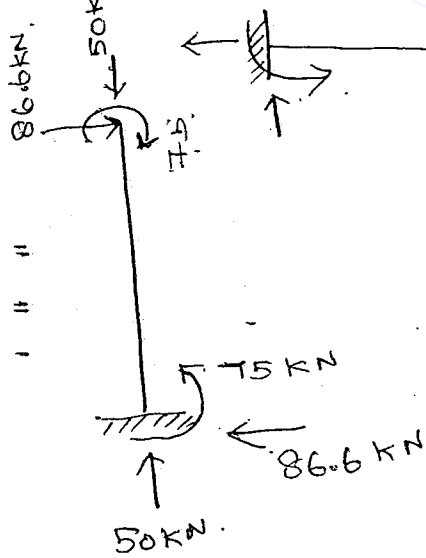
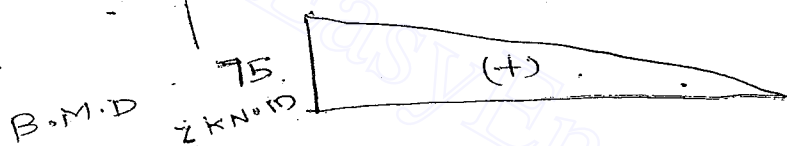
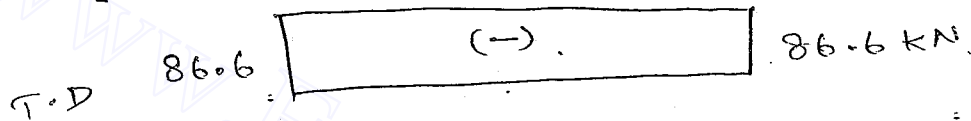
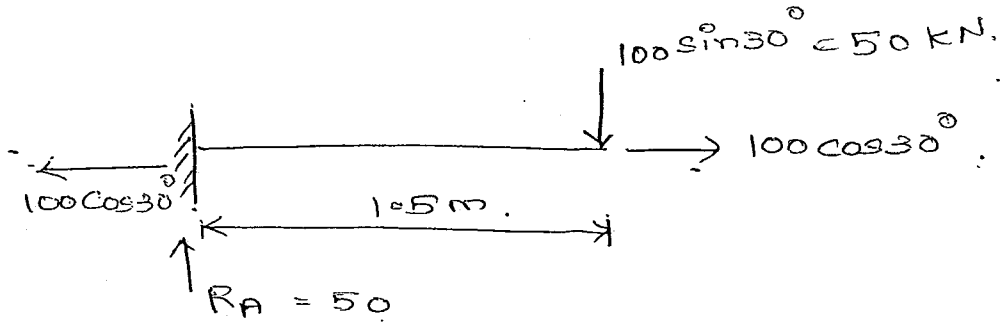
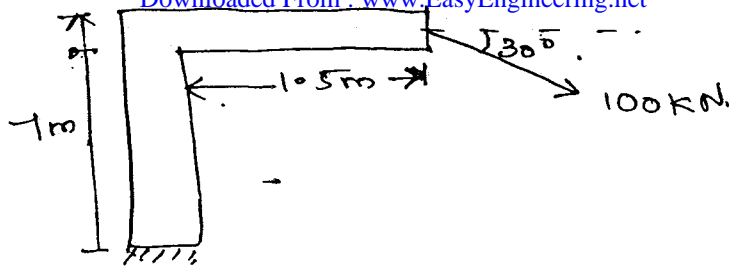
$$x = 1.02 \text{ m.}$$

B.M @ $x = 1.02$.

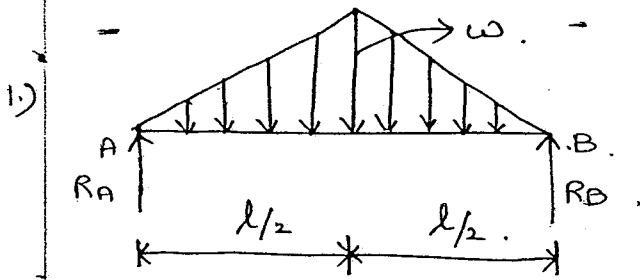
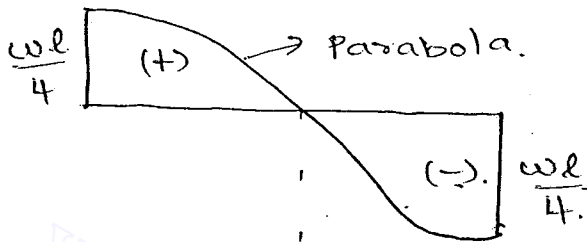
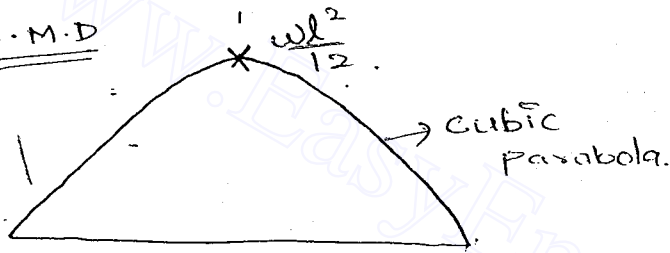
$$2.04 \times 1.02 - \frac{2 \times 1.02^2}{2}$$

$$= 10.44 \text{ kNm.}$$

B.)



S.F.D and B.M.D.

S.F.DB.M.D

$$R_A + R_B = \frac{1}{2} \times w \times l.$$

$$\sum M_A = 0.$$

$$R_B \times l - \frac{1}{2} \times w \times l \times \frac{l}{2} = 0.$$

$$R_B l = \frac{1}{2} \times w \times l \times \frac{l}{2}$$

$$R_B = \frac{wl}{4}$$

$$R_A = \frac{wl}{4}$$

Taking moment about centre.

$$= R_A \times \frac{l}{2} - \left(\frac{1}{2} \times w \times \frac{l}{2} \times \frac{l}{2} \right)$$

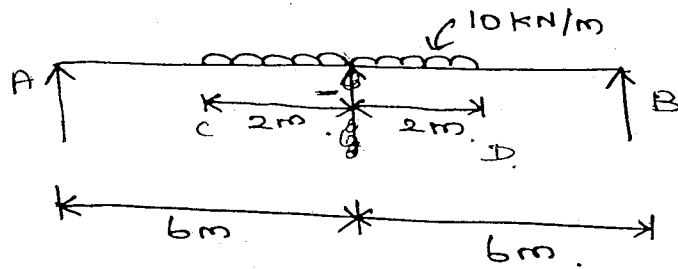
$$= \frac{wl}{4} \times \frac{l}{2} - \left[\frac{wl^2}{24} \right]$$

$$= \frac{wl^2}{8} - \frac{wl^2}{24}$$

$$= \frac{2wl^2}{24}$$

$$M_{\max} = \frac{wl^2}{12}$$

2) Draw S.F.D and B.M.D for the beam.



$$M_A = 0$$

$$R_B \times 12 - 10 \times 4 \times (2+4) = 0$$

$$R_B = \frac{240}{12} = 20 \text{ kN}$$

$$R_A = 40 - 20$$

$$\boxed{R_A = 20 \text{ kN}}$$

$$M_C = R_A \times 4$$

$$= 20 \times 4$$

$$= 80 \text{ kN}\cdot\text{m}$$

$$M_D = R_B \times 4$$

$$= 20 \times 4$$

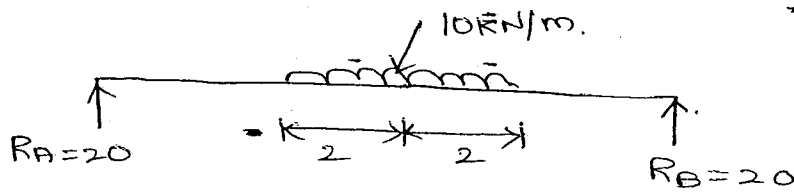
$$= 80 \text{ kN}\cdot\text{m}$$

$$M_{\max} = R_A \times 6 - 10 \times 2 \times 1$$

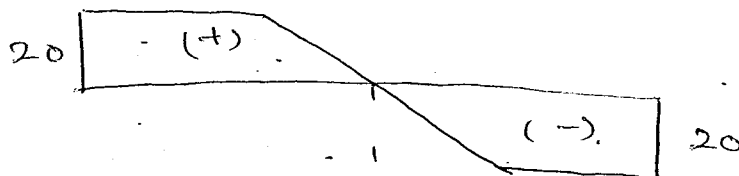
$$= (20 \times 6) - 20$$

$$= 120 - 20$$

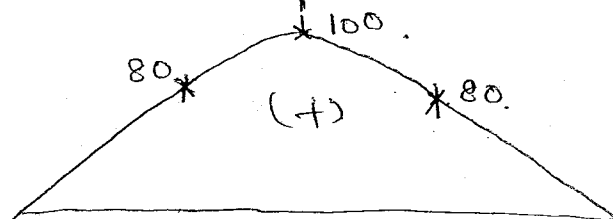
$$= 100 \text{ kN}\cdot\text{m}$$



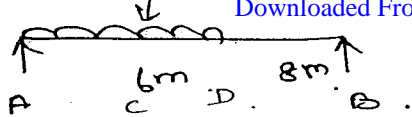
S.F.D



B.M.D



3.)



$$\sum M_A = 0$$

$$R_B \times 14 = \frac{20 \times 6^2}{2}$$

$$R_B = 25.71 \text{ kN}$$

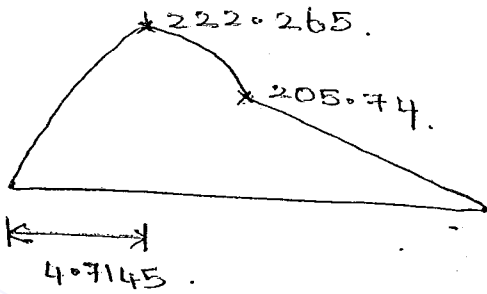
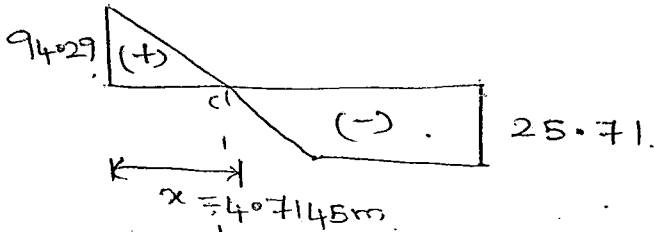
$$R_A = 94.29 \text{ kN}$$

$$V_{xx} = 0$$

$$R_A - 20x = 0$$

$$94.29 = 20x$$

$$x = 4.7145 \text{ m}$$



$$M_{max} = R_A x - \left(\frac{20x^2}{2} \right)$$

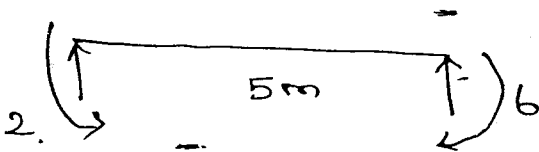
$$= 222.265 \text{ kNm}$$

$$M_D = (94.29 \times 6)$$

$$- \frac{20 \times 6^2}{2}$$

$$= 205.74 \text{ kNm}$$

4.)



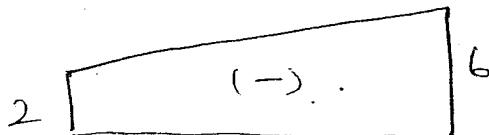
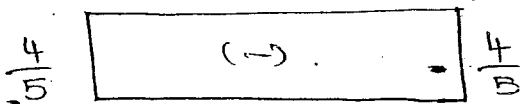
$$R_A + R_B = 0$$

$$\sum M_A = 0$$

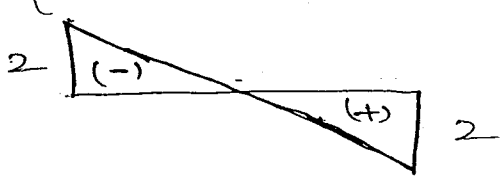
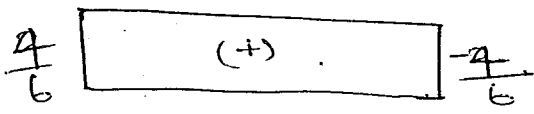
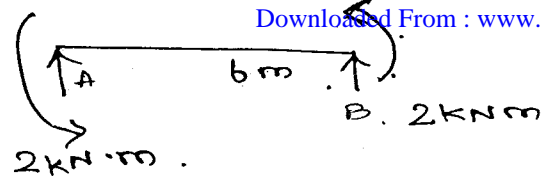
$$R_B \times 5 - 6 \times 2 = 0$$

$$R_B = \frac{4}{5}$$

$$R_A = -\frac{4}{5}$$



5.)



$$R_A + R_B = 0$$

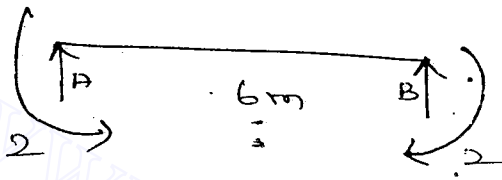
$$\sum M_A = 0$$

$$R_B \times 6 + 2 + 2 = 0$$

$$R_B = -\frac{4}{6}$$

$$R_A = \frac{4}{6}$$

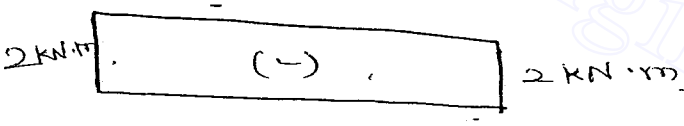
6.)



S.F.D.



B.M.D.



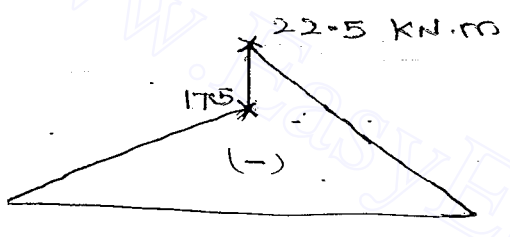
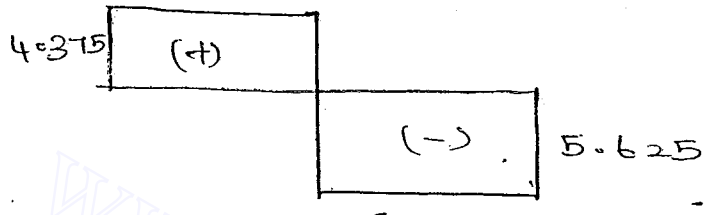
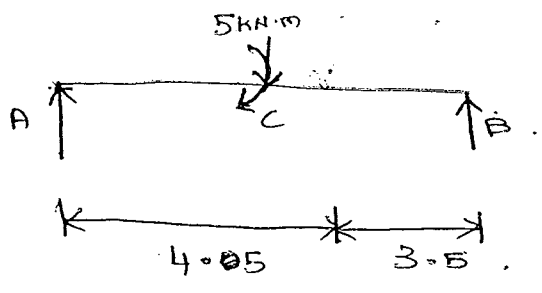
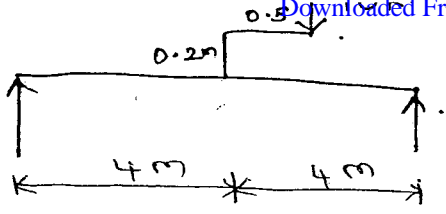
$$\sum M_A = 0$$

$$R_B \times 6 + 2 + 2 = 0$$

$$R_B = 0$$

$$R_A = 0$$

7.)



$$\sum M_A = 0$$

$$R_B \times 8 = (10 \times 4) + 5$$

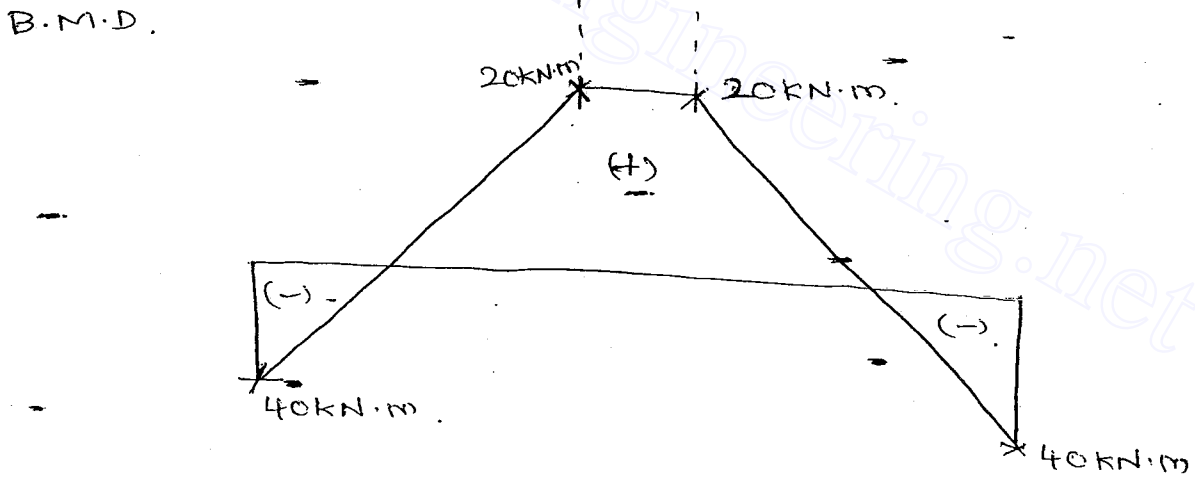
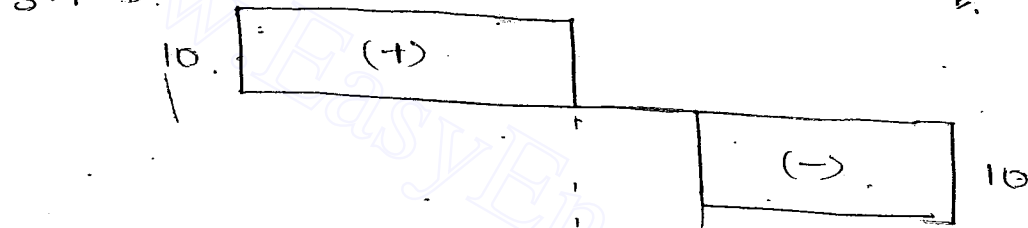
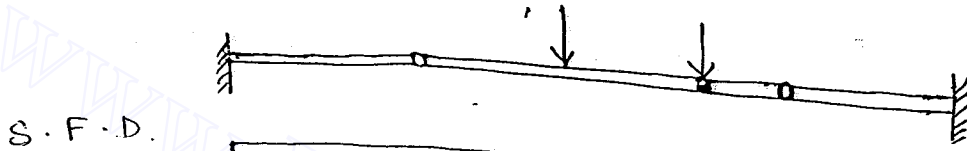
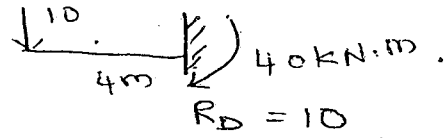
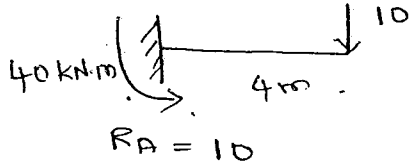
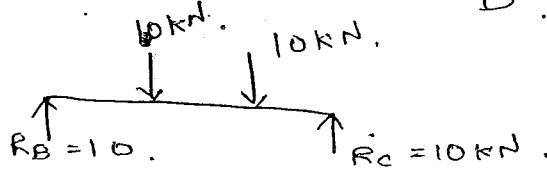
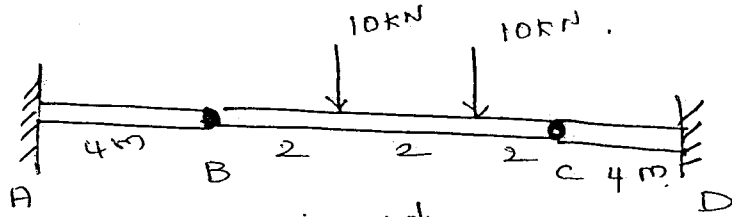
$$R_B = \frac{45}{8}$$

$$R_B = 5.625 \text{ KN}$$

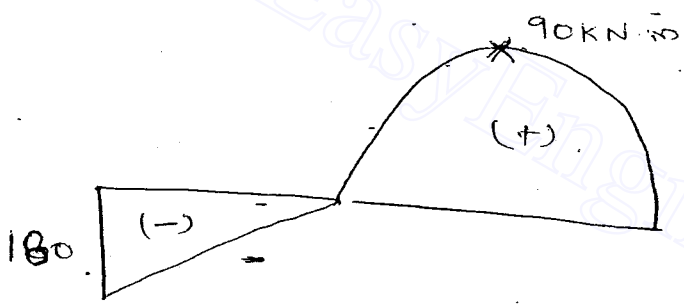
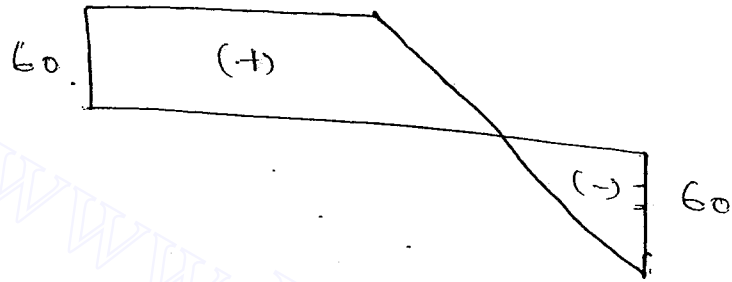
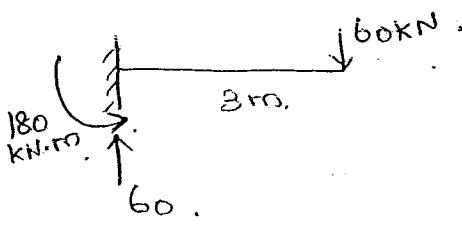
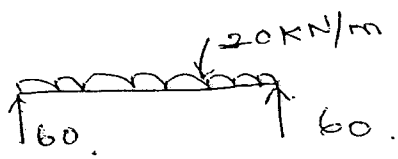
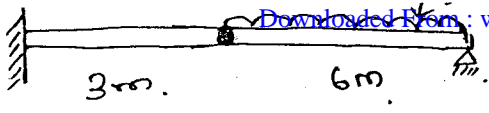
$$R_A = 10 - 5.625 = 4.375 \text{ KN}$$

$$M_C = R_A \times 4 + 5 = (4.375 \times 4) + 5 = 22.5 \text{ KN}\cdot\text{m}$$

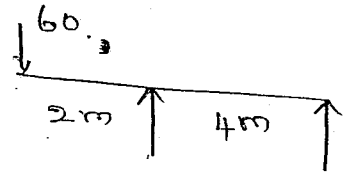
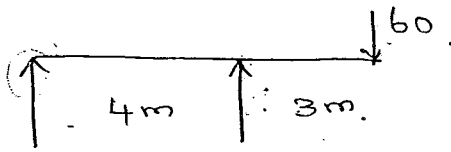
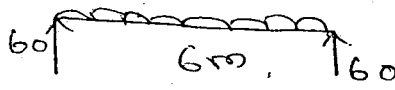
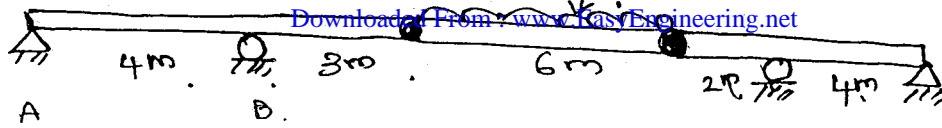
8.) Determine Max S.F and B.M for the compound beam given below.



9.)



10)



R_A R_B

R_C R_D

$\sum M_B = 0$

$\sum M_D = 0$

$R_A \times 4 = 60 \times 3$

$R_C \times 4 = 60 \times 6$

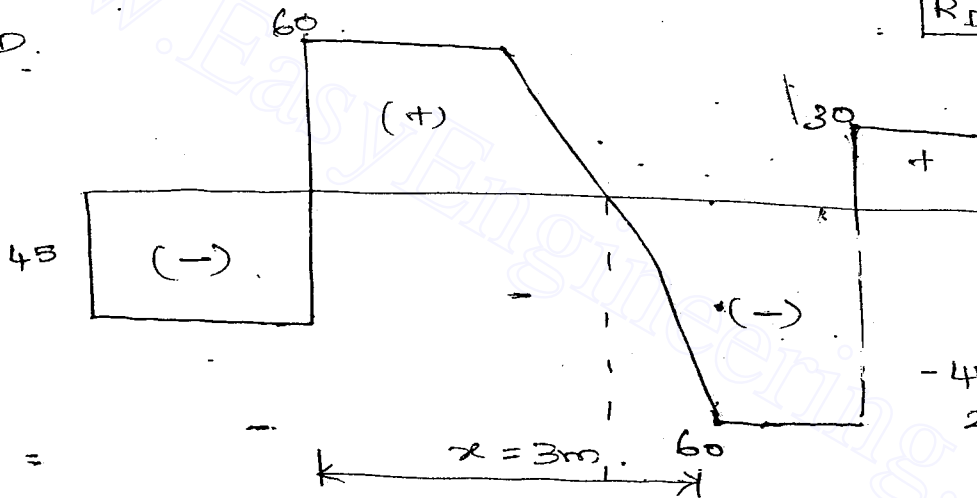
$R_A = 45 \text{ kN}$

$R_C = 90 \text{ kN}$

$R_B = 105 \text{ kN}$

$R_D = 30 \text{ kN}$

S.F.D.



$V_{xx} = 0$

$-45 + 105 - 20x = 0$

$20x = 60$

$x = 3 \text{ m}$

M@3m

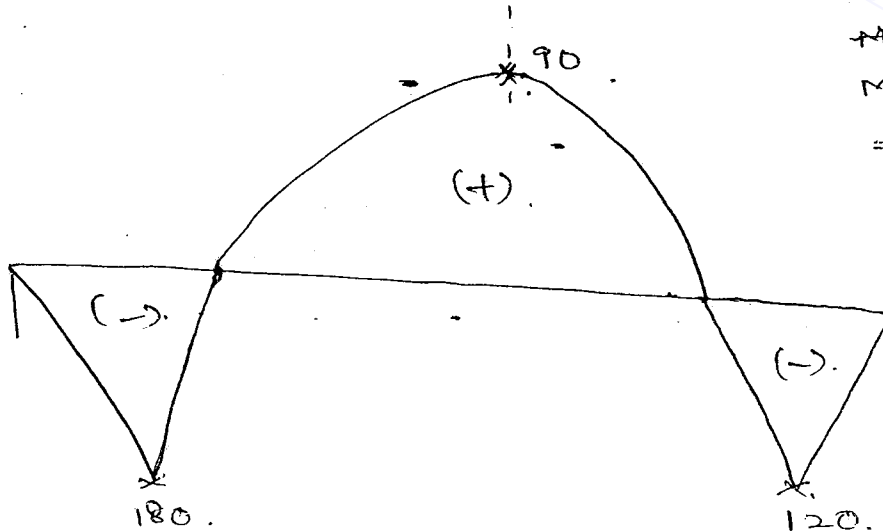
M_{max}

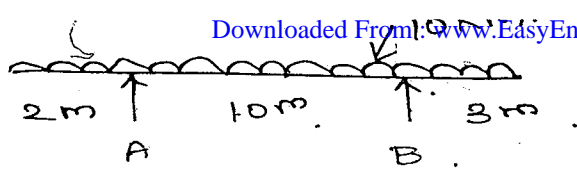
$= (-45 \times 10)$

$+ (105 \times 6)$

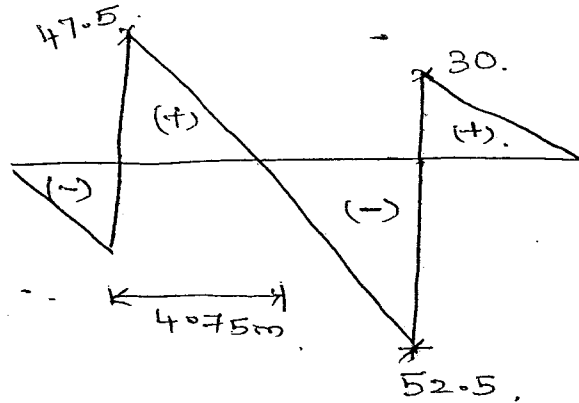
$- (20 \times \frac{3^2}{2})$

$= 90 \text{ kNm}$





$$R_A + R_B = 150$$



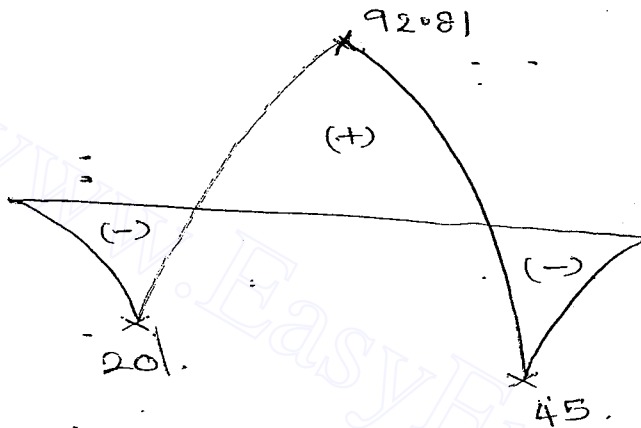
$$\sum M_A = 0$$

$$R_B \times 10 - \left(\frac{10 \times 13^2}{2} \right) + \left(\frac{10 \times 2^2}{2} \right) = 0$$

$$10 R_B = 825$$

$$R_B = 82.5 \text{ kN.}$$

$$R_A = 67.5 \text{ kN.}$$



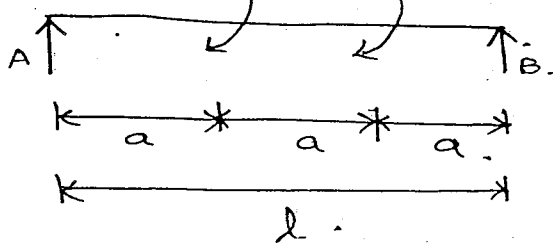
$$V_{xx} = 0$$

$$R_A - 10 \times (2 + x) = 0$$

$$67.5 - 20 = 10x$$

$$x = 4.75 \text{ m}$$

12.)

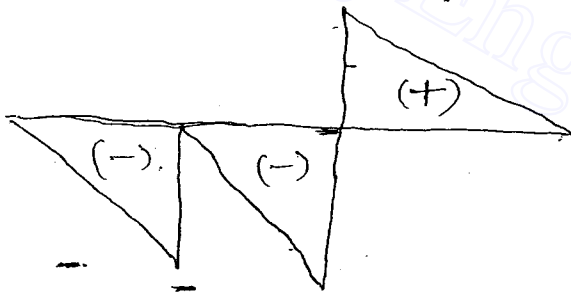
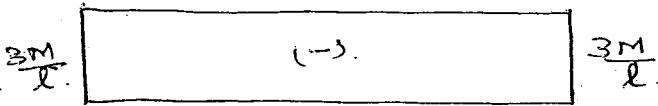


$$\sum M_A = 0.$$

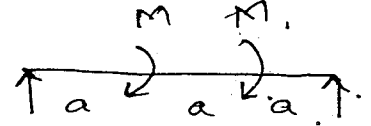
$$R_B \cdot 3a = 2M + M.$$

$$R_B = \frac{3M}{l}.$$

$$R_A = -\frac{3M}{l}.$$



(13.)

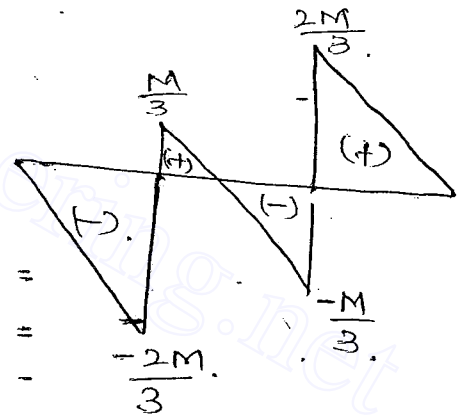
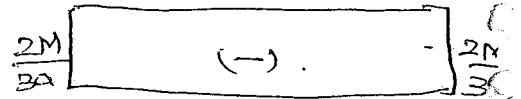


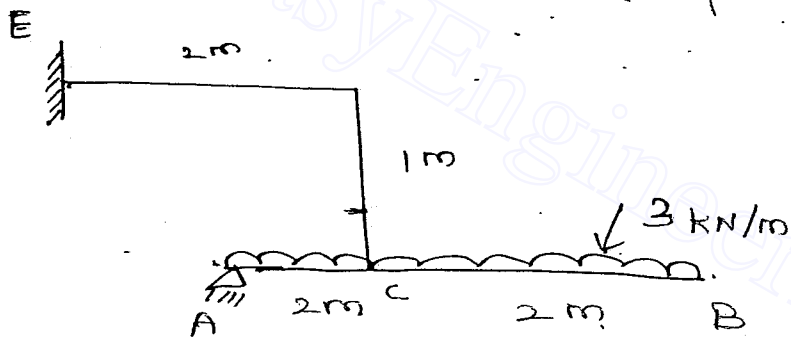
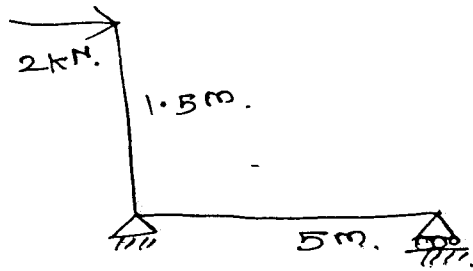
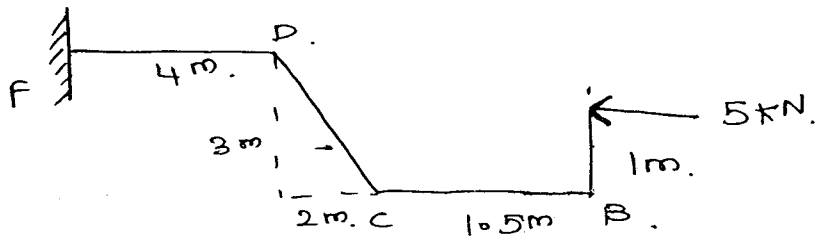
$$\sum M_A = 0.$$

$$3R_B a = 2M.$$

$$R_B = \frac{2M}{3a}.$$

$$R_A = -\frac{2M}{3a}.$$





28/10/2015

COMPLEX STRESSES

* To ensure the safety of the structural component, we not only have to ensure that the structural component is under equilibrium due to external forces, but also each and every point inside the volume of the structural component must be in equilibrium and must have stresses less than the maximum permissible stress.

Thus we need to know on which plane maximum normal stress will act, on which plane maximum shear stress will act, what is the magnitude of normal and shear stress.

As the magnitude of normal and shear stress varies with inclination of plane, stress on a plane can be calculated from stress on other plane using method of transformation of stress.

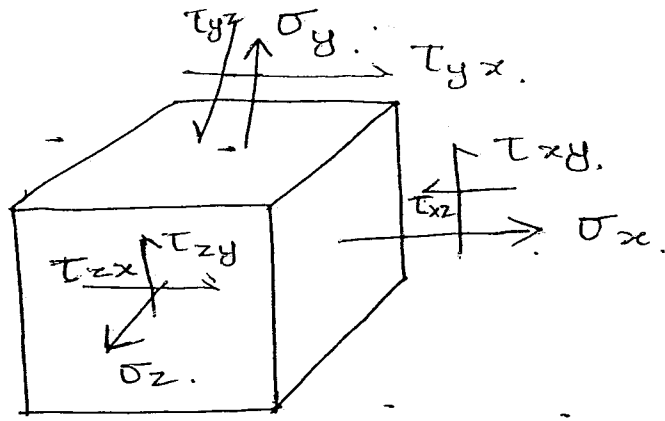
At any point most general state of stress is represented by 6 components

$$\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}, \tau_{zx}$$

But in particular case not all of these 6 stresses act simultaneously.

Here we will consider only 3 stress components ($\sigma_x, \sigma_y, \tau_{xy}$).

PLANE STRESS:



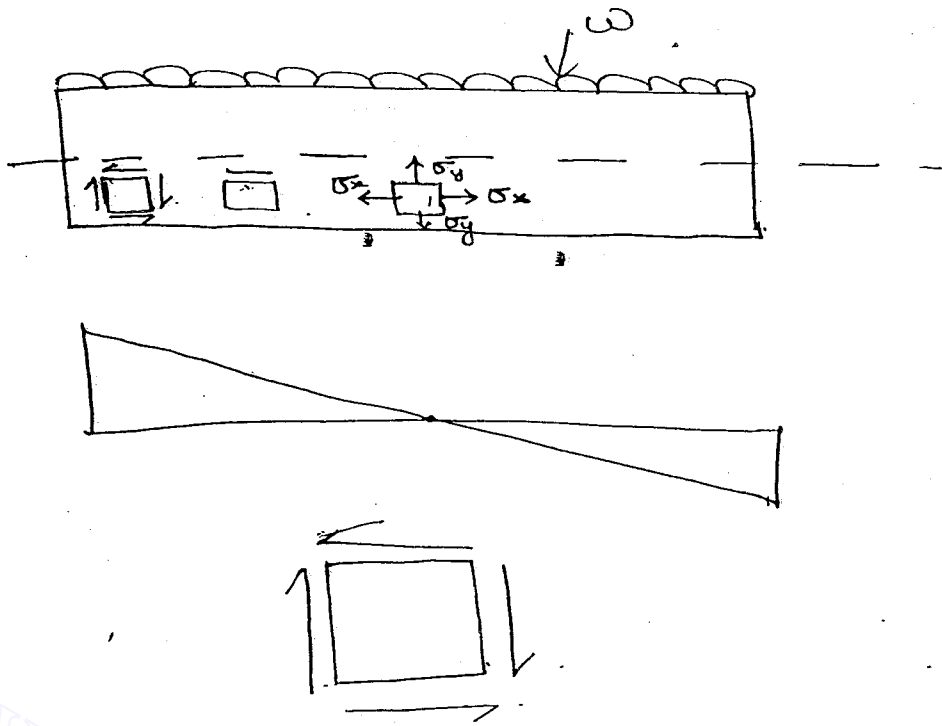
when, two faces of cubic elements are free from any stress condition is called plane stress condition.;

For eg: if z-axis is chosen \perp to the face on which no stress is acting then. (i.e)

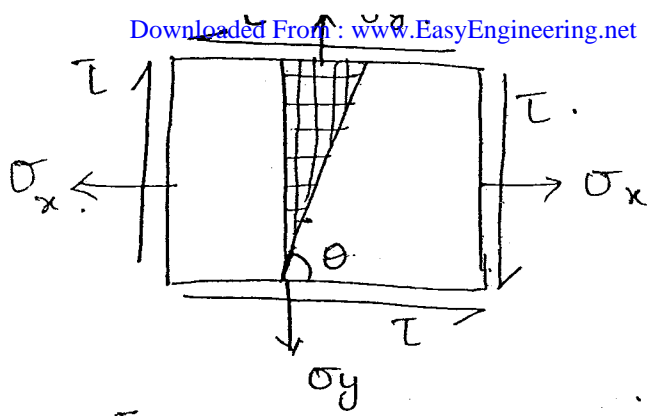
$$\sigma_{zx} = \tau_{yz} = \tau_{zx} = 0$$

remaining component $\sigma_x, \sigma_y, \tau_{xy}$.
Examples of plane stress condition are

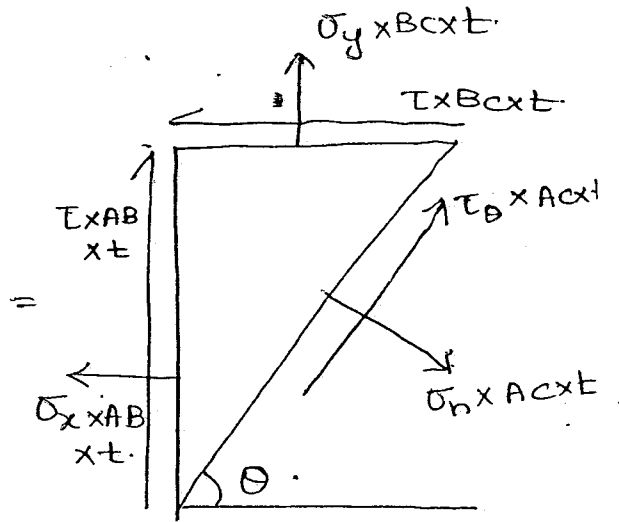
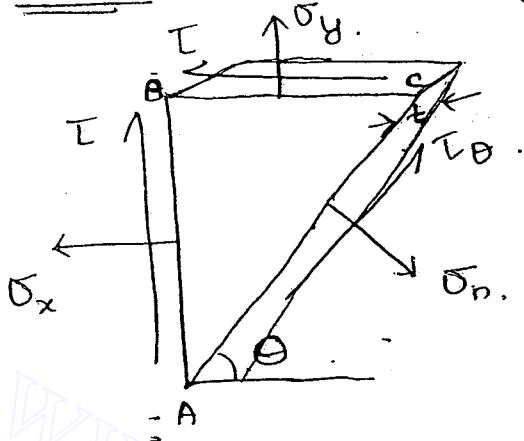
- (i) Bar in tension and compression.
- (ii) shaft in torsion.
- (iii) Beam in bending
- (iv) plates subjected to forces acting in the plane of the plate.
- (v) stress on the surface of structural element that is not subjected to external forces etc.,



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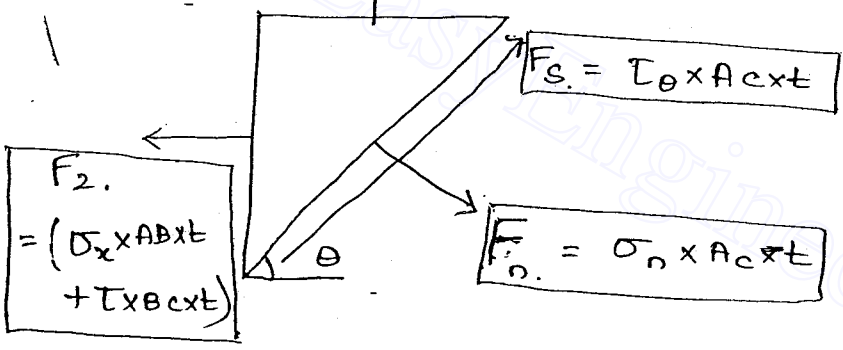


Stress



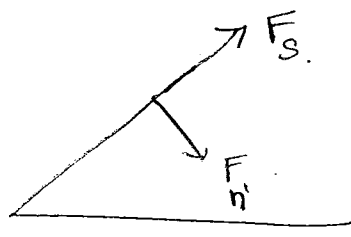
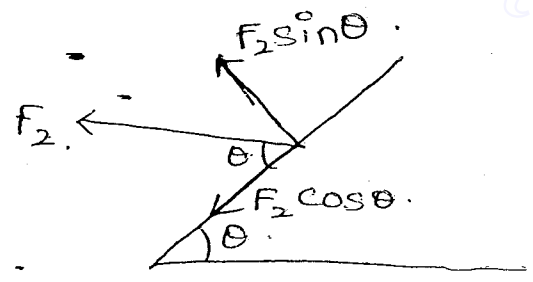
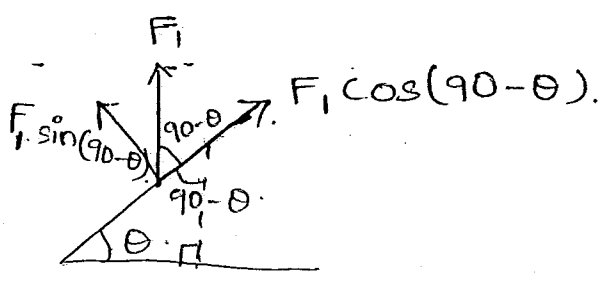
Force

$$F_1 = (\sigma_y \times BC \times t + \tau \times AB \times t)$$



$$AB = AC \sin \theta$$

$$BC = AC \cos \theta$$



ΣF along the plane = 0.

$$F_1 \cos(90-\theta) - F_2 \cos\theta + F_s = 0.$$

$$F_1 \sin\theta - F_2 \cos\theta + F_s = 0.$$

$$(\sigma_y \times Bcxt + \tau \times ABxt) \sin\theta - (\sigma_x \times ABxt + \tau \times Bcxt) \cos\theta + T_\theta \times A_c \times t = 0.$$

$$[\sigma_y \times A_c \cos\theta \times t + \tau \times A_c \sin\theta \times t] \sin\theta.$$

$$- [\sigma_x \times A_c \sin\theta \times t + \tau \times A_c \cos\theta \times t] \cos\theta + (T_\theta \times A_c \times t) = 0.$$

$$(\sigma_y \cos\theta + \tau \sin\theta) \sin\theta - (\sigma_x \sin\theta + \tau \cos\theta) \cos\theta + T_\theta = \frac{0}{A_c \times t}$$

$$\sigma_y \cos\theta \sin\theta + \tau \sin^2\theta - \sigma_x \sin\theta \cos\theta - \tau \cos^2\theta + T_\theta = 0$$

$$T_\theta = \sigma_x \sin\theta \cos\theta + \sigma_y \sin\theta \cos\theta + \tau (\cos^2\theta - \sin^2\theta)$$

$$T_\theta = \frac{(\sigma_x - \sigma_y)}{2} \sin 2\theta + \tau \cos 2\theta$$

$$\sum F \text{ across plane} = 0.$$

$$F_n = F_2 \sin \theta - F_1 \sin(90 - \theta) = 0.$$

$$F_n - F_2 \sin \theta - F_1 \cos \theta = 0.$$

$$(\sigma_n \times AC \times t) - (\sigma_x \times AB \times t + t \times BC \times t) \sin \theta - (\sigma_y \times BC \times t + t \times AB \times t) \cos \theta = 0$$

$$(\sigma_n \times AC \times t) - (\sigma_x \times AC \sin \theta \times t + t \times AC \cos \theta \times t) \sin \theta - (\sigma_y \times AC \cos \theta \times t + t \times AC \sin \theta \times t) \cos \theta = 0$$

$$\sigma_n - \sigma_x \sin^2 \theta - T \cos \theta \sin \theta - \sigma_y \cos^2 \theta - T \sin \theta \cos \theta = \frac{0}{AC \times t}$$

$$\sigma_n - \sigma_x \sin^2 \theta - \sigma_y \cos^2 \theta - 2T \sin \theta \cos \theta = 0$$

$$\boxed{\sigma_n = \sigma_x \sin^2 \theta + \sigma_y \cos^2 \theta + T \sin 2\theta}$$

when $\theta = 45^\circ$ $T_\theta = T_{\max}$.

$$T_\theta = \frac{\sigma_x - \sigma_y}{2} \sin 2(45^\circ) + T \cos 2(45^\circ)$$

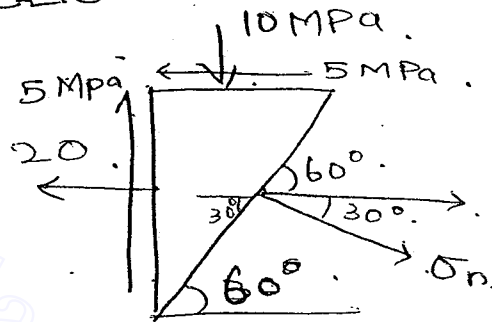
$$= \frac{\sigma_x - \sigma_y}{2} (1) + (T \times 0)$$

$$\boxed{T_{\max} = \frac{\sigma_x - \sigma_y}{2}}$$

$$\sigma_y, \text{ Resultant stress} = \sqrt{T_\theta^2 + \sigma_n^2}$$

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1. Determine normal and tangential stress which is subjected to two \perp^r stress 20 MPa tensile along x direction and 10 MPa compressive along y direction shear stress 5 MPa. The normal stress on the inclined plane makes an angle 30° with the direction of tensile stress. Determine σ_r and direction.



$$\tau_\theta = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau \cos 2\theta$$

$$= \frac{20 - 10}{2} \sin 2(60) + 5 \cos 2(60)$$

$$\tau_\theta = 10.49 \text{ MPa}$$

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2}$$

$$\sigma_n = \sigma_x \cos^2 \theta + \sigma_y \sin^2 \theta + \tau \sin 2\theta$$

$$= 20 \cdot \sin^2 60 - 10 \sin^2 60 + 5 \sin 2(60)$$

$$\sigma_n = 1.83 \text{ MPa}$$

$$\begin{aligned}\sigma_n &= \sigma_x \sin^2 \theta + \sigma_y \cos^2 \theta + \tau \sin 2\theta \\ &= 20 \sin^2 60 + 10 \cos^2 60 + 5 \sin 2(60)\end{aligned}$$

$$\boxed{\sigma_n = 16.83 \text{ MPa}}$$

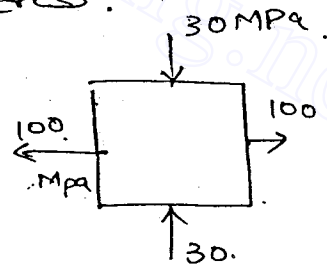
$$\begin{aligned}\sigma_x &= \sqrt{\sigma_n^2 + \tau_\theta^2} \\ &= \sqrt{16.83^2 + 10.49^2}\end{aligned}$$

$$\sigma_x = 19.83 \text{ MPa}$$

$$\begin{aligned}\alpha &= \tan^{-1} \left(\frac{\sigma_n}{\tau_\theta} \right) \\ &= \tan^{-1} \left(\frac{16.83}{10.49} \right)\end{aligned}$$

$$\boxed{\alpha = 58.06^\circ} \text{ w.r.t } \sigma_n$$

2) Determine max shear stress and its plane of load.



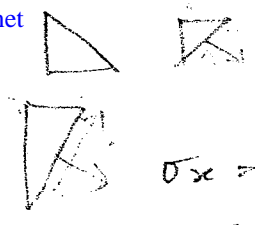
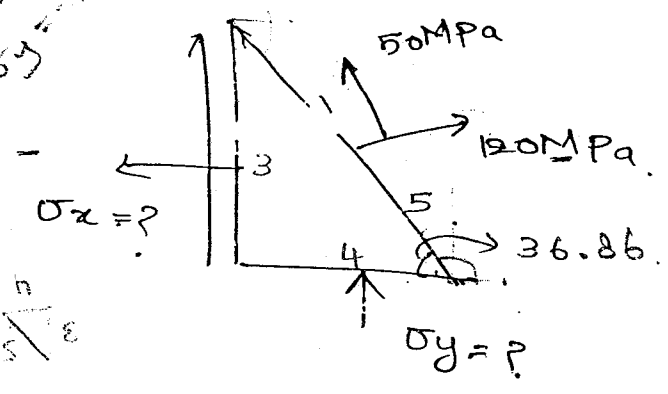
$$\begin{aligned}\tau_{\max} &= \frac{\sigma_x - \sigma_y}{2} \\ &= \frac{100 + 30}{2}\end{aligned}$$

$$\boxed{\tau_{\max} = 65 \text{ MPa}}$$

Plane is @ 45°

3.)

$\sigma_x = 3$
 $\sigma_y = -15$



$\sigma_x = 186.67$
 $\sigma_y = 82.5$

$\frac{50}{0.96} = \sigma_x - 0$

$\sigma_n = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta$

$\sigma_x - \sigma_y = 104.17$

$\sigma_n = \sigma_x \sin^2 \theta + \sigma_y \cos^2 \theta$

$\theta = 143.13$

$-120 = 0.36 \sigma_x + 0.64 \sigma_y$

$50 = \left(\frac{\sigma_x - \sigma_y}{2} \right) \sin 2(36.87)$

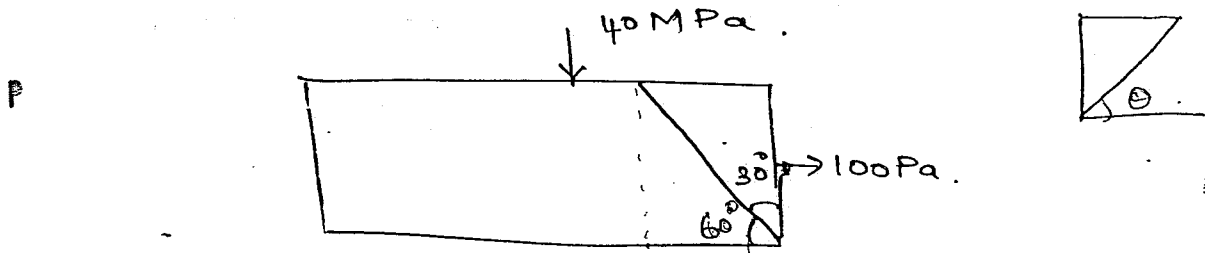
$\sigma_x - \sigma_y = 104.17$

$= 0.36 \sigma_x + 0.64 \sigma_y = -120$

$\sigma_x - \sigma_y = 104.17$

$0.36 \sigma_x + 0.64 \sigma_y = -120$

4.) Determine σ_r for given plane.



$$\tau_{\theta} = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta$$

$$= \frac{100 + 40}{2} \sin 120$$

$$= 60.62 \text{ MPa}$$

$$\sigma_n = \sigma_x \sin^2 \theta + \sigma_y \cos^2 \theta$$

$$= 100 (\sin 60)^2 + 40 (\cos 60)^2$$

$$\boxed{\sigma_n = 65 \text{ MPa}}$$

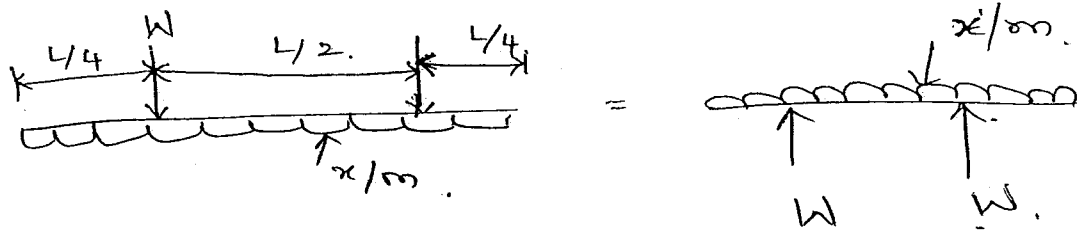
$$\sigma_r = \sqrt{\tau_{\theta}^2 + \sigma_n^2}$$

$$= \sqrt{(60.62)^2 + 65^2}$$

$$\boxed{\sigma_r = 88.88}$$

28/10/2015

- 1.) Determine the B.M @ centre of plank when floating in water with 2 person of weight W standing @ distance $\frac{L}{4}$ from the free end. Length of plank L



$$\sum Y = 0$$

$$W + W - x \times L = 0$$

$$x \times L = 2W$$

$$x = \frac{2W}{L}$$

B.M @ centre

$$= W \times \frac{L}{4} - \left(\frac{2W}{L} \times \frac{L}{2} \times \frac{L}{4} \right)$$

$$= \frac{WL}{4} - \frac{WL}{4}$$

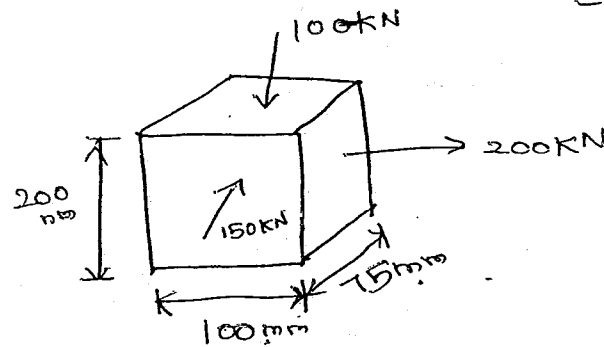
$$\boxed{\text{B.M @ centre} = 0}$$

2) Determine change in volume of an object as shown in figure below.

$$\mu = 0.3$$

$$E = 200 \times 10^9 \text{ N/m}^2$$

$$E = 2 \times 10^5 \text{ N/mm}^2$$



$$\epsilon_v = \frac{\delta v}{v}$$

$$\delta v = \epsilon_v \times v$$

$$\epsilon_v = \frac{\sigma_x + \sigma_y + \sigma_z}{E} (1 - 2\mu)$$

$$= -0.1 \times 100 \times 75 \times 200$$

$$= -150 \times 10^3 \text{ mm}^3$$

$$= -1.5 \times 10^{-4} \text{ m}^3$$

$$\epsilon_v = \frac{(200 - 100 - 150) \times 10^3}{2 \times 10^5} [1 - 2(0.3)]$$

$$= -0.1$$

$$\sigma_y = \frac{-100 \times 10^3}{100 \times 75} = -13.33 \text{ N/mm}^2$$

$$\sigma_x = \frac{200 \times 10^3}{75 \times 200} = 13.33 \text{ N/mm}^2$$

$$\sigma_z = \frac{-150 \times 10^3}{100 \times 200} = -7.5 \text{ N/mm}^2$$

$$\epsilon_v = \frac{13.33 - 13.33 - 7.5}{2 \times 10^5} (1 - 2(0.3))$$

$$\epsilon_v = -1.5 \times 10^{-5}$$

$$\delta v = \epsilon_v \times v$$

$$= -1.5 \times 10^{-5} \times 100 \times 75 \times 200$$

$$\delta v = -22.5 \text{ mm}^3$$

3. A metallic bar of size $l = 300\text{mm}$, $40 \times 40\text{mm}$ c/s. subjected to axial load 160kN .
 $\delta l = 0.12\text{mm}$. $\delta b = 0.005\text{mm}$. Determine E, μ
 Bulk modulus K, G, E_v, δ_v .

$$\delta l = \frac{Pl}{AE}$$

$$0.12 = \frac{160 \times 10^3 \times 300}{40 \times 40 \times E}$$

$$E = 2.5 \times 10^5 \text{ N/mm}^2$$

$$E_v = \frac{\sigma_x + \sigma_y + \sigma_z}{E} (1 - 2\mu)$$

$$\mu = \left(\frac{\delta b/b}{\delta l/l} \right) = \frac{(0.005/40)}{(0.12/300)} = 0.3125$$

$$K = \frac{E}{3(1 - 2\mu)}$$

$$K = \frac{E}{3(1 - 2\mu)} = \frac{2.5 \times 10^5}{3(1 - 2(0.3125))}$$

$$K = 22.22 \times 10^3 \text{ N/mm}^2$$

$$G = \frac{E}{2(1 + \mu)} = \frac{2.5 \times 10^5}{2(1 + 0.3125)}$$

$$G = 95.238 \times 10^3 \text{ N/mm}^2$$

$$E_v = \sigma_x \quad K = \frac{\text{Direct stress}}{E_v}$$

$$E_v = \frac{\text{Direct stress}}{K}$$

$$= \frac{100}{22.22 \times 10^3}$$

$$E_v = 4.5 \times 10^{-4}$$

$$\epsilon_v = \frac{\delta_v}{v}$$

$$\delta_v = \epsilon_v \times v$$

$$= 4.5 \times 10^{-4} \times 300 \times 40 \times 40$$

$$\boxed{\delta_v = 216 \text{ mm}}$$

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PRINCIPLE STRESSES AND THEORIES, OF FAILURE

- * Principle stress is the normal stress acting on any plane where shear stress is zero ($\tau = 0$)
- * The plane where only normal stress acts (shear stress or tangential stress is zero) is called principal plane.
- * There are 2 principle plane.
 - * Major principal plane
 - * Minor principal plane.
- * Principal planes must be always @ 90° to each other.
- * Max. shear stress $\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}$
- * Plane along which maximum shear stress acts, make any angle 45° from the principle plane.
- * To determine the location of principal plane, the tangential stress (or) shear stress is made equal to zero from where θ can be determined.

$$\tan(180 - 2\theta) = \frac{2\tau}{\sigma_x - \sigma_y}$$

* The major and minor principle stresses are given by the formula:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

$$= \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$$

* Location of principal plane:

$$\tau_\theta = 0.$$

$$\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau \cos 2\theta = 0.$$

$$\frac{\sin 2\theta}{\cos 2\theta} = \frac{-2\tau}{\sigma_x - \sigma_y}.$$

$$\tan 2\theta = \frac{-2\tau}{\sigma_x - \sigma_y}.$$

$$\tan (180 - 2\theta) = \frac{2\tau}{\sigma_x - \sigma_y}.$$

$$180 - 2\theta = \tan^{-1} \left(\frac{2\tau}{\sigma_x - \sigma_y} \right)$$

$$\theta_{P.P.} = 90 - \frac{1}{2} \tan^{-1} \left(\frac{2\tau}{\sigma_x - \sigma_y} \right).$$

$$\rightarrow \text{If } \sigma_x > \sigma_y, \quad \theta_{P.P.} = 45^\circ - 90^\circ \text{ from } \sigma_x.$$

$$\rightarrow \text{If } \sigma_x = \sigma_y, \quad \theta_{P.P.} = 45^\circ \text{ from } \sigma_x.$$

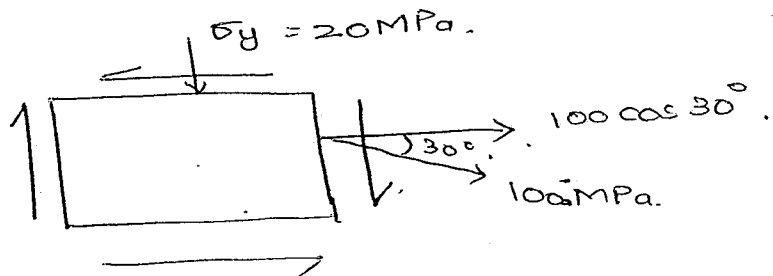
$$\rightarrow \text{If } \sigma_x < \sigma_y, \quad \theta_{P.P.} = 0^\circ - 45^\circ \text{ from } \sigma_x.$$

$$\tan 2\theta = \frac{-2\tau}{\sigma_x - \sigma_y} = \frac{-2(-\tau)}{\sigma_x - \sigma_y}.$$

$$\tan 2\theta = \frac{2\tau}{\sigma_x - \sigma_y}.$$

$$\theta = \frac{1}{2} \tan^{-1} \left(\frac{\tau}{\sigma_x - \sigma_y} \right)$$

1. A steel material is subjected to direct stress @ an angle 30° with the horizontal having magnitude of 100 kPa and compressive stress of 20 MPa . In \perp^r direction, Determine principal stress, principal plane, max. shear stress.



$$\tau = 100 \sin 30 = 50 \text{ MPa.}$$

$$\sigma_x = 100 \cos 30 = 86.6 \text{ MPa.}$$

$$\sigma_y = 20.$$

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau^2}$$

$$= 106.4 \text{ MPa.}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau^2}$$

$$= 39.75 \text{ MPa.}$$

$$\tau_{\max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{106.4 - 39.75}{2} = 33.325 \text{ MPa}$$

$$\theta_{\text{P.P.}} = 90 - \frac{1}{2} \tan^{-1} \left(\frac{2\tau}{\sigma_x - \sigma_y} \right)$$

$$\theta_{\text{P.P.}} = 68.46^\circ$$

→ Concrete Technology

- * Properties of concrete
- * Basics of mix design.

→ Concrete Design :

- * Basic working stresses and limit state design concepts
- * Analysis of ultimate load capacity
- * Design of members subjected to flexure, shear, compression and torsion by limit state method.

→ Prestressed Concrete -

- * Basic elements of prestressed concrete
- * Analysis of beam sections at transfer and service load.

STRUCTURAL ANALYSIS

Structure:

A structure refers to a system of connected parts used to support a load.

When any elastic body is subjected to a system of loads and deformation takes place and the resistance is set up against the deformation. Then the elastic body is known as structure.

eg: simply supported beam is a stable structure.



Mechanism:

If no resistance is set up in the body against the deformation then it is known as an ^{un}stable structure or mechanism.

The simply supported beam with an internal hinge is an unstable structure.

Classification of structure:

Skeletal structure:

Structures which can be idealized to a series of straight or curved lines

eg: Roof trusses (or) building frames

Surface structure:

Structure which can be idealized to plane or curved surface

eg: slabs (or) and shells.

Solid structure:

Structure which can neither be idealized to a skeleton nor to a plane or curved surface

eg: Massive foundation.

Classification of skeletal structure:

(i) Based on type of joint:

→ Pin jointed frames.

when members are connected by means of pin-joint. These frames support the load by developing only axial forces, which the external load acts at the joints and members are straight.

(ii) Rigid Jointed Frames:Assumptions:

→ The joints of rigid jointed frame are assumed to be rigid, so that the angles between the members meeting at a joint remain unchanged.

→ These frames resist external forces by developing bending moments, shear forces, axial forces and twisting moment in the members of the frame.

Based on Dimension:(i) Plane Frames.

All members of the plane frame as well as the external loads are assumed to be in one plane.

→ Pinned plane frame:

Members carrying only axial forces.

→ Rigid jointed plane frame:

Members carry Axial force, Bending moment and shear force.

Note: If loaded in its own plane any cross section of member is subjected to 3 internal forces. (Axial force, shear force, Bending moment).

If loading is away from the plane torsional moment also acts on the member.

(ii) Space Frame:

All members do not lie in one plane.

Every often it is also a combination of series of frames.

→ Pin jointed space frame

→ Rigid jointed space frame.

Note: Any cross section of a member of a skeletal space structure there are six internal force component

(Axial force, Biaxial shear force (V_x, V_y).

Twisting moment (T) and biaxial moment (M_x, M_y).

Equations of static Equilibrium:

* For plane frame: subjected to in plane external force in xy plane.

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum M_z = 0$$

(or)

$$\sum H = 0$$

$$\sum V = 0$$

$$\sum M = 0$$

* For space frame

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum F_z = 0$$

$$\sum M_x = \sum M_y = \sum M_z = 0$$

Statically Determinate Structure:

* Structure that can be analysed with the help of equations of static equilibrium alone.

* It undergoes finite equilibrium deformation before the condition of equilibrium is satisfied.

eg: A cantilever beam, a simply supported beam, a 3 hinged arch, a suspension cable.

Statically Indeterminate Structure:

* Any structure whose reaction components (or) internal stresses cannot be established by using the equation of static equilibrium alone.

* The no. of forces $>$ greater than no. of equilibrium equations.

* For complete analysis additional equations based on conditions of compatibility (or) consistent displacement can be used.

* D

Degree of static indeterminacy (or)

Redundancy:

* Equations in addition to static equilibrium equation necessary to complete analysis statically indeterminate structure.

$$D_s = \text{No. of unknowns} - \text{No. of static equilibrium equations}$$

* Formulation of static indeterminacy

$$D_s = D_{se} + D_{si}$$

where,

D_{se} - External indeterminacy.

$$D_{se} = r - 6 \quad (\text{space frame})$$

$$D_{se} = r - 3 \quad (\text{plane frame})$$

D_{si} = Internal indeterminacy.

$$D_{si} = m - (2j - 3) \quad (\text{Pin jointed plane frame})$$

$$D_{si} = m - (3j - 6) \quad (\text{Pin jointed space frame})$$

$$D_{si} = 3C \quad (\text{Rigid jointed plane frame})$$

$$D_{si} = 6C \quad (\text{Rigid jointed space frame})$$

j - No. of joints.

C - No. of cuts required for obtaining an open configuration.

Simplified Formula:

$$\begin{aligned}
 D_s &= m + r - 2j \quad (\text{Pin jointed Plane frame}) \\
 &= (m + r) - 3j \quad (\text{Pin jointed space frame}) \\
 &= (3m + r) - 3j \quad (\text{Rigid jointed plane frame}) \\
 &= (6m + r) - 6j \quad (\text{Rigid jointed space frame})
 \end{aligned}$$

m - No. of member forces.

r - No. of reaction.

Internal pin (or) Internal Hinge.

* A pin provided anywhere in the structure cannot transmit the moment from one part to another part of the structure and thus provides one additional condition equation.

$$\sum M = 0$$

Internal Link:

* A link (consisting of a short bar with a pin at each end) provided anywhere in the structure is incapable of transferring a moment as well as horizontal force from one part to another part of the structure and thus provides to addition equation condition.

$$\sum M = 0, \quad \sum H = 0$$

statically Indeterminate beams: (4)

Eg: Propped cantilever, Continuous beam

Fixed beam.

* General loading (which has both horizontal or vertical component.

$$\sum H = \sum V = \sum M = 0$$

No. of. equilibrium equation = 3.

Vertical loading:

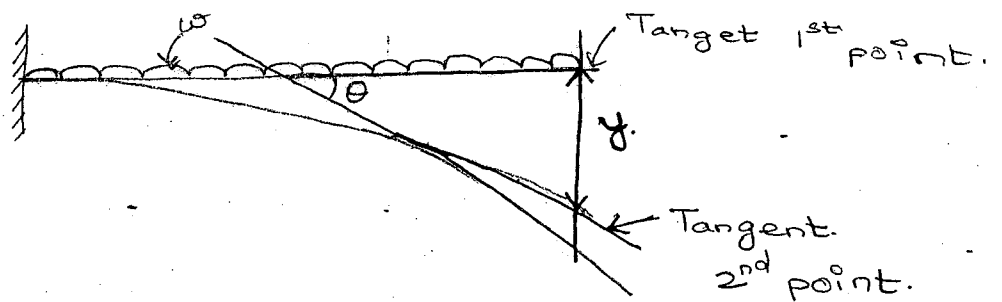
No. of. equilibrium equation = 2.

$$\sum V = \sum M = 0$$

only for beams not for frames.

STRUCTURAL ANALYSIS

MOMENT AREA THEOREM:



Theorem 1:

The angle made by intersection of two tangents drawn at two places of elastic curve (deflected shape of beam) is equal to area of B.M diagram b/w two point of tangence divided by EI

(or)

The difference of angle b/w two point of tangency of elastic curve is equal to Area of B.M divided by EI.

Theorem 2:

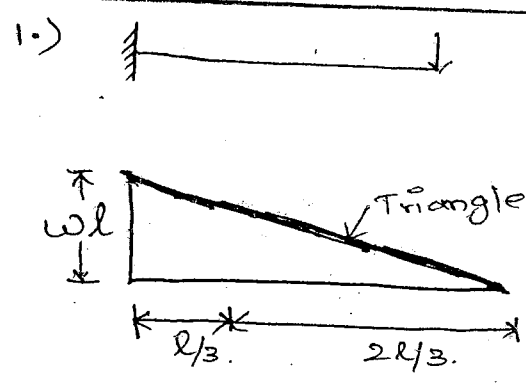
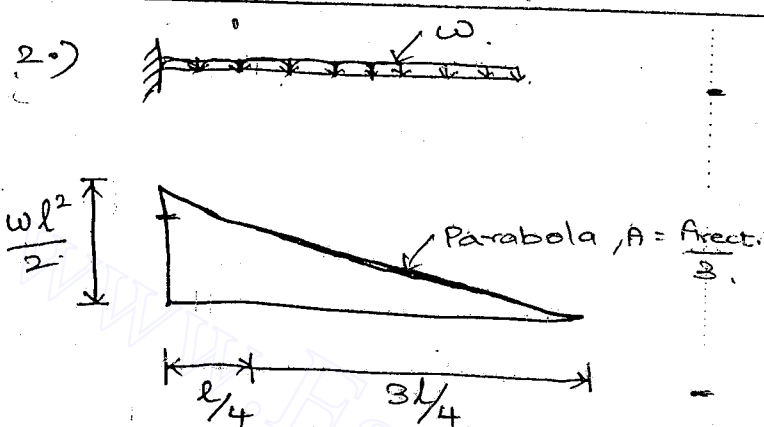
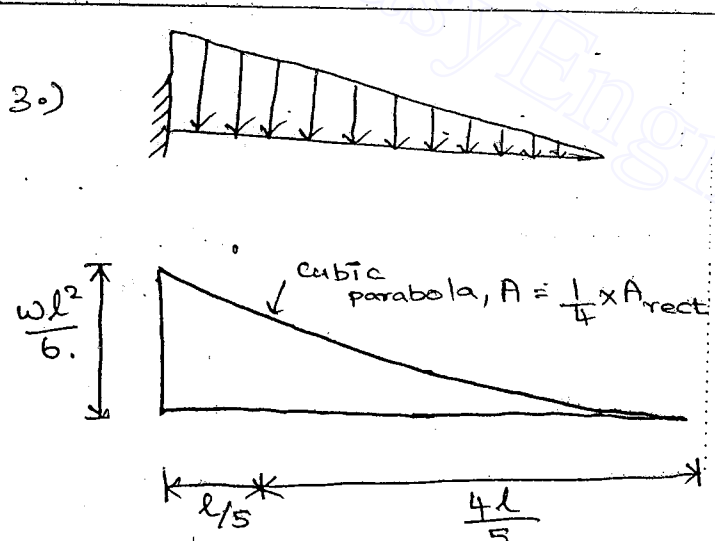
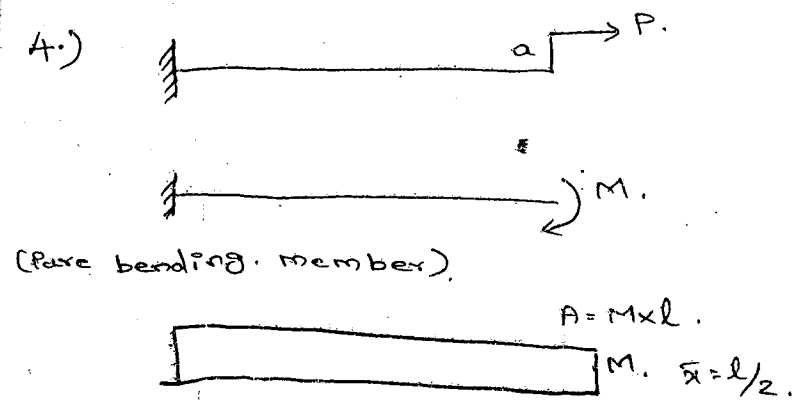
$$y = \frac{A\bar{x}}{EI}$$

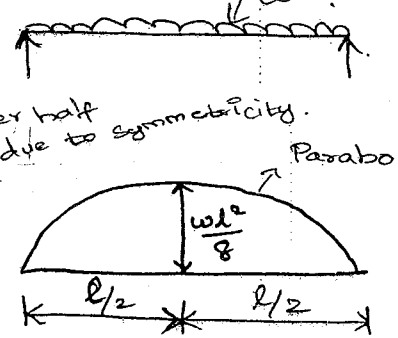
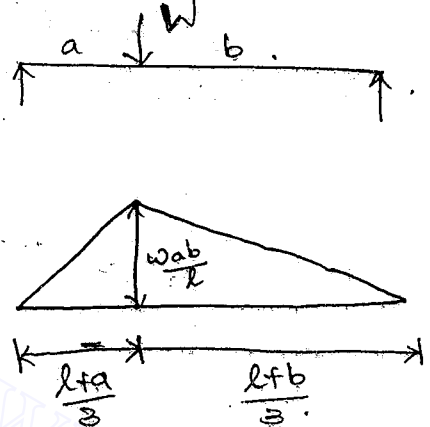
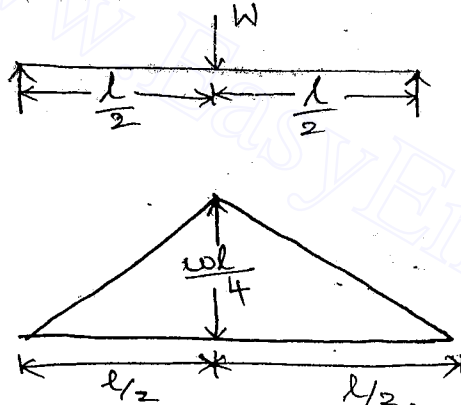
The intercept made by 2 tangents drawn on two points of elastic curve on a vertical reference line is equal to Moment of Area of B.M diagram b/w 2 points of tangency about the vertical reference line divided by EI.

(or)

The vertical distance of second point with respect to first point is equal to the moment of Area of M/EI diagram about the second point.

Standard Cases:

	θ	Δ
<p>1.) </p>	A \bar{x} θ	Δ
<p>2.) </p>	A_{rect} \bar{x} θ	Δ
<p>3.) </p>	A_{rect} \bar{x} θ	Δ
<p>4.)  (Pure bending member) $A = M \times l$ $M, \bar{x} = l/2$ </p>	A_{rect} \bar{x} θ	Δ

5.)	A	\bar{x}	θ	y
<p>Consider half area due to symmetry.</p>  <p>Parabola = $\frac{2}{3} \times A_{rect.}$</p>	$\frac{2}{3} \times A_{rect.}$	$\frac{5}{8} \times \frac{l}{2}$ from s.s end	$\frac{wl^3}{24EI}$	$\frac{5wl^4}{384EI}$
<p>6.)</p> 	$\frac{A_{rect}}{2}$	$\frac{2}{3} \left(\frac{la}{3} \right)$ (or) $\frac{2}{3} \left(\frac{lb}{3} \right)$ from s.s end.	$\frac{wab(l+a)}{6EI}$ (or) $\frac{wab(l+b)}{6EI}$	$\frac{wab(l+a)}{27EI}$ (or) $\frac{wab(l+b)}{27EI}$
<p>7.)</p> 	$\frac{A_{rect}}{2}$	$\frac{l}{3}$ from s.s end	$\frac{wl^2}{16EI}$	$\frac{wl^3}{48EI}$

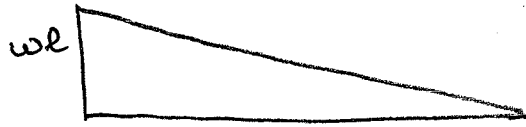
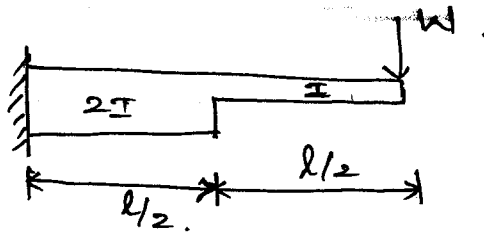
GEO TECHNICAL ENGINEERING

→ SOIL MECHANICS

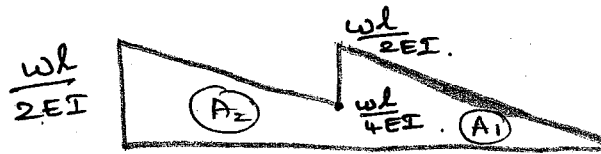
→ FOUNDATION ENGINEERING

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2.) Determine θ and y for the following beam:



B.M.D.



$\frac{wl}{EI}$

$\theta =$ Area of B.M. diagram.

$$= A_1 + A_2$$

$$= \left(\frac{1}{2} \times \frac{l}{2} \times \frac{wl}{2EI} \right) + \left[\left(\frac{l}{2} \right) \times \left(\frac{wl}{4EI} + \frac{wl}{2EI} \right) \right]$$

$$= \frac{wl^2}{8EI} + \left(\frac{l}{4} \times \frac{3wl}{4EI} \right)$$

$$= \frac{wl^2}{8EI} + \frac{3wl^2}{16EI}$$

$$\theta = \frac{5wl^2}{16EI}$$

$$y = A_1 \bar{x}_1 + A_2 \bar{x}_2$$

$$= \left[\frac{wl^2}{8EI} \times \frac{l}{3} \right] + \left[\frac{3wl^2}{16EI} \times \frac{7l}{9} \right]$$

$$= \frac{wl^3}{24EI} + \frac{wl^3}{48EI}$$

$$= \frac{3wl^3}{48EI}$$

$$y = \frac{wl^3}{16EI}$$

$$\bar{x}_1 = \frac{2\left(\frac{l}{2}\right)}{3} = \frac{l}{3}$$

$$\bar{x}_2 = \frac{l}{3}$$

$$\bar{x}_2 = \left(\frac{a+2b}{a+b} \right) \frac{b}{3}$$

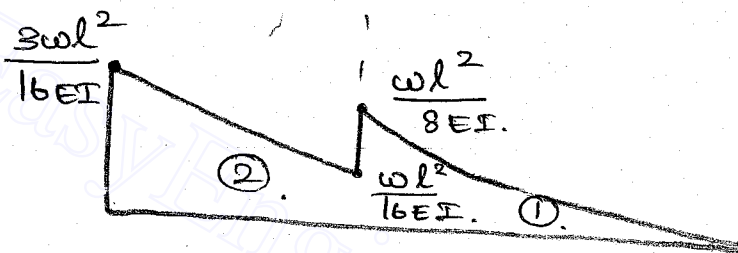
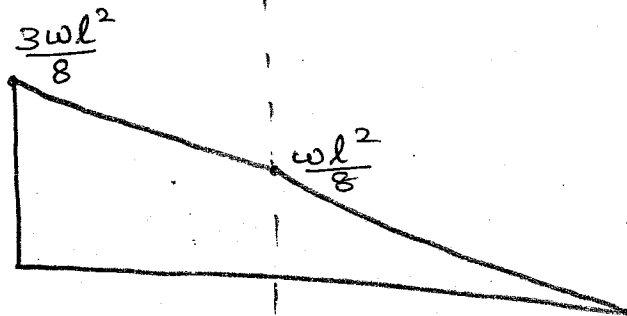
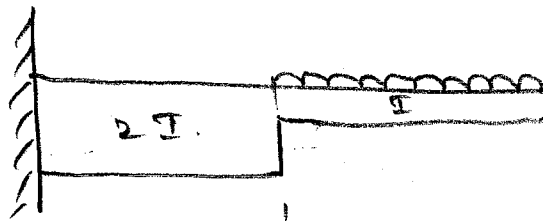
$$= \frac{l}{2} + \left[\frac{\left(\frac{wl}{4EI} + \frac{2wl}{2EI} \right)}{\frac{wl}{4EI} + \frac{wl}{2EI}} \right] \times$$

$$= \frac{l}{2} + \left[\frac{5wl^2}{4EI} \times \frac{l}{\frac{3wl}{4EI}} \right]$$

$$= \frac{l}{2} + \frac{5l}{18}$$

$$\bar{x}_2 = \frac{14l}{18} = \frac{7l}{9}$$

1. find θ_{max} and y_{max} for the beam:
(Mohr's Method)



$$\begin{aligned}\theta &= A_1 + A_2 \\ &= \frac{wl^3}{48EI} + \frac{wl^3}{16EI} \\ &= \frac{4wl^3}{48EI} \\ \theta &= \frac{wl^3}{12EI}\end{aligned}$$

$$\begin{aligned}y &= A_1 \bar{x}_1 + A_2 \bar{x}_2 \\ &= \left[\frac{wl^3}{48EI} \times \frac{3l}{8} \right] + \left[\frac{wl^3}{16EI} \times \frac{19l}{24} \right] \\ &= \frac{wl^4}{128EI} + \frac{19wl^4}{384EI} \\ &= \frac{3wl^4 + 19wl^4}{384EI}\end{aligned}$$

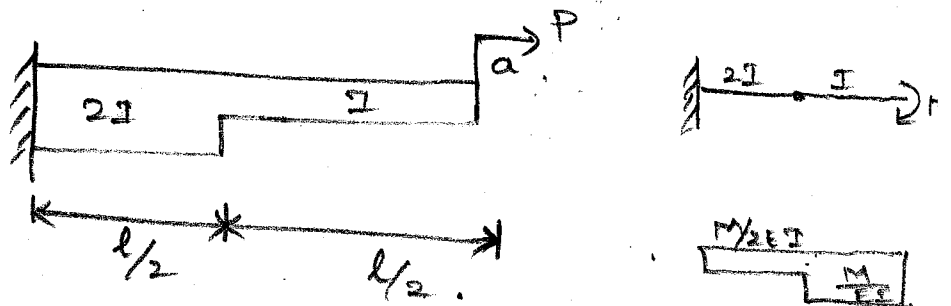
$$y = \frac{22wl^4}{384EI} \quad y = \frac{11wl^4}{192EI}$$

$$\begin{aligned}A_1 &= \frac{1}{3} \text{Arect.} \\ &= \frac{1}{3} \times l \times \frac{wl^2}{8EI} \\ &= \frac{wl^3}{48EI} \\ A_2 &= \frac{4wl^2}{16EI} \times \frac{l}{4} \\ &= \frac{wl^3}{16EI}\end{aligned}$$

$$\begin{aligned}\bar{x}_1 &= \frac{3l \times \frac{3}{8}}{4 \times 2} = \frac{3l}{8} \\ \bar{x}_2 &= \frac{l}{2} + \left(\frac{\frac{wl^2}{16EI} + 6wl^2}{\frac{4wl^2}{16EI}} \right) \\ &= \frac{l}{2} + \frac{7wl^2}{4} \times \frac{l}{6} \\ &= \frac{l}{2} + \frac{7l}{24}\end{aligned}$$

$$\bar{x}_2 = \frac{19l}{24}$$

- b) Determine slope and Deflection of the beam: given below.



θ :

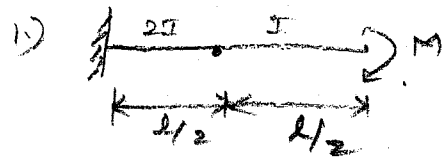
$$\text{Area of B.M diagram} = \frac{M \times l}{4EI} + \frac{M \times l}{2EI}$$

$$\theta = \frac{3Ml}{4EI}$$

y :

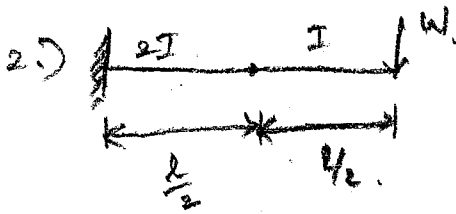
$$\text{Slope of beam} = \left[\frac{Ml}{4EI} \times \left(\frac{l+l}{4} \right) \right] + \left[\frac{Ml}{2EI} \times \frac{l}{4} \right]$$

$$y = \frac{5Ml^2}{16EI}$$



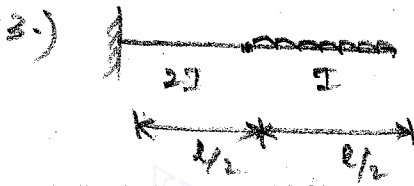
$$\frac{3ML^2}{4EI}$$

$$\frac{5ML^2}{16EI}$$



$$\frac{5Wl^2}{16EI}$$

$$\frac{Wl^3}{16EI}$$



$$\frac{wl^3}{12EI}$$

$$\frac{11wl^4}{192EI}$$

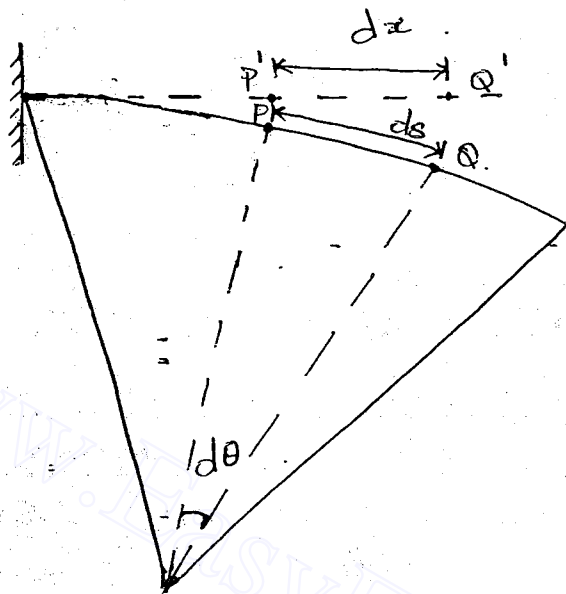
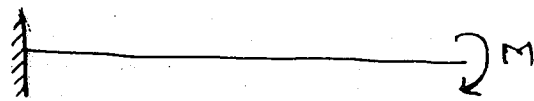
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- Analysis of statically determinate trusses, arches, beams, cables and frames
- Displacement in statically determinate structures.
- Analysis of statically indeterminate structures by force/energy method.
- ⇒ Analysis by displacement method (slope deflection and moment distribution method)
- Influence lines for determinate and indeterminate structures.
- Basic concept of matrix method of structural analysis.

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DOUBLE INTEGRATION:



$$\theta = \frac{\text{Arc length}}{\text{Radius of curvature}}$$

$$d\theta = \frac{ds}{R}$$

$$ds = dx$$

$$d\theta = \frac{dx}{R}$$

$$\theta = \frac{dy}{dx}$$

$$d\left(\frac{dy}{dx}\right) = \frac{dx}{R}$$

$$\frac{d^2y}{dx^2} = \frac{1}{R} \quad \text{--- (1)}$$

According to Bending Theory.

$$\frac{M}{I} = \frac{f}{y} = \frac{E}{R}$$

$$\frac{1}{R} = \frac{M_{xx}}{EI} \quad \text{--- (2)}$$

$$\frac{d^2 y}{dx^2} = \frac{M_{xx}}{EI}$$

$$EI \frac{d^2 y}{dx^2} = M_{xx}$$

Integrating above equation

$$EI \frac{dy}{dx} = \int M_{xx}$$

$$\theta = \frac{dy}{dx} = \int \frac{M_{xx}}{EI}$$

Integrating above equation

$$y = \iint \frac{M_{xx}}{EI}$$

$$EI \frac{d^2 y}{dx^2} = M_{xx}$$

Differentiating above equation

$$E.I \frac{d^3 y}{dx^3} = \frac{dM_{xx}}{dx} = \text{Shear Force}$$

Differentiating above equation

$$EI \frac{d^4 y}{dx^4} = \frac{dV}{dx} = \text{Load}$$

- * If B.M is -ve (i.e) hogging then the curvature is +ve. but if B.M is +ve. (sagging) then the curvature is -ve. And hence correction is applied to above equation.

$$EI \frac{d^2y}{dz^2} = -M_{xx}$$

Note:

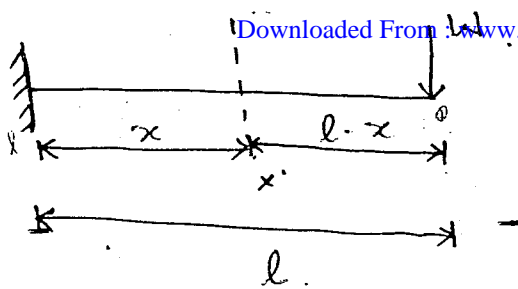
- * At fully rigid end the product of EI is ∞ infinity.

$\therefore \frac{1}{EI} = \frac{1}{\infty} = 0$
and hence θ and y is zero @ fixed end.

- * At simple support, δ is maximum. @ mid span but $\theta = 0$ @ mid span where as @ support $\theta = \theta_{max}$, $\delta = 0$.

- * Cantilever with one point load @ free end max. deflection will be at the free end.

- * Cantilever with one point load anywhere in the beam δ_{max} of elastic curve will be under point load. Beyond point load deflection will vary linearly.



$$EI \frac{d^2y}{dx^2} = -M_{xx}$$

$$M_{xx} = -w(l-x)$$

$$EI \frac{d^2y}{dx^2} = -[-w(l-x)]$$

$$EI \frac{d^2y}{dx^2} = w(l-x) \quad \text{--- (1)}$$

Integrating equation (1).

$$\int EI \frac{d^2y}{dx^2} = \int w(l-x)$$

$$EI \frac{dy}{dx} = w\left(lx - \frac{x^2}{2}\right) + C_1$$

⊙ when $x=0$ $\frac{dy}{dx} = 0$

$$C_1 = 0$$

$$\frac{dy}{dx} = \frac{w}{EI} \left(lx - \frac{x^2}{2}\right)$$

For max. slope $x=l$

$$\frac{dy}{dx} = \frac{w}{EI} \left(l^2 - \frac{l^2}{2}\right)$$

$$\theta_{max} = \frac{dy}{dx} = \frac{wl^2}{2EI}$$

$$EI \frac{dy}{dx} = \omega \left(lx - \frac{x^2}{2} \right) \quad \text{--- (2)}$$

Integrating equ (2) . -

$$\int EI \frac{dy}{dx} = \int \omega \left(lx - \frac{x^2}{2} \right)$$

$$EI y = \omega \left(\frac{lx^2}{2} - \frac{x^3}{6} \right) + C_2$$

When $x = 0$ $y = 0$.

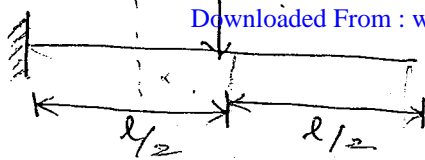
$$EI y = \frac{\omega}{EI} \left[\frac{lx^2}{2} - \frac{x^3}{6} \right] + C_2 = 0 \quad \boxed{C_2 = 0}$$

For y_{max} put $x = l$.

$$y = \frac{\omega}{EI} \left(\frac{l^3}{2} - \frac{l^3}{6} \right)$$

$$\boxed{y_{max} = \frac{\omega l^3}{3EI}}$$

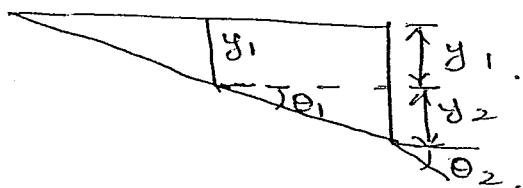
2.3



$$\frac{1}{2} \times \frac{wl}{2} \times \frac{l}{2} \times \left[\frac{2l}{3} + \frac{l}{3} \right]$$

$$\frac{wl^2}{8EI} \left[\frac{5l}{6} \right]$$

$$\frac{5wl^3}{48EI}$$



$$\theta_1 = \frac{wa^2}{2EI}$$

$$= \frac{w(l/2)^2}{2EI}$$

$$\theta_1 = \frac{wl^2}{8EI}$$

$$\theta_1 = \theta_2 = \frac{wl^2}{8EI}$$

$$y_1 = \frac{wa^3}{3EI} = \frac{w(l/2)^3}{3EI} = \frac{wl^3}{24EI}$$

$$\theta_1 = \frac{y_2}{l/2}$$

$$\frac{wl^2}{8EI} = \frac{y_2}{l/2}$$

$$y_2 = \frac{wl^3}{16EI}$$

$$y_{max} = y_1 + y_2 = \frac{wl^3}{24EI} + \frac{wl^3}{16EI}$$

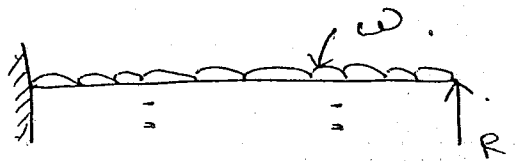
$$= \frac{2wl^3 + 3wl^3}{48EI}$$

$$y_{max} = \frac{5wl^3}{48EI}$$

$$\frac{12 \times 4}{2 \times 6 \times 4} = \frac{3}{2}$$

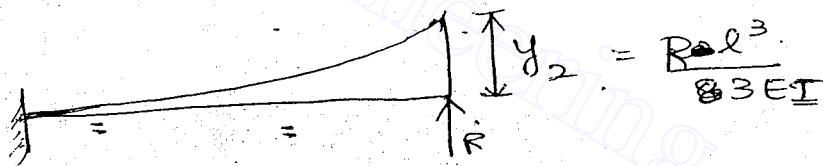
3) A cantilever of span "l" subjected to UDL "w" on yielding support is provided at free end. Determine

- (i) Prop reaction.
- (ii) Max. hogging B.M.
- (iii) Max. sagging B.M.
- (iv) Position of sagging B.M.
- (v) Point of contraflexure.



$$R \times l = \frac{wl^2}{2}$$

$$R = \frac{wl}{2}$$



$$y_1 - y_2 = 0$$

$$\frac{Rl^3}{3EI} - \frac{wl^4}{8EI} = 0$$

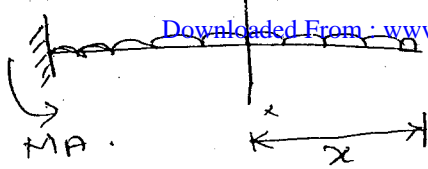
$$\frac{Rl^3}{3EI} = \frac{wl^4}{8EI}$$

$$R = \frac{3wl}{8}$$

Max. hogging B.M.

$$= R \times l - \frac{wl^2}{2}$$

$$= \frac{3wl^2}{8} - \frac{wl^2}{2} = -\frac{wl^2}{2}$$



$$R_A x - M_A - \frac{wx^2}{2} = 0$$

$$\frac{5wl}{8} x - \frac{wl^2}{2} - \frac{wx^2}{2}$$

Point of Contra flexure.

$$R x - \frac{wx^2}{2} = 0$$

$$\frac{3wl}{8} x = \frac{wx^2}{2}$$

$$x = \frac{3l}{4}$$

Max. Sagging B.M.

B.M. di?

$$V_{xx} = 0$$

$$R - wx = 0$$

$$\frac{3wl}{8} = wx$$

$$x = \frac{3l}{8}$$

Max. Sagging B.M. @ $x = \frac{3l}{8}$

$$M = R x - \frac{wx^2}{2}$$

$$= \frac{3wl}{8} x - \frac{wx^2}{2}$$

$$= \left(\frac{3wl}{8} \times \frac{3l}{8} \right) - \frac{w}{2} \left(\frac{3l}{8} \right)^2$$

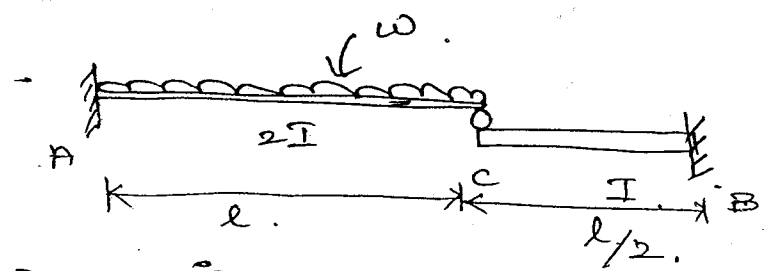
$$= \frac{9wl^2}{64} - \frac{9wl^2}{128}$$

$$\text{Max. Sagging B.M.} = \frac{9wl^2}{128}$$

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STRUCTURAL ANALYSIS

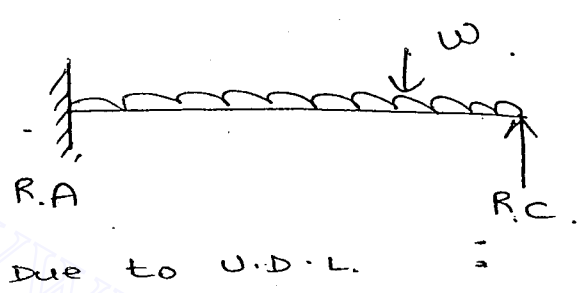
10)



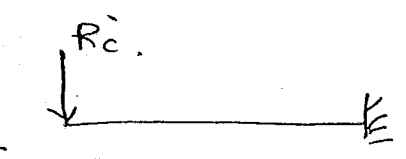
$$R_c = \frac{3}{10} wl$$

$$R_A = \frac{7}{10} wl$$

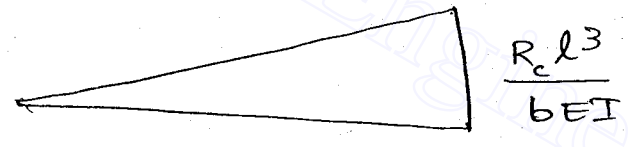
Determine S.F @ hinge Deflection @ hinge
Support reaction and support B.M.



Due to U.D.L.



Due to point load.



$$S_{net} = \frac{R_c l^3}{3EI}$$

$$= \frac{R_c l^3}{24EI} \quad \text{--- (2)}$$

$$S_{net} = \frac{wl^4}{16EI} - \frac{R_c l^3}{6EI} \quad \text{--- (1)}$$

Equating (1) and (2)

$$\frac{wl^4}{16EI} - \frac{R_c l^3}{6EI} = \frac{R_c l^3}{6EI}$$

$$\frac{wl^4}{16EI} = \frac{2R_c l^3}{6EI}$$

$$R_c = \frac{wl}{8}$$

$$\frac{2R_c l^3}{6EI} = \frac{3}{16 \times 2}$$

$$\frac{R_c l^3}{3EI} = \frac{wl}{8} = wl$$

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$$\frac{wl^4}{16EI} - \frac{R_c l^3}{6EI} = \frac{R_c l^3}{24EI}$$

$$\frac{wl^4}{16EI} = \frac{R_c l^3}{24EI} + \frac{R_c l^3}{6EI}$$

$$\frac{wl^4}{16EI} = \frac{5R_c l^3}{24EI}$$

$$R_c = \frac{24wl}{16 \times 5}$$

$$= \frac{6wl}{4 \times 5}$$

$$R_c = \frac{3wl}{10}$$

$$\sum V = 0$$

$$R_A + R_c = wl$$

$$R_A = wl - \frac{3wl}{10}$$

$$R_A = \frac{7wl}{10}$$

$$M_A = \frac{3wl}{10} \times l - \frac{wl^2}{2}$$

$$= \frac{3wl^2}{10} - \frac{wl^2}{2}$$

$$= -\frac{2wl^2}{10}$$

$$M_A = -\frac{wl^2}{5}$$

$$M_B = R_c \times l/2$$

$$= \frac{3wl}{10} \times \frac{l}{2}$$

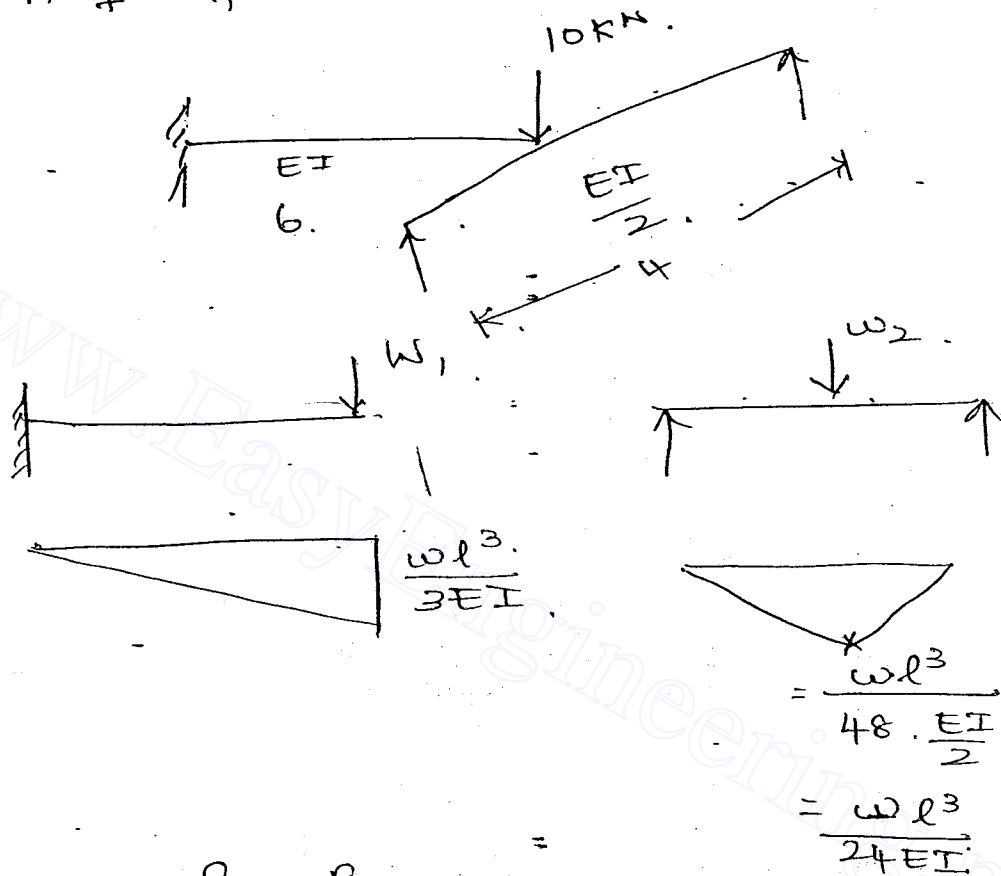
$$M_B = \frac{3wl^2}{20}$$

$$\delta = \frac{R_c l^3}{24EI}$$

$$= \frac{3wl \times l^3}{10 \times 24EI}$$

$$\delta_{net} = \frac{3wl^4}{80EI}$$

- 2.) A cantilever beam, $l = 6\text{m}$, is supported by s.s. Beam in transverse direction @ free end & @ the mid span of s.s beam. Point load of 10kN is applied at the junction point. Determine support reaction s.s span = 4m . Assume EI of s.s. is half of EI of cantilever.



$$\delta_1 = \delta_2$$

$$\frac{w_1 l^3}{24EI} = \frac{w_2 l^3}{3EI}$$

$$w_1 = \frac{3w_2}{24}$$

$$\delta_1 = \delta_2$$

$$\frac{w_1 l^3}{3EI} = \frac{w_2 l^3}{24EI}$$

$$\frac{w_1 \times 6^3}{3} = \frac{w_2 (4)^3}{24}$$

$$w_1 = \frac{3}{6^3} \times \frac{4^3}{24} \times w_2$$

$$\omega_1 = \frac{64 \times 8}{24 \times 216} \omega_2$$

$$\omega_1 = \frac{\omega_2}{27}$$

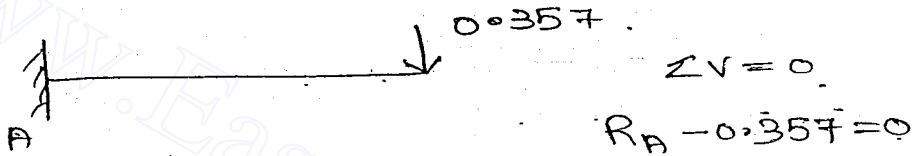
$$\omega_1 + \omega_2 = 10$$

$$\frac{\omega_2}{27} + \omega_2 = 10$$

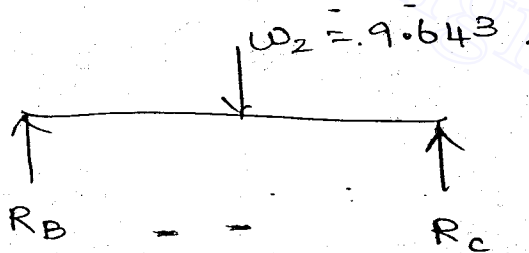
$$\frac{28\omega_2}{27} = 10$$

$$\omega_2 = 9.643 \text{ kN}$$

$$\omega_1 = 0.357 \text{ kN}$$



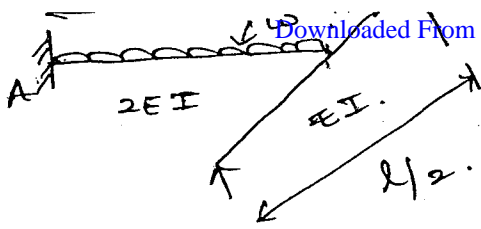
$$R_A = 0.357$$



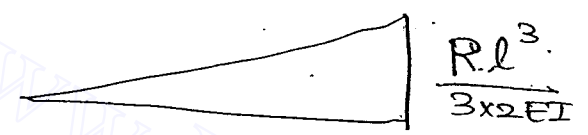
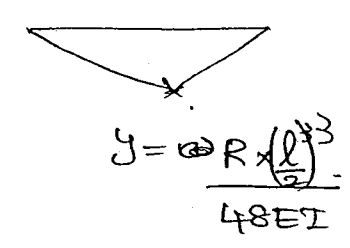
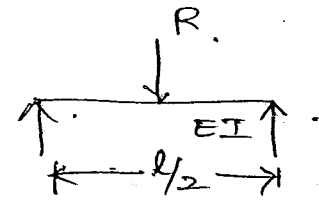
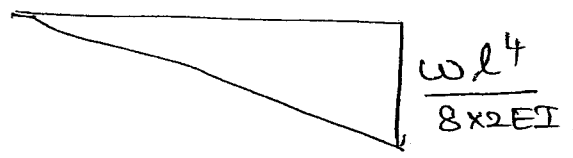
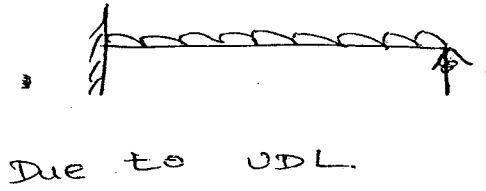
$$R_B = R_C = \frac{9.643}{2}$$

$$R_B = R_C = 4.8215 \text{ kN}$$

3.)



Determine support reaction and B Support B.M.



$$y = \frac{Rl^3}{384EI}$$

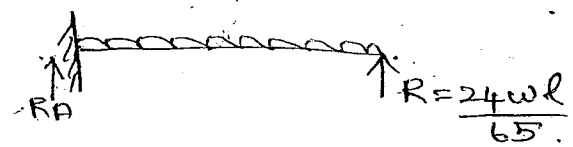
$$\frac{wl^4}{16EI} + \frac{Rl^3}{6EI} = \frac{Rl^3}{384EI}$$

$$\frac{wl^4}{16EI} = \frac{Rl^3}{384EI} + \frac{Rl^3}{6EI}$$

$$\frac{wl^4}{16EI} = \frac{65Rl^3}{384EI}$$

$$R = \frac{wl \times 384}{16 \times 65}$$

$$R = \frac{24wl}{65}$$



$$\sum V = 0$$

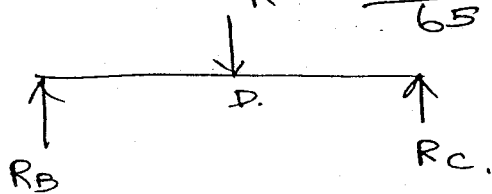
$$R_A = wl - \frac{24wl}{65}$$

$$R_A = \frac{41wl}{65}$$

$$M_A = \frac{24wl}{65} \times l - \frac{wl^2}{2}$$

$$= \frac{24wl^2}{65} - \frac{wl^2}{2}$$

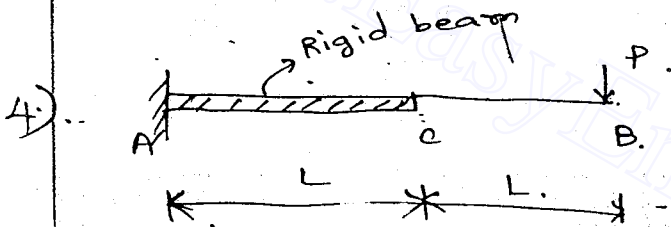
$$M_A = -\frac{17wl^2}{130EI}$$



$$R_B = R_C = \frac{24wl}{65 \times 2} = \frac{24wl}{130}$$

$$\begin{aligned} M_D &= R_B \times \frac{l}{4} \\ &= \frac{24wl}{130} \times \frac{l}{4} \\ &= \frac{6wl^2}{130} \end{aligned}$$

$$M_D = \frac{3wl^2}{65}$$



In rigid beam deflection is infinity.

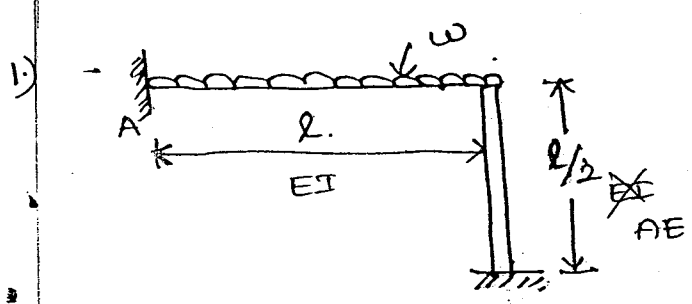
$$y = \frac{Pl^3}{3}$$

$$B.M. @ A = P \times (L+L)$$

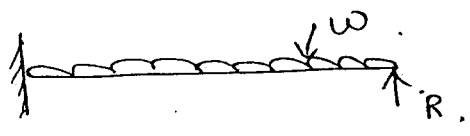
$$= P \times 2L$$

$$B.M @ A = 2PL$$

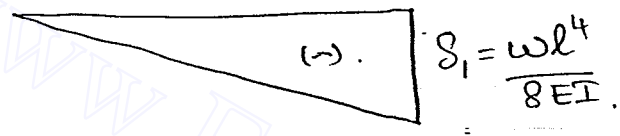
30/10/2019



Determine support reactions and support moment.

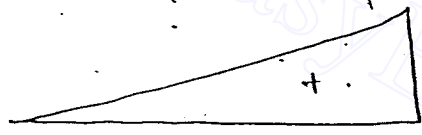


Due to U.D.L



$$\delta_1 = \frac{wl^4}{8EI}$$

Due to point load



$$\delta_2 = \frac{Rl^3}{3EI}$$

$$\begin{aligned} \delta_{net} &= \frac{Rl}{AE} \\ &= \frac{R(l/2)}{AE} \\ \delta_{net} &= \frac{Rl}{2AE} \end{aligned}$$

$$\delta_{net} = \delta_2 - \delta_1$$

$$= -\frac{Rl^3}{3EI} + \frac{wl^4}{8EI}$$

$$-\frac{Rl^3}{3EI} + \frac{wl^4}{8EI} = \frac{Rl}{2AE}$$

$$\frac{Rl^3}{3EI} + \frac{Rl}{2AE} = \frac{wl^4}{8EI}$$

$$\frac{2ARl^3 + 3IRl}{6EAI} = \frac{wl^4}{8EI}$$

$$R \left(\frac{2Al^3 + 3Il}{6EAI} \right) = \frac{wl^4}{8EI}$$

$$R = \frac{wl^4}{8EI} \times \frac{6EAI}{(2Al^3 + 3Il)}$$

$$R = \frac{3A\omega l^3}{4(2Al^2 + 3I)}$$

$$M_A = R \times l - \frac{\omega l^2}{2}$$

$$= \frac{3A\omega l^3 \times l}{4(2Al^2 + 3I)} - \frac{\omega l^2}{2}$$

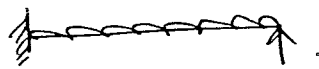
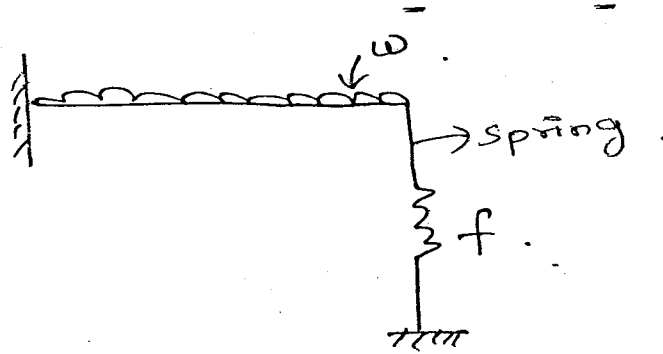
$$= \frac{3A\omega l^4}{4(2Al^2 + 3I)} - \frac{\omega l^2(2Al^2 + 3I)2}{4(2Al^2 + 3I)}$$

$$= \frac{3A\omega l^4 - 4A\omega l^4 - 6\omega l^2 I}{4(2Al^2 + 3I)}$$

$$= \frac{-A\omega l^4 - 6\omega l^2 I}{4(2Al^2 + 3I)}$$

$$M_A = - \left[\frac{A\omega l^4 + 6\omega l^2 I}{4(2Al^2 + 3I)} \right]$$

2. Analysis of a cantilever beam supported by spring @ free end. Flexibility f , stiffness k . Determine reaction:

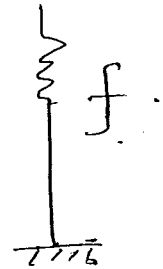


Due to U.D.L

$$\delta_1 = \frac{wl^4}{8EI}$$

Due to point load

$$\delta_2 = \frac{Rl^3}{3EI}$$



$$f = \frac{\text{Force}}{\text{Displacement}}$$

$$\text{Displacement} = f \times R$$

$$\delta_{\text{net}} = f \times R$$

$$\begin{aligned} \delta_{\text{net}} &= \delta_1 - \delta_2 \\ &= \frac{wl^4}{8EI} - \frac{Rl^3}{3EI} \end{aligned}$$

$$\frac{wl^4}{8EI} - \frac{Rl^3}{3EI} = f \times R$$

$$R \left[\frac{l^3}{3EI} + f \right] = \frac{wl^4}{8EI}$$

$$R \left[\frac{l^3 + 3fEI}{3EI} \right] = \frac{wl^4}{8EI}$$

$$R = \frac{wl^4}{8EI} \times \frac{3EI}{(l^3 + 3fEI)}$$

$$R = \frac{\frac{wl^4}{8EI}}{f + \frac{l^3}{3EI}}$$

$$k = \frac{R}{\delta_{net}}$$

$$\delta_{net} = \frac{R}{k}$$

$$\frac{wl^4}{8EI} - \frac{Rl^3}{3EI} = \frac{R}{k}$$

$$\frac{R}{k} + \frac{Rl^3}{3EI} = \frac{wl^4}{8EI}$$

$$R \left[\frac{1}{k} + \frac{l^3}{3EI} \right] = \frac{wl^4}{8EI}$$

$$R \left[\frac{3EI + kl^3}{3kEI} \right] = \frac{wl^4}{8EI}$$

$$R = \frac{wl^4}{8EI} \times \frac{3kEI}{(3EI + kl^3)}$$

$$R = \frac{3kwl^4}{8(3EI + kl^3)}$$

CONJUGATE BEAM METHOD

Conjugate beam is an imaginary beam has same span as the original beam subjected to load equal to $\frac{M}{EI}$ diagram. The support condition of conjugate beam may or may not same as that of the original beam.

There are 2 theorems:

Theorem 1:

The slope @ any section of original beam is equal to the S.F of its conjugate beam @ the corresponding section. ($\theta_{xx} = V_{xx}$)

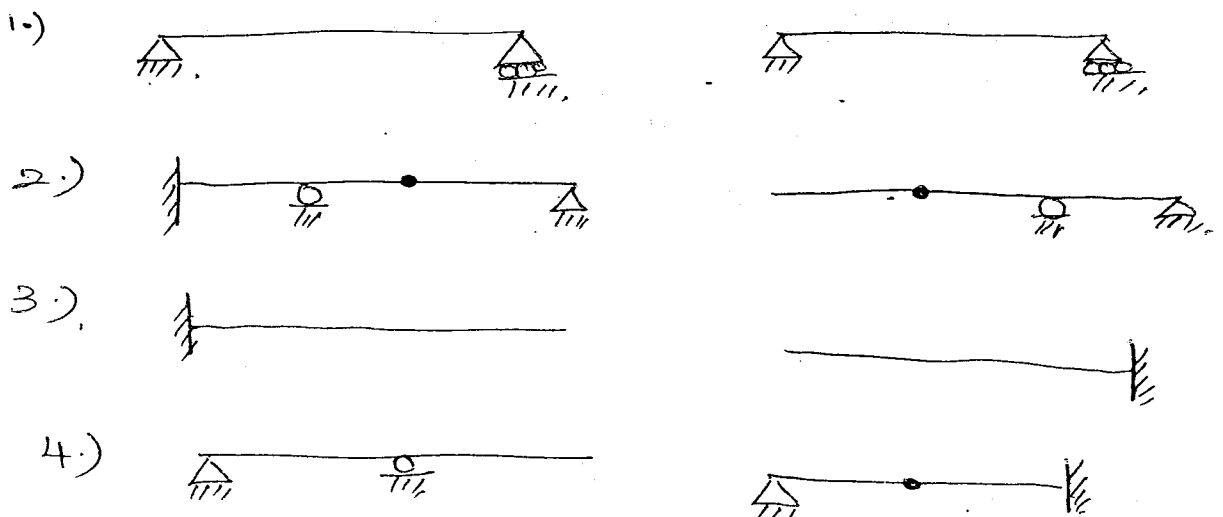
Theorem 2:

The deflection @ any section of an original beam is equal to the B.M of its conjugate beam @ the corresponding section. ($\delta_{xx} = M_{xx}$)

Support conditions:

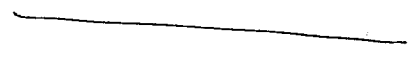
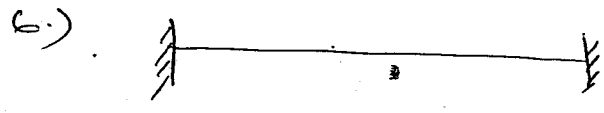
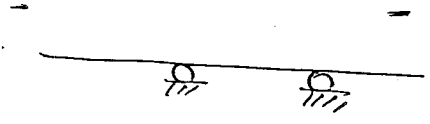
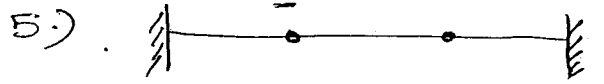
Real beam

Conjugate Beam



Real beam

Conjugate Beam



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STRAIN ENERGY METHOD (Castigliano's Theorem)

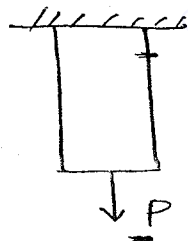
* Energy theorem is based on law of conservation of energy, which states that "External work done on a structure due to displacement = Internal Energy stored"

$$\text{Strain energy} = \text{External work done}$$

$$\text{External work done} = \text{Average Load} \times \text{displacement}$$

$$\text{Strain Energy} = \frac{P}{2} \times \delta l.$$

Strain Energy Due to Axially applied Load:



$$U = EWD.$$

$$EWD = \left(\text{Average Load} \right) \times \text{displacement}$$

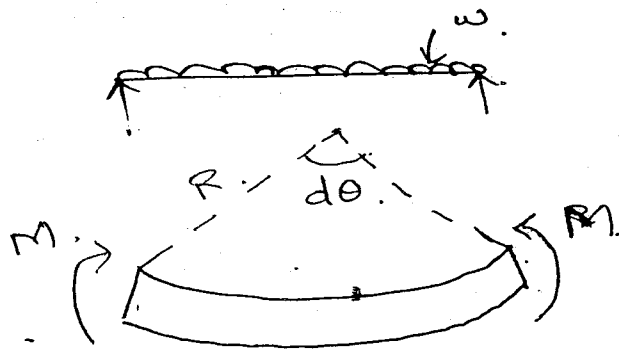
$$= \left(\frac{0+P}{2} \right) \times \delta l.$$

$$= \frac{P}{2} \times \frac{Pl}{AE}.$$

$$= \frac{P^2 l}{2 AE} \times \frac{A}{A}.$$

$$= \left(\frac{P}{A} \right)^2 \times \left(\frac{Al}{2E} \right)$$

$$\text{Strain Energy} = f^2 \times \frac{V}{2E}.$$



$$U = EWD.$$

= Moment \times Rotation

$$du = \left(\frac{0 + M}{2} \right) \times d\theta.$$

$$\boxed{du = \frac{M}{2} d\theta} \quad \text{--- (A)}$$

Bending Theory:

$$\frac{M}{I} = \frac{E}{R}$$

$$\boxed{\frac{1}{R} = \frac{M}{EI}} \quad \text{--- (1)}$$

$$d\theta = \frac{\text{Arc length}}{\text{Radius}} = \frac{ds}{R}$$

$$\boxed{\frac{1}{R} = \frac{d\theta}{ds}} \quad \text{--- (2)}$$

Equating (1) and (2)

$$\boxed{d\theta = \frac{M}{EI} ds} \quad \text{--- (3)}$$

Substitute (3) in (A)

$$du = \frac{M}{2} \times \frac{M}{EI} ds$$

Strain energy due to Bending

$$\boxed{U = \int \frac{M^2}{2EI} ds}$$

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Strain energy due to shear,

$$U = \frac{T^2}{2GA} ds.$$

strain energy due to Torsion,

$$U = \frac{T^2}{2GI} ds.$$

Theorem 1:

The partial derivative of total strain energy of a loaded structure with respect to applied Load is equal to the deflection δ under that Load.

where,

$$\delta = \frac{\partial U}{\partial P}.$$

The partial derivative of total strain energy with respect to applied moment is equal to the slope in the direction of applied moment.

$$\theta = \frac{\partial U}{\partial M}.$$

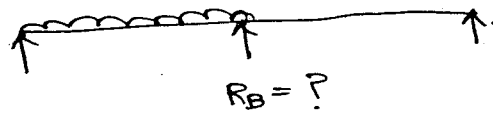
Theorem 2: [Least Energy Theorem].

Total strain energy of a loaded indeterminate structure is the minimum.

The partial derivative of total strain Energy of a loaded indeterminate structure with respect to redundant force is zero.

$$\frac{\partial U}{\partial R} = 0.$$

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$$\frac{\partial U}{\partial R_B} = 0$$

For a bending member G, C and N is given.

$$U_{\text{total}} = \int \frac{M^2}{2EI} ds + \int \frac{F_s^2}{2GA} ds$$

If G is not given.

$$U_{\text{total}} = \int \frac{M^2}{2EI} ds$$

If I is very small and I is neglected.

$$U_{\text{total}} = \int \frac{M^2}{2EI} ds$$

Strain Energy Method

* External work done on a structure due to displacement } Internal Energy Stored.

* Strain energy due to axially applied load

$$U = \frac{f^2}{2E} \times V.$$

* Strain energy due to moment.

$$U = \int \frac{M^2}{2EI} ds.$$

* Strain energy due to shear stress

$$U = \int \frac{\tau^2}{2G\theta} ds$$

* Strain energy due to torsion

$$U = \int \frac{T^2}{2GJ} ds$$

Max Shear Stress

$$U = \frac{\tau_{max}^2}{4G}$$

Hollow circular

Rod.

$$U = \frac{\tau_{max}^2}{4G} \left(\frac{D^2 - d^2}{D^2} \right)$$

$$U_{Spring} = \frac{R^2}{2}$$

* Theorem 1: (i) $\frac{\partial U}{\partial P} = \delta$

Partial derivative of total strain energy of a loaded structure with applied load is equal to deflection under that load.

$$(ii) \frac{\partial U}{\partial M} = \theta$$

Partial derivative of total strain energy of a loaded structure with moment is equal to slope in the direction of applied moment.

* Theorem 2: $\frac{\partial U}{\partial P} = 0$

Partial derivative of total strain energy of a loaded indeterminate structure with redundant force is equal to zero.

* For a given beam, $U_{total} = \int \frac{M^2}{2EI} ds + \int \frac{\tau^2}{2G\theta} ds$

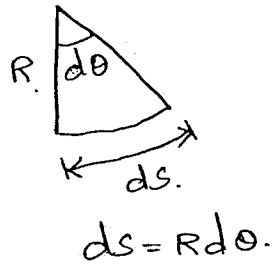
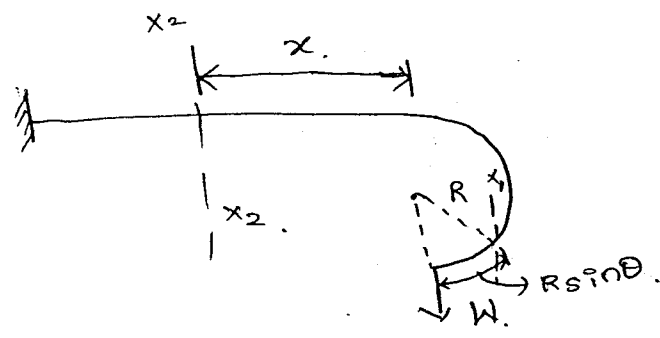
25/11/2015

b)



- find Deflection.

$$\delta = \frac{W^2}{2EI} \left[\frac{R^3 \pi}{2} + \frac{l^3}{3} \right]$$



$$ds = R d\theta$$

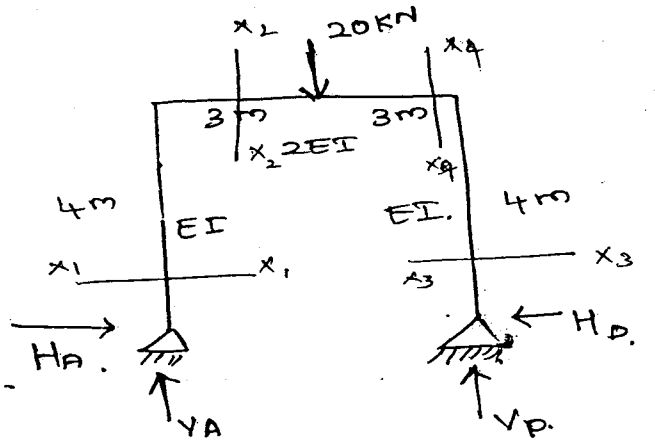
$$M_{xx1} = WR \sin \theta$$

$$M_{xx2} = Wx$$

$$\begin{aligned}
 U &= \int_0^{\pi} \frac{M_{xx1}^2 ds}{2EI} + \int_0^l \frac{M_{xx2}^2 dx}{2EI} \\
 &= \int_0^{\pi} \frac{(WR \sin \theta)^2 R d\theta}{2EI} + \int_0^l \frac{(Wx)^2 dx}{2EI} \\
 &= \int_0^{\pi} \frac{W^2 R^3 \sin^2 \theta d\theta}{2EI} + \int_0^l \frac{W^2 x^2 dx}{2EI} \\
 &= \frac{W^2 R^3}{2EI} \int_0^{\pi} \left(\frac{1 - \cos 2\theta}{2} \right) d\theta + \frac{W^2}{2EI} \int_0^l x^2 dx \\
 &= \frac{W^2 R^3}{4EI} \left[\theta - \frac{\sin 2\theta}{2} \right]_0^{\pi} + \frac{W^2}{2EI} \left[\frac{x^3}{3} \right]_0^l \\
 &= \frac{W^2 R^3}{4EI} \left[\pi - 0 \right] + \frac{W^2}{6EI} l^3 \\
 &= \frac{W^2 R^3 \pi}{4EI} + \frac{W^2 l^3}{6EI} = \frac{W^2}{2EI} \left[\frac{R^3 \pi}{2} + \frac{l^3}{3} \right]
 \end{aligned}$$

2.)

Determine the horizontal Reaction:



$$M_{xx1} = -H_A x x$$

Limit

0 - 4

$$M_{xx2} = (V_A x x) \mp (H_A x 4)$$

0 - 3

$$M_{xx3} = -H_D x x$$

0 - 4

$$M_{xx4} = (V_D x x) - (H_D x 4)$$

0 - 3

1.98

$$\frac{\partial U}{\partial R} = 0$$

$$-\int_0^4 \frac{H_A x x}{2EI} dx + \int_0^3 \frac{V_A x - 4H_A}{4EI} dx \mp \int_0^4 \frac{H_D x x}{2EI} dx \mp$$

$$+ \int_0^3 \frac{V_D x - 4H_D}{4EI} dx = 0$$

$\frac{-H_A}{2EI}$

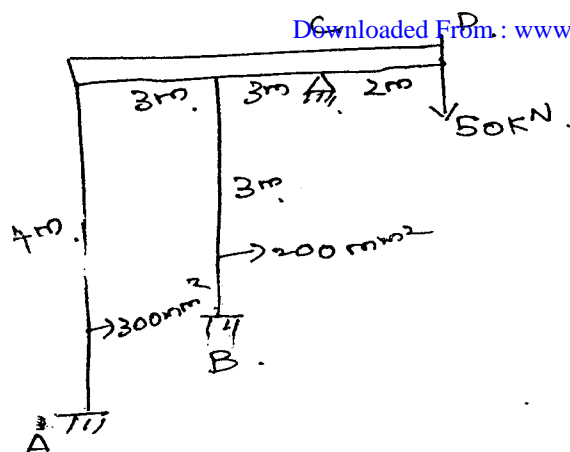
$$\left[\frac{x^2}{2} \right]_0^4 + \frac{1}{4EI} \left[\frac{V_A x^2}{2} - 4H_A x \right]_0^3 \mp \left[\frac{x^2}{2} \right]_0^4 \times \frac{H_D}{2EI}$$

$$+ \frac{1}{4EI} \left[\frac{V_D x^2}{2} - 4H_D x \right]_0^3 = 0$$

$$-\frac{16H_A}{4EI} + \frac{1}{4EI} \left[\frac{9V_A}{2} - 12H_A \right] - \frac{16H_D}{4EI} + \frac{1}{4EI} \left[\frac{9V_D}{2} - 12H_D \right]$$

$$-\frac{32H}{4EI} + \frac{9V_A}{8EI} - \frac{12H_A}{4EI} + \frac{9V_D}{8EI} - \frac{12H_D}{4EI} = 0$$

$$-\frac{56H}{4EI} + \frac{18V_A}{8EI} = 0$$



$$\sum V = 0$$

$$R_A + R_B + R_C = 50$$

$$\sum M_C = 0$$

$$(R_A \times 6) + (R_B \times 3) = -50 \times 2$$

$$6R_A + 3R_B = -100$$

$$R_B = -\left(\frac{100 + 6R_A}{3}\right)$$

$$U_{rod} = U_{RA} + U_{RB}$$

$$= \frac{R_A^2 l}{2AE} + \frac{R_B^2 l}{2AE}$$

$$= \frac{R_A^2 l}{2AE} + \frac{\left(\frac{100 + 6R_A}{3}\right)^2 l}{2AE}$$

$$U = \frac{R_A^2 l}{2AE} + \frac{(100^2 + 1200R_A + 36R_A^2) l}{18AE}$$

$$\frac{\partial U}{\partial R} = 0$$

$$\frac{R_A^2}{2AE} \int_0^4 l dx + \int_0^3 l dx \left[\frac{100^2 + 1200R_A + 36R_A^2}{18AE} \right] = 0$$

$$\frac{R_A^2}{2AE} \left(\frac{l^2}{2}\right)_0^4 + \frac{10000 + 1200R_A + 36R_A^2}{18AE} \left(\frac{l^2}{2}\right)_0^3 = 0$$

$$\frac{16R_A^2}{4AE} + \frac{(10000 + 1200R_A + 36R_A^2) \cdot 9}{36AE} = 0$$

$$2 \times 144 R_A^2 + 3(90000 + 10800 R_A + 324 R_A^2) = 0$$

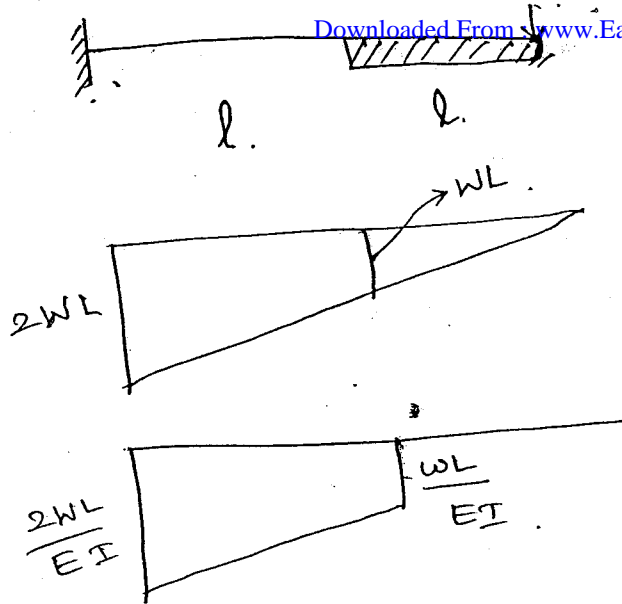
$$468 R_A^2 + 10800 R_A + 90000 = 0$$

$$R_A = -11.53 \text{ kN}$$

$$288 R_A^2 + 270000 + 32400 R_A + 972 R_A^2 = 0$$

$$1260 R_A^2 + 32400 R_A + 270000 = 0$$

4.)



$$A = b \left(\frac{a+b}{2} \right)$$

$$\bar{x} = \frac{b}{3} \left(\frac{a+2b}{a+b} \right)$$

$$\delta_B = A \times \bar{x}$$

$$= \left(\frac{WL}{EI} (2+l) \times \frac{l}{2} \right) \times \left[\frac{l}{3} \cdot \left[\frac{5WL}{EI} \right] \right]$$

$$= \frac{3WL^2}{2EI} \left[\frac{l}{3} \times \frac{5}{3} \right]$$

$$\delta_B = \frac{5WL^3}{6EI}$$

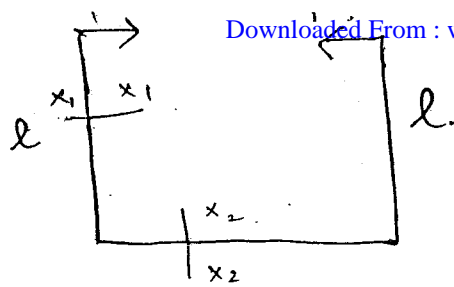
$$\delta_C = \frac{3WL}{EI} \cdot \frac{l}{2} \times \left[\frac{l}{3} \times \frac{5}{3} + l \right]$$

$$= \frac{3WL^2}{2EI} \times \left[\frac{5l}{9} + l \right]$$

$$= \frac{3WL^2}{2EI} \times \frac{14l}{9}$$

$$\delta_C = \frac{7WL^3}{3EI}$$

5.)



$M_{xx1} = Px$

$M_{xx2} = Pl$

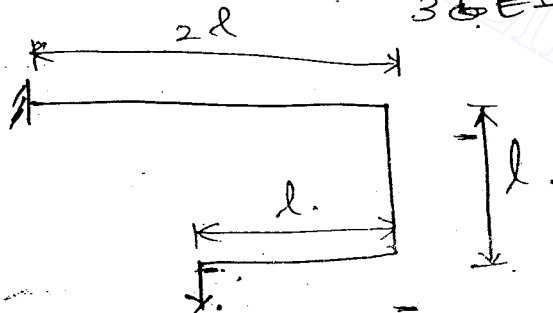
$$\begin{aligned} \delta &= \frac{\partial U}{\partial P} \\ &= 2 \int_0^l \frac{(Px)^2}{2EI} dx + \int_0^l \frac{(Pl)^2}{2EI} dx \\ &= \frac{2}{2} \left[\frac{P^2 x^3}{3EI} \right]_0^l + \left[\frac{P^2 l^2 x}{2EI} \right]_0^l \\ &= \frac{Pl^3}{3EI} + \frac{P^2 l^3}{2EI} \end{aligned}$$

$U = \frac{5P^2 l^3}{6EI}$

$\frac{\partial U}{\partial P} = \frac{10P^2 l^3}{6EI} = \frac{5Pl^3}{3EI}$

$\delta = \frac{5Pl^3}{3EI}$

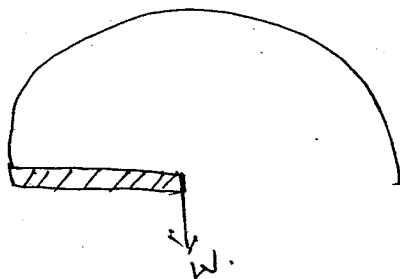
7.)



$\delta_{vertical} = \frac{3}{2} \frac{Ml^2}{EI}$

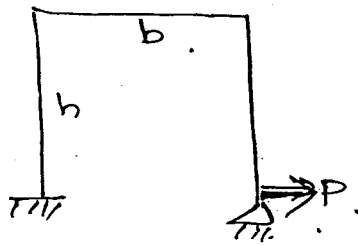
$\theta = \frac{4Ml}{EI}$

8.)



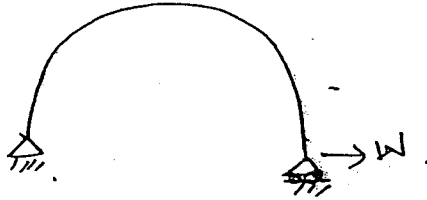
$\delta = \frac{WR^3 \pi}{2EI}$

9.)



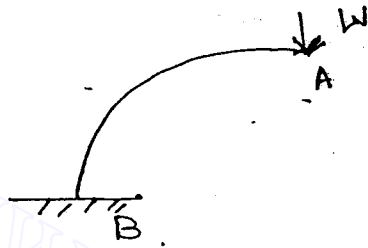
$$\delta_D = \frac{Ph^2(2b+3h)}{3EI}$$

10.)



$$\delta_B = \frac{WR^3\pi}{2EI}$$

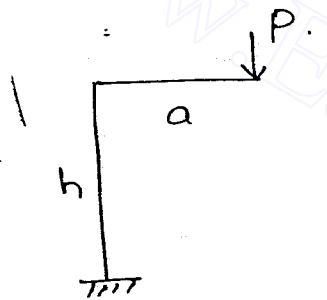
11.)



$$\delta_v @ A = \frac{Wr^3\pi}{4EI}$$

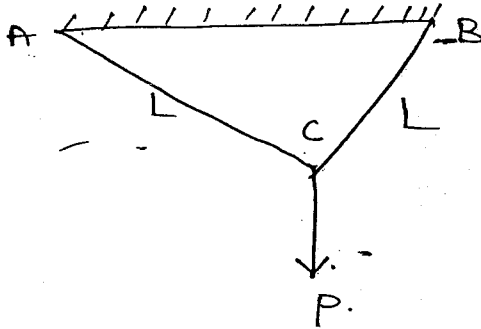
$$\delta_h @ A = \frac{Wr^3}{2EI}$$

12.)



$$\delta_h = \frac{Pab^2}{2EI}$$

13.)



$$\delta_c = \frac{PL}{AE}$$

STRUCTURAL ANALYSIS.

COLUMNS.

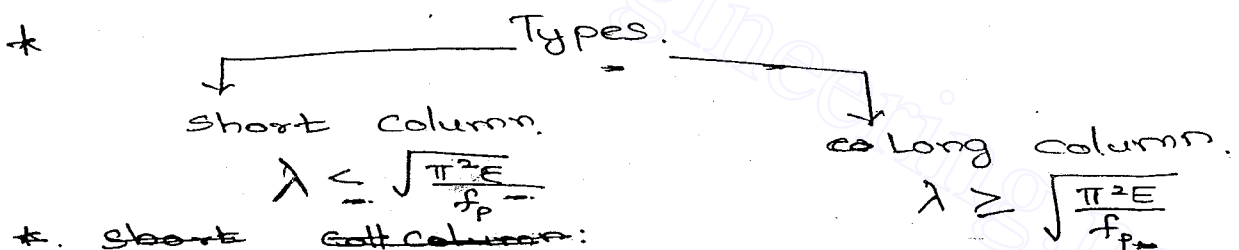
* A structural member which is subjected to axial compressive load and the lateral dimensions are very smaller than the longitudinal dimension then it is called a strut.

* A strut is a compression member.

* A strut which is truly vertical is known as a column or stanchion.

* Following are the examples of compression member are:

strut, column, Boom (cranes), web of an I-section @ a support, web of I-section under point load away from the support, thin narrow depth girder, thin shaft subjected to heavy torsion.



* Short column:

f_p - yield stress.

* Short column
→ Always fails in crushing where actual compressive stress exceeds yield stress.

* Long column.
→ fails in buckling even though actual compressive stress may be less than yield stress.

* Buckling:

The lateral deflection of compression member due to axially applied load.

* A long column may buckle due to following reasons.

- No column is truly vertical.
- No load is purely axial.
- No column is homogenous throughout.
- Some occasional lateral load may occur.

$$r = \sqrt{\frac{I}{A}}$$

$$I = A \times r \times r$$

$$= A r^2$$

$$r = \sqrt{\frac{I}{A}}$$

* If a column section has one effective length; in both the direction, then only r_{min} shall be considered.

where $r_{min} = \sqrt{\frac{I_{min}}{A}}$

* If column has two effective lengths the r_{min} shall not be considered

In case individual radius of gyration shall be considered in either direction and then maximum slenderness ratio may be applied.

Example:

A cantilever provided with two lateral type.

A column having bolted connection at top and bottom:

* The slenderness ratio of left of compression member to the radius of gyration.

$$1.) \lambda = \frac{l_{eff}}{r_{min.}}$$

$$2.) \lambda_x = \frac{l_{ex}}{r_x}, \quad \lambda_y = \frac{l_{ey}}{r_y}$$

Greater value.

* If a compression member is safe in buckling then sure it will be safe in crushing.

But if it is safe in crushing then it may or may not be safe in buckling.

(i.e.) If a column is designed as a long column, then it is safe as a short column also.

∴ In practise every column is designed as a long column.

* The effective length of column is independent on end condition only.

l_{eff} is the distance b/w zero B.M to zero B.M

* There are 2 methods to analysis the buckling load of a column.

1. Euler's theory's

2. Empirical formulae.

* There are 5 Empirical

* There are 5 Empirical Formulae.

Rankine formula.

ISI Formula. (or) (Merchant Rankine)

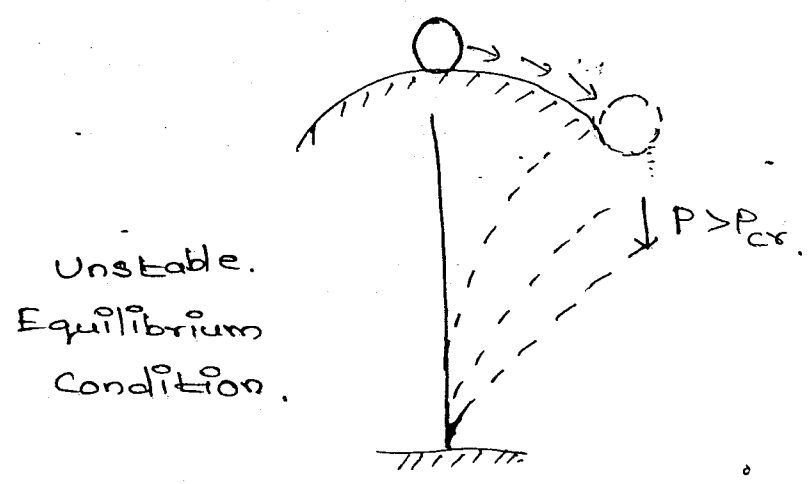
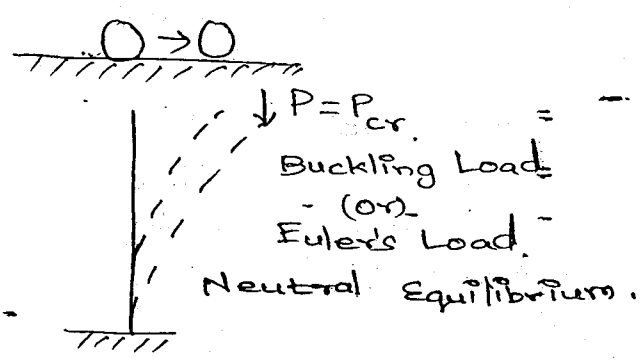
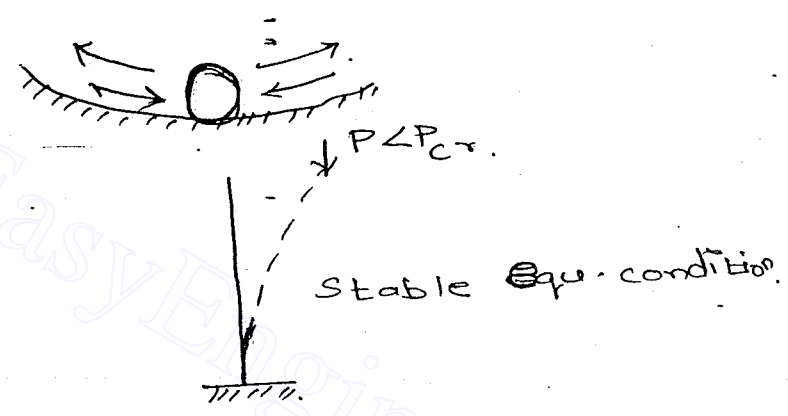
Prof. Perry (secant formula)



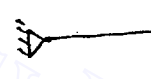

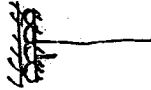
Straight Line formula

Parabolic formula

* Critical Load (or) Buckling Load (or) crippling

Load is the load applied on a column @ neutral equilibrium condition.



<p>End Condition</p>	<p>Both end hinged. (held in position) and not restrained against rotation.</p>		<p>$l_{eff} = l$</p>	
	<p>Both end fixed. (held in position) and restrained against rotation.</p>		<p>$l_{eff} = \frac{l}{2}$</p>	
	<p>One end is fixed other end hinged. (Both ends held in position one end restrained against rotation)</p>		<p>$l_{eff} = \frac{l}{\sqrt{2}}$</p>	
	<p>One end is fixed other end free. (one end held in position and other restrained against rotation and other end not held in position but restrained against rotation)</p>		<p>$l_{eff} = 2l$</p>	
	<p>One end fixed other end is guided roller. (one end held in position and restrained against rotation and other end not held in position but restrained against rotation)</p>			

Diagram

l_{eff}

Column

Subjected to axial and lateral dimension is smaller than length

Column or struts are vertical strut

Short
 $\lambda < 3$

Short
 $\lambda < 12$
 $\lambda < \sqrt{\frac{12E}{f_y}}$

Long
 $\lambda \geq 12$
 $\lambda > \sqrt{\frac{12E}{f_y}}$

Radius of gyration

$$r = \sqrt{\frac{I}{A}}$$

Case (i) only one left

$$\lambda = \sqrt{\frac{I_{min}}{A}}$$

Case (ii) more than one left

$$\lambda_x = \sqrt{\frac{I_{xx}}{A}} \quad \lambda_y = \sqrt{\frac{I_{yy}}{A}}$$


$\lambda = \text{greater of } \lambda_x \text{ (or) } \lambda_y$


Left depends on end condition


All columns are designed as long column in general
Methods to design


Assumptions

- * long column
- * fails by buckling
- * Initially straight
- * Axial load applied
- * loaded upto elastic limit
- * Both ends are pin end

Case (i)  $l_e = l \quad P_c = \frac{\pi^2 EI}{l^2}$

Case (ii)  $l_e = \frac{l}{2} \quad P_c = \frac{4\pi^2 EI}{l^2}$

Case (iii)  $l_e = \frac{l}{2} \quad P_c = \frac{4\pi^2 EI}{l^2}$

Case (iv)  $l_e = \frac{l}{2} \quad P_c = \frac{4\pi^2 EI}{l^2}$

Critical (or) crippling (or) crushing load: load applied @

Neutral equilibrium condition

Stable

Neutral

Unstable

CORE (OR) KERNEL OF THE SECTION:

* Core is the central zone of column.

where P Load is applied then tension doesnot takes place anywhere in the column.

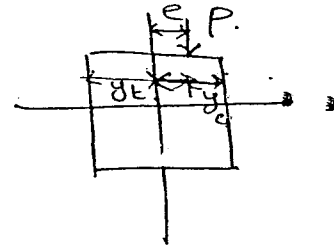
$$-\frac{P}{A} - \frac{M}{Z} = 0$$

$$\frac{P}{A} - \frac{Pe}{Z} = 0$$

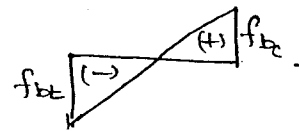
$$P \left(\frac{P}{A} - \frac{e}{Z} \right) = 0$$

$$\frac{1}{A} = \frac{e}{Z} y$$

$$e = \frac{I}{Ay}$$



$$f = \frac{P}{A} \quad +$$



$$\frac{P}{A} - \frac{M}{Z} \quad \frac{P}{A} + \frac{M}{Z}$$

Solid circular.

$$e = \frac{I}{Ay_t} = \frac{\frac{\pi}{64} D^4}{\frac{\pi D^2}{4} \times \frac{D}{2}} = \frac{D}{8} \quad e = \frac{D}{8}$$

$$\text{DIA OF CORE} = \frac{D}{4}$$

Hollow circular.

$$e = \frac{I}{Ay_t} = \frac{\frac{\pi}{64} (D^4 - d^4)}{\frac{\pi}{4} (D^2 - d^2) \times \frac{D}{2}} = \frac{D^2 + d^2}{8}$$

$$\text{DIA OF CORE} = \frac{D^2 + d^2}{4}$$

Square.

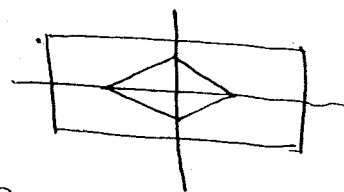
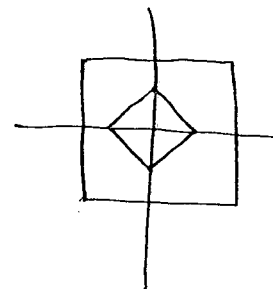
$$e = \frac{I}{Ay_t} = \frac{\frac{b^4}{12}}{b^2 \times \frac{b}{2}} = \frac{b}{6}$$

$$\text{DIA OF CORE} = \frac{b}{3}$$

Rectangle.

$$e_x = \frac{d}{6} \quad e_y = \frac{b}{6}$$

$$\text{DIA OF CORE along } x = \frac{d}{3} \quad \text{along } y = \frac{b}{3}$$



$$\frac{\pi}{8} \frac{(D^2 + d^2)(D^2 - d^2)}{b^4} \frac{(a^2 - b^2)}{(a^2 + b^2)} = \frac{\pi}{4} \frac{D^2 - d^2}{D} \frac{D}{R}$$

$$\frac{D^2 + d^2}{8}$$

$$\frac{e_y}{r} - \frac{e_y}{\frac{I_{xx}}{DA}} = 0$$

$$e_y = \frac{2I_{xx}}{A \cdot D}$$

$$e_x = \frac{2I_{yy}}{A b_f}$$

Core (or) kernel of the section

Radius $e = \frac{r}{A y}$

Solid circle

$$e = d/8$$

$$D_{ia} = d/4$$

Hollow circle

$$e = \frac{D^2 - d^2}{8D}$$

$$e = \frac{D^2 - d^2}{4D}$$

Square

$$e = \frac{b}{6}$$

$$e = b/3$$

Rectangle

$$e_y = \frac{b}{6}$$

$$e_x = \frac{d}{6}$$

$$d_{ia} = \frac{b}{3}, \frac{d}{3}$$

I

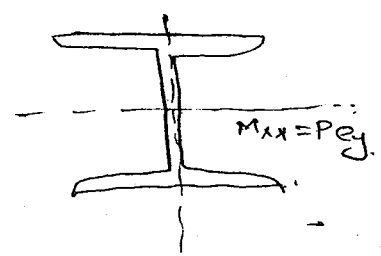
$$e_x = \frac{I_{yy}}{A b_f}$$

$$e_y = \frac{I_{xx}}{A D}$$

$$\frac{P}{A} - \frac{M}{Z} = 0$$

$$\frac{P}{A} - \frac{Pe}{\left(\frac{I}{y}\right)} = 0$$

$$\frac{P}{A} = \frac{Pe_y}{\frac{I_{xx}}{D/2}} = 0$$



$$e_y = \frac{2I_{xx}}{AD}$$

$$e_x = \frac{2I_{yy}}{Ab_f}$$

RANKINE FORMULA (EMPIRICAL FORMULA)

$$\frac{1}{P_R} = \frac{1}{P_c} + \frac{1}{P_E} = \frac{P_c + P_c}{P_c P_E}$$

$$P_R = \frac{P_c P_E}{P_c + P_c}$$

$$= \frac{P_c}{\frac{P_c}{P_E} + \frac{P_c}{P_E}}$$

$$= \frac{f_c A}{1 + f_c A \cdot \frac{\pi^2 EI}{l_{eff}^2}}$$

$$= \frac{f_c A}{1 + f_c A \cdot \frac{\pi^2 EA r^2}{l_{eff}^2}}$$

$$= \frac{f_c A}{1 + f_c \left(\frac{l_{eff}}{r}\right)^2 \frac{\pi^2 E}{\pi^2 E}}$$

$$= \frac{f_c A}{1 + \frac{f_c}{\pi^2 E} \lambda}$$

$$= \frac{f_c A}{1 + \alpha \lambda}$$

$$P_R = \frac{f_c A}{1 + \alpha \lambda}$$

α - Rankine's constant.

$$\alpha = \frac{f_c}{\pi^2 E}$$

Assumptions for Euler's Theory:

- * column is a large long column.
- * column fails only due to buckling.
- * column is subjected concentric load.
- * column is initially straight (No initial curvature)
- * column is loaded upto limit of proportionality.
- * Both ends of column are pinned (Ideal condition):

case (i) Both ends are pinned:

$$P_{\text{euler}} = \frac{\pi^2 EI}{l_{\text{eff}}^2} = \frac{\pi^2 EI}{l^2}$$

case (ii) Both ends are fixed.

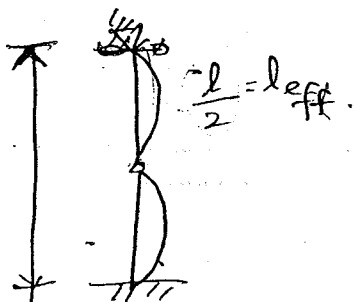
$$P_{\text{euler}} = \frac{4\pi^2 EI}{l^2}$$

case (iii) one end fixed other end pinned.

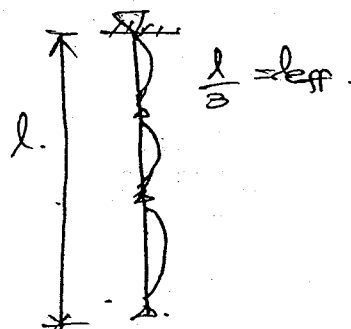
$$P_{\text{euler}} = \frac{2\pi^2 EI}{l^2}$$

case (iv) one end fixed other end free.

$$P_{\text{euler}} = \frac{\pi^2 EI}{4l^2}$$



$$P_{\text{euler}} = \frac{4\pi^2 EI}{l^2}$$



$$P_{\text{euler}} = \frac{9\pi^2 EI}{l^2}$$

CONCRETE STRUCTURES

→ Concrete Technology

- * Properties of concrete
- * Basics of mix design.

→ Concrete Design :

- * Basic working stresses and limit state design concepts
- * Analysis of ultimate load capacity
- * Design of members subjected to flexure, shear, compression and torsion by limit state method.

→ Prestressed concrete

- * Basic elements of prestressed concrete
- * Analysis of beam sections at transfer and service load.

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REINFORCED CONCRETE STRUCTURE

- 1) WSM - Beams
- (i) singly reinforced beam
 - (ii) doubly reinforced beam
 - (iii) T-beam

- 2) LSM - Beams
- (i) singly reinforced beam
 - (ii) doubly reinforced beam
 - (iii) T-beams.

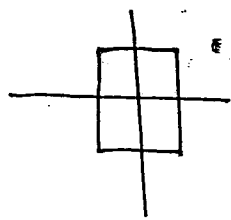
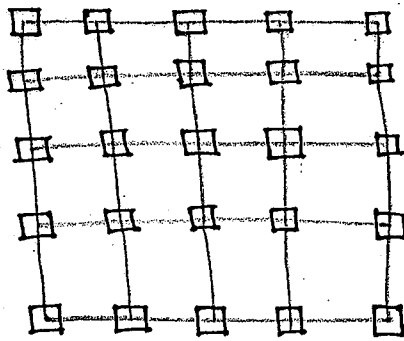
3) shear bond and Torsion.

4) Columns

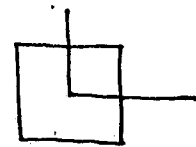
5) Foundation.

* Beam is a flexural member (or) bending member.

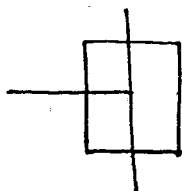
* slab is also a flexural member.



- Axial bending



- Biaxial bending



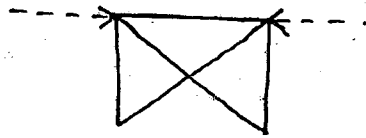
- Uniaxial bending

Father of Cement : Joseph Aspidin

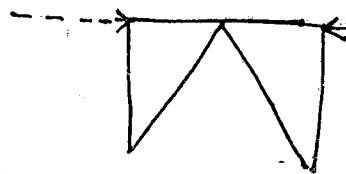
f_{ck} - characteristics compressive strength in concrete.

Base Isolation technique : To prevent from seismic forces

Bracings :



X - Bracing



K - Bracing

Beam \rightarrow steel failure \rightarrow spalling of concrete

Minimum % of Reinforcement :

slab - 0.12% of A_g for Fe 415
0.15% of A_g for Fe 250

Beam -
$$P_t = \frac{0.85 b d}{f_y}$$

column - 0.8% of A_g

Maximum % of Reinforcement

Beam - 4% of A_g in compression

4% of A_g in tension

column - 4% of A_g (with overlapping)

6% of A_g (without overlapping)

RCC:

* RCC stands for Reinforced Cement Concrete.

* Concrete is a mixture of cement, fine aggregate, coarse aggregate, and admixture and water in a pre-determined proportion.

* Concrete is very strong in compression and weak in tension.

* Tensile strength of concrete } = $\frac{1}{10} \times$ compressive strength

* Concrete is designated by M_{25}, M_{15}, M_{20} etc., where

M → Design mix

20 → f_{ck} @ 28 days.

* f_{ck} test result of sample shall not be less than 5%.

* f_{ck} @ 7 days = $\frac{2}{3} \times f_{ck}$ @ 28 days.

* f_{ck} @ 365 days = $(1.1 - 1.2) \times f_{ck}$ @ 28 days.

* Cracking (or) Bending tensile strength of concrete } $f_{cr} = 0.7 \sqrt{f_{ck}}$

In PSC,

$E_c = 5700 \sqrt{f_{ck}}$ (According to IS 1343-19)

$E_c = 5000 \sqrt{f_{ck}}$ (According to IS 456-2)

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Two major classification of RC

section:

- * Cracked section
- * Uncracked section.

Cracked section:

RC section where the cracks are permitted or in other words the crack does not affect the structure.

Eg: Beam, column, wall, slab etc.

Uncracked section:

RC sections where hair line cracks are not permitted is called as uncracked section.

Eg: water tank, retaining wall and chimney (storage structures)

- * Maximum permissible stress in concrete in bending compression.

$$\sigma_{cbc} = \frac{1}{3} f_{ck}$$

- * Maximum permissible stress in concrete in direct compression.

$$\sigma_{cc} = \frac{1}{4} f_{ck}$$

Grade of Steel

$$\sigma_{st} = \frac{f_y}{1.8} = 0.55 f_y$$

 σ_{sc} (N/mm²)(N/mm²)

(factor of Safety of steel) = 1.8

* Mild steel	140 ($\phi \leq 20\text{mm}$)	130
Fe 250		
Plain steel	130 ($\phi > 20\text{mm}$)	
Grade-I steel		
* HYSD		
Fe 415	230	190
TORSION STEEL		
* HYSD		
Fe 500		
TORSION STEEL	275	190

→ Bond strength of HYSD bars is 60% more than the plain bars.

Neutral Axis:

When a beam is subjected to a transverse load there will be bending compression @ top and bending tension @ bottom fibre and at the point where there is zero stress (Neither bending ^{tension} (nor) Bending compression) is called neutral axis.

* In uncracked section the tension and compression acts all over the section.

Hence whole depth of the section is considered.

N.A. lies @ the C.G. of the section.

* In the cracked section compression is taken by concrete and tension is taken by steel alone. concrete in tension side is neglected, and then N.A. is derived.

* Code book

IS 3370	water tank.
IS 1343.	Prestressed concrete
IS 1893.	seismic Load.
IS 13920	Ductility Load.

* If the actual bending tensile stress developed in concrete due to applied bending moment is less than f_{cr} the section is called uncracked section.

$$f_{bt} \leq f_{cr}$$

$$f_{bt} = \frac{M}{I} \times y_t \leq f_{cr}$$

f_{bt} - Bending tension.

* Maximum permissible B.M in concrete based on f_{cr} for is called cracking moment. (i.e) beyond this B.M beam cracks.

As per IS 456 : 2000 there are 3 grades of concrete.

* ordinary - M_{10} , M_{15} , M_{20}

* standard - M_{25} , M_{30} , M_{35} , M_{40} , M_{45}
 M_{50} , M_{55}

* High quality - M_{60} - M_{80}

For concrete grade higher than M_{20} the mix proportion is based on mix design rule.

where,

Mean target strength, $f_t = f_{cm} = f_{ck} + k s$

As per IS 10262 - guidelines for concrete mix design:

where,

k → probability factor (or) tolerance factor. which depends upon percentage of error permitted in concrete.

* If error increases value of "k" decreases.

According to IS 456 - 2000 for 5% of

error the value of k is 1.65 -

s - standard deviation.

Depending upon concrete grade σ_c is in the range b/w 3.5 and 5.

Concrete grade	standard deviation
M ₁₀ , M ₁₅ .	3.5 N/mm ²
M ₂₀ , M ₂₅ .	4 N/mm ²
M ₃₀ , M ₃₅ .	5 N/mm ²

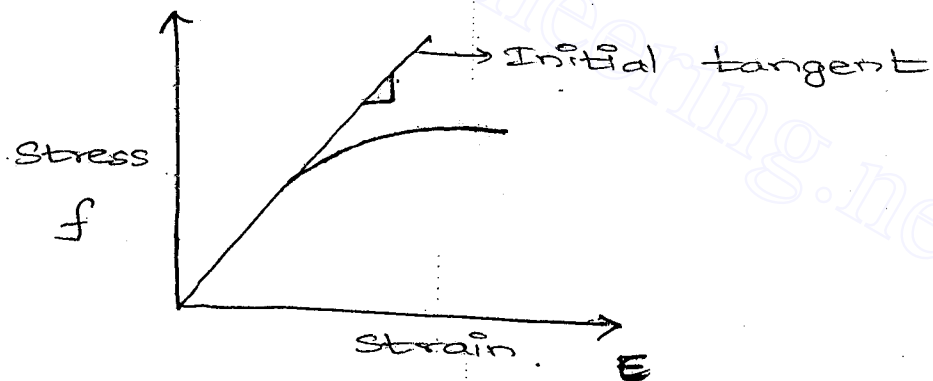
There are 3 methods to determine modulus of elasticity of concrete.

* Initial tangent Method

* Tangent method

* Secant method.

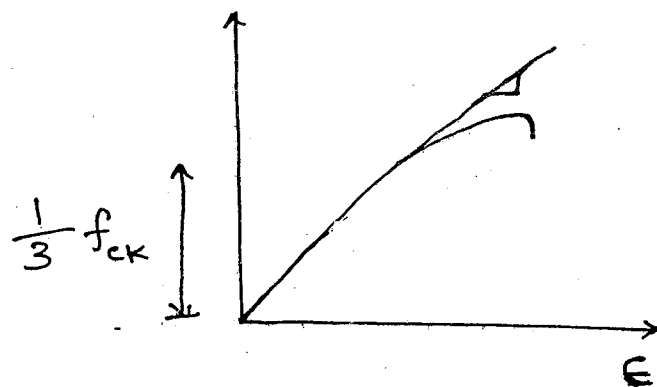
Initial Tangent Method:



Slope of initial tangent

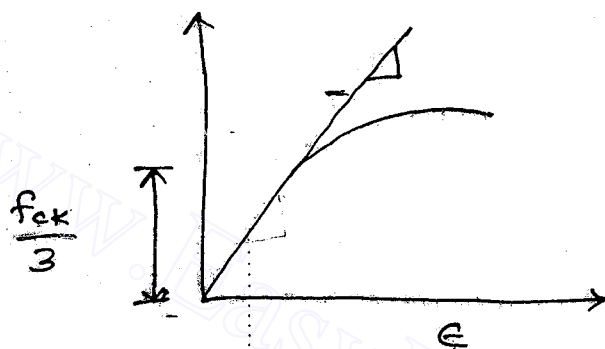
$$\tan \theta = \frac{\Delta f}{\Delta E}$$

Tangent Method:



$$\left. \begin{array}{l} \text{Slope of} \\ \text{tangent} \end{array} \right\} = \frac{\Delta f}{\Delta \epsilon}$$

Secant Method:



$$\begin{aligned} \text{Slope of straight line} \\ = \frac{\Delta f}{\Delta \epsilon} \end{aligned}$$

$$E_c = 5000 \sqrt{f_{ck}}$$

where,

E_c - Modulus of elasticity of concrete
(or)

short term elastic Modulus.

(or)

short term static Young's modulus

* E_c is arrived from secant method

* Long term (or) effective Modulus of elasticity of concrete due to creep - effect

$$E_{eff} = \frac{E_c}{1+\theta}$$

θ - creep coefficient

Modular Ratio (m):

* It is defined as the ratio of modulus of elasticity of steel to that of modulus of elasticity of concrete.

* Its symbol is m.

* It is very important to convert steel area into equivalent concrete area in WSM for analysis purpose.

* $m = \frac{E_s}{E_c}$ is not considered in RCC analysis based on WSM because Young's modulus of concrete does not take creep and shrinkage into account.

* Above value is used in pre-stress only where losses due to creep and shrinkage are considered separately.

* For RCC based on WSM the modular ratio is given by,

$$m = \frac{280}{3\sigma_{cbc}}$$

* Above formula partially takes into account long term effects such as creep.

* According to IS: 456-2000 the minimum clear cover for steel bar shall be as below.

Type of Exposure	PCC grade	RCC grade	Minimum clear cover
Mild	M ₁₀	M ₂₀	20 mm
Moderate	M ₁₅	M ₂₅	30 mm
Severe	M ₂₀	M ₃₀	45 mm
Very severe	M ₂₀	M ₃₅	50 mm
Extreme	M ₂₅	M ₄₀	75 mm

Exposure : As per IS 456-2000 cl-8.2.2 and 35.3.2 (Table -3)

Mild Exposure:

structure protected from rain, heat, etc., (weather condition)

For eg: Beams, slabs.

Moderate Exposure:

structure exposed to rain, heat etc.,

* structure buried in non-aggressive soil and water.

Severe Exposure:

* structure exposed to severe rain

* under sea structure.

* surface exposed to alternate wetting and drying.

* Exposed to coastal environment.

Very Severe Exposure:

- * Concrete surface exposed to sea spray.
- * Concrete to be buried or in contact with aggressive sub soil / ground water.

Ex: coastal areas.

Extreme Exposure:

- * Surface of members in tidal zone.
- * Members in direct contact with solid / liquid aggressive chemicals.

Minimum clear cover for structural units:

For slabs. = 20mm

For Beams = 25 mm

For column = 25mm, 40mm
(≤ 200 mm) (> 200 mm)

For Foundation = 50 mm

For Retaining wall = 40 - 50 mm

Quantity of cement.

Exposure Types	Qty of cement		W/C ratio	
	PCC	RCC	PCC	RCC
Mild	220kg	300kg	0.6	0.55
Moderate	240 kg	300kg	0.6	0.5
Severe	250kg	320kg	0.5	0.45
Very Severe	260 kg	340kg	0.45	0.45
Extreme	280kg	360 kg	0.4	0.4

To avoid corrosion of steel bars: | (1)

- (i) Adequate concrete cover.
- (ii) More quantity of cement.
- (iii) Maximum compact of concrete to reduce permeability.
- (iv) Coating of steel bars.
- (v) cathodic protection.

Dia of steel bars: -

6, 8, 10, 12, 14, 16, 18, 20, 22, 25, 28, 32, 36, 40, 45, 50

Minimum dia of column - 12mm

- * Isolated footing same as strays
- * Combined footing same as beam
- * Retaining wall same as slab

Analysis Of Section By WSM

Statement :

* Analysis of uncracked section and cracked section based on WSM are based on the principle of strain compatibility.

* Due to good bond b/w concrete and steel (i.e) the strain developed in concrete in the tension zone in the vicinity of tension steel is equal to the strain developed in the tension steel.

Area equivalent stress

$$f_{ce} E_c = E_s$$

$$\frac{f_c}{E_c} = \frac{f_s}{E_s}$$

$$f_c = \frac{f_s}{(E_s/E_c)}$$

$$f_c = \frac{f_s}{m}$$

$$\sigma_c = \frac{\sigma_{st}}{m}$$

Area equivalent concrete

$$F_c = F_s$$

$$f_c A_c = f_s A_{st}$$

$$E_c/E_s \times E_c \times A_c$$

$$= E_s \times E_s \times A_c$$

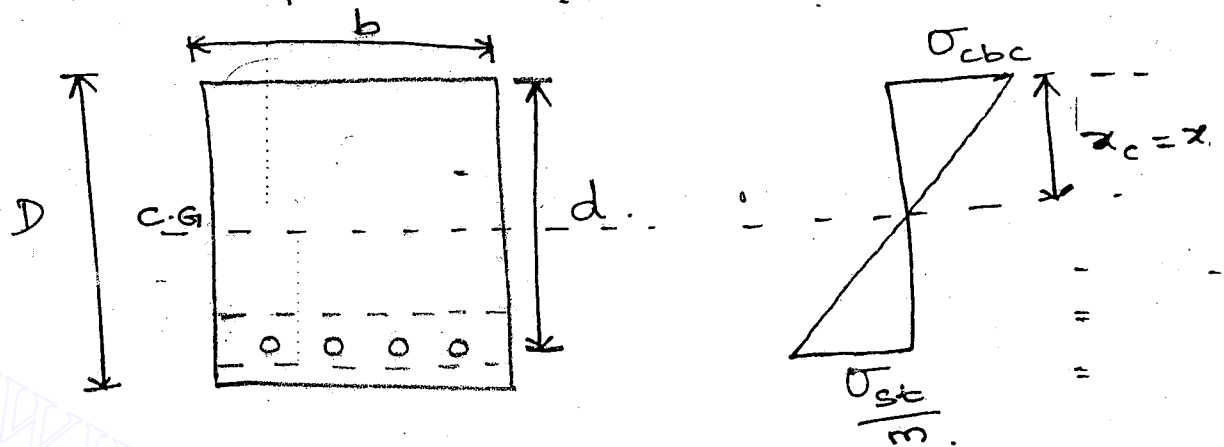
$$A_c = \frac{E_s}{E_c} \times A_{st}$$

$$A_c = m A_{st}$$

Analysis Of Uncracked Section (1)

* Neutral Axis of uncracked section lies at C.G. of the section.

Analysis of Cracked Section:



Depth of actual neutral axis:

Area of \times C.G. of Comp = Area of \times C.G. of tensi

$$b \times x_a \times \frac{x_a}{2} = m \times A_{st} \times (d - x_a)$$

$$\boxed{\frac{b x_a^2}{2} = m A_{st} (d - x_a)}$$

Depth of critical N.A :

From stress - block diagram, by similar triangle method.

$$\frac{\sigma_{cbc}}{(\sigma_{st}/m)} = \frac{x_a}{(d - x_a)} \quad \boxed{x_a = x_c}$$

$$\sigma_{cbc} (d - x_c) = x_c \frac{\sigma_{st}}{m}$$

$$\sigma_{cbc} d - \sigma_{cbc} x_c = x_c \frac{\sigma_{st}}{m}$$

$$m \sigma_{cbc} d - m \sigma_{cbc} x_c = x_c \frac{\sigma_{st}}{m}$$

$$m \sigma_{cbc} d = x_c (m \sigma_{cbc} + \sigma_{st})$$

$$x_c = \frac{m \sigma_{cbc} d + \sigma_{st}}{m \sigma_{cbc} d}$$

$$x_c = \frac{m \sigma_{cbc} d}{m \sigma_{cbc} d + \sigma_{st}}$$

$$x_c = k \cdot d$$

$$k = \frac{m \sigma_{cbc}}{m \sigma_{cbc} + \sigma_{st}}$$

$$= \frac{280}{30} \times \sigma_{cbc}$$

$$= \frac{280}{30} \times \sigma_{cbc} + \sigma_{st}$$

$$= \frac{280}{3} \times \frac{\sigma_{cbc}}{280 + 3 \sigma_{st}}$$

$$k = \frac{280}{280 + 3 \sigma_{st}}$$

Compressive stress:

$$F_c = \text{stress} \times \text{Area}$$

$$= \frac{0 + \sigma_{cbc}}{2} \times b \times x_c$$

$$= \frac{\sigma_{cbc} b x_c}{2}$$

$$F_c = \frac{1}{2} c b x_c$$

Tensile stress:

$$F_t = \frac{\sigma_{st}}{\gamma} \times A_{st}$$

$$F_t = t A_{st}$$

Lever Arm:

$$z = d - \frac{x_a}{3}$$

$$x_a = x_c$$

$$= d - \frac{x_c}{3}$$

$$= d - \frac{kd}{3}$$

$$= d \left[1 - \frac{k}{3} \right]$$

$$z = d_j$$

$$z = d_j$$

Moment of Resistance (Comp):

$$M = F \times L.A.$$

$$= \frac{c b x_a}{2} \left(d - \frac{x_a}{3} \right)$$

$$= \frac{c b x_c}{2} \left(d - \frac{x_c}{3} \right)$$

$$= \frac{c b k d}{2} \left(d - \frac{k d}{3} \right)$$

$$= \frac{c b k d^2}{2} \left(1 - \frac{k}{3} \right)$$

$$= \frac{c k j}{2} b d^2$$

$$M = Q b d^2$$

Moment of Resistance: (Tension)

$$\begin{aligned}
 M &= \text{Force} \times \text{Lever Arm} \\
 &= t \times A_{st} \times \left(d - \frac{x_a}{3} \right) \\
 x_a &= x_c \\
 &= t \times A_{st} \left(d - \frac{x_c}{3} \right) \\
 &= t \times A_{st} \left(d - \frac{kd}{3} \right) \\
 &= t A_{st} d \left(1 - k/3 \right) \\
 &= t A_{st} d_j
 \end{aligned}$$

This is the reason why depth is always greater than its breadth.

Note: (cracking) tensile stress
(Repln)

$$f_{cr} = 0.7 \sqrt{f_{ck}}$$

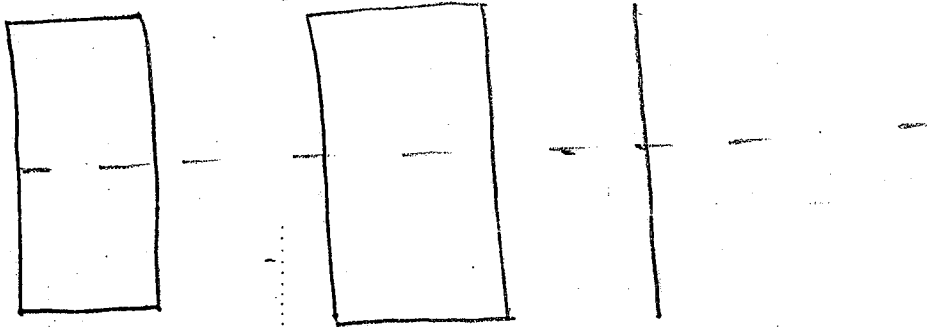
$$\begin{aligned}
 \text{cracking tensile strain} &= \frac{f_{cr}}{E_c} \\
 &= \frac{0.7 \sqrt{f_{ck}}}{5000 \sqrt{f_{ck}}}
 \end{aligned}$$

$$\text{strain in steel} = 1.04 \times 10^{-4}$$

$$\begin{aligned}
 \text{stress in steel} &= E \times \epsilon \\
 &= 1.04 \times 10^{-4} \times 2 \times 10^5 \\
 &= 20.8 \text{ N/mm}^2
 \end{aligned}$$

Types of section:

- * Under reinforced section.
- * Balanced section
- * Over reinforced section.



Balanced section.

- * $A_{st \text{ required}} = A_{st \text{ provided}}$
- * $x_a = x_c$
- * x_a and x_c lies on N.A.
- * Failure is balanced one.
- * $C = \sigma_{cbc}$ $t = \sigma_{st}$

URS:

- * $A_{st \text{ pro}} < A_{st \text{ required}}$
- * $x_a < x_c$ x_a lies below
- * Ductile failure.

* $A_{st\ pro} > A_{st\ requir}$.

* $x_a > x_c$ x_a lies below x_c

+ Brittle failure.

1. Given data:

$M_{external} = 60\text{ KN}\cdot\text{m}$.

M_{20} , Fe 415. $b = ?$ $d = ?$

$M = Q b d^2$.

$Q = \frac{C j k}{2}$

$C = \sigma_{cbc} = \frac{f_{ck}}{3} = \frac{20}{3}$

$C = 7$

$k = \frac{280}{280 + 3 \sigma_{st}}$
 $= \frac{280}{280 + 3(230)}$

$k = 0.28$

$j = 1 - \frac{k}{3}$
 $= 1 - \frac{0.28}{3}$

$j = 0.906$

$Q = \frac{7 \times 0.28 \times 0.906}{2}$

$60 \times 10^6 = \frac{7 \times 0.28 \times 0.906}{2} \times \frac{d}{2} \times d^2$

$d = 513\text{ mm}$

$b = \frac{d}{2}$ $b =$

$$D = 513 + 20 + 16/2$$

$$= 541$$

$$\boxed{D = 550 \text{ mm}}$$

$$d = 550 - 20 - \frac{16}{2}$$

$$\boxed{d = 522 \text{ mm}}$$

$$M.R = t A_{st} \left(d - \frac{x_u}{3} \right)$$

$$= t A_{st} d \left(1 - \frac{k}{3} \right)$$

$$60 \times 10^6 = 230 \times A_{st} \times 513 \times \left(1 - \frac{0.28}{3} \right)$$

$$x_u \approx A_{st} = 560 \text{ mm}^2$$

$$\text{No. of bars} = \frac{560}{\frac{\pi \times 16^2}{4}}$$

$$= 3 \text{ nos.}$$

* pure B. M is there,
Shear force = zero.

∴ provide 3 # 16mm ϕ bar.

Min % of reinforcement.

$$= \frac{0.85 b d}{f_y}$$

$$= \frac{0.85 \times \frac{522}{2} \times 522}{415}$$

$$= 280 \text{ mm}$$

Max % of reinforcement

$$= 4\% \text{ of } A_g$$

$$= \frac{4}{100} \times b \times D$$

$$= \frac{4}{100} \times \frac{522}{2} \times 550$$

$$A_{st} = 5742 \text{ mm}^2$$

2) Given data :

$$b = 230 \text{ mm} \quad \text{and} \quad D = 500 \text{ mm}$$

$$M_{20}, \quad Fe 415$$

$$M.R = Q b d^2$$

$$k = \frac{280}{280 + 3 \sigma_{st}}$$

$$Q = \frac{C j k}{2}$$

$$= \frac{7 \times 0.28 \times 0.906}{2}$$

$$k = 0.28$$

$$j = 1 - \frac{k}{3}$$

$$= 1 - \frac{0.28}{3}$$

$$j = 0.906$$

$$M.R = \frac{7 \times 0.28 \times 0.906}{2} \times 230 \times 472$$

$$M = 45.49 \text{ kN.m}$$

$$D = 550 - 2 \times 25$$

$$= 472 \text{ mm}$$

$$\frac{b x_a^2}{2} = m A_{st} (d - x_a)$$

$$m = t A_{st} d j$$

$$A_{st} = 603 \text{ mm}^2$$

$$b \cdot \frac{x_a^2}{2} = m A_{st} (d - x_a)$$

$$x_a = 150 \text{ mm}$$

$$x_c = k \times d$$

$$x_c = 132.16$$

$$x_a > x_c$$

It is over reinforced.

REINFORCED CEMENT CONCRETE

calc

1. If M_{20} and Fe 45 calculate the

ratio N.A depth factor. and Lever

arm factor.

$$\sigma_{cbc} = \frac{f_{ck}}{3} =$$

$$m = 14.$$

$$k = \frac{m \sigma_{cbc}}{m \sigma_{cbc} + \sigma_{st}}$$

$$= \frac{14 \times 7}{14 \times 7 + 230}$$

$$k = 0.2987$$

$$j = 1 - \frac{k}{3}$$

$$j = 0.9$$

$$\frac{k}{j} = \frac{0.2987}{0.9}$$

$$\frac{k}{j} = 0.3317$$

Percentage of tension Reinforcement

$$F_c = F_t$$

$$\frac{1}{2} \times c \times b \times x_a = t \times A_{st}$$

$$\text{Ass. } \boxed{x_a = x_c}$$

$$\frac{1}{2} \times c \times b \times x_c = t \times A_{st}$$

$$\frac{1}{2} \times c \times b \times k \times d = t \times A_{st}$$

$$\boxed{\frac{A_{st}}{bd} = \frac{kc}{2t}}$$

$$\boxed{P_t = \frac{kc}{2t} \times 100}$$

2) Given: $b = 300\text{mm}$, $D = 600\text{mm}$, $d = 550\text{mm}$
 $4 \# 25\text{mm}$ Fe 415. $M_{max} = 50 \text{ kN.m}$, M_u

$$M = t \times A_{st} \times d$$

$$t \times A_{st} = \frac{M}{d} = \frac{50 \times 10^6}{550}$$

$$t = 25.46 \times 10^3 \text{ N/mm}^2 < 230 \text{ N/mm}^2$$

$$M = \frac{Qbd^2}{2}$$

$$Q = \frac{ckj}{2}$$

$$k = \frac{m \sigma_{cb}}{\sigma_{cb} + t}$$

$$j = \frac{m \sigma_{cb}}{m \sigma_{cb} + t}$$

$$m = \frac{E_s}{E_c}$$

$$M = \frac{c}{2} \times b \times x_a \times \left(d - \frac{x_a}{3}\right)$$

$$= \frac{c}{2} \times b \times x_a \times \left(d - \frac{x_a}{3}\right)$$

$$\frac{b x_a^2}{2} = m A_{st} (d - x_a)$$

$$m = \frac{280}{300}$$

$$m = 14$$

$$150 x_a^2 = 14 \times 4 \times \frac{\pi \times 25^2}{4} (550 - x_a)$$

$$150 x_a^2 + 27488.93 x_a - 15.1184 \times 10^6 = 0$$

$$x_a = 238.807 \text{ mm}$$

$$50 \times 10^6 = \frac{c}{2} \times 300 \times 238.81 (550 - 238.81)$$

$$c = 2.967 \text{ N/mm}^2 < \sigma_{cbc}$$

3.) R.C.C beam size 300 x 600 mm is reinforced with 4 # 16 mm ϕ in compression zone. 5 # 20 mm ϕ in tension zone. Determine (i) M.R.

(ii) M.R = 60 kNm c, t

Used M_{20} Fe 415.

Given: $d = 560 \text{ mm}$.

$$\frac{b x_a^2}{2} = m A_{st} (d - x_a)$$

$$\frac{300 x_a^2}{2} =$$

$$M.R = \sigma_{st} \times A_{st} \left(d - \frac{x_a}{3} \right)$$

$$= 230 \times 5 \times \left(\frac{20 \times \pi}{4} \right)$$

$$= 46968 \text{ kNm}$$

$$= 361.28 \text{ Nmm}$$

$$M.R = \frac{c k_j}{2} \times b$$

$$\frac{b x_a^2}{2} = m A_{st} (d - x_a)$$

300

$$M.R = \frac{Ck_j}{2} \times b \times d^2$$

$$C = \frac{20}{3}$$

$$= 7$$

$$= \frac{7 \times 0.298 \times 0.9 \times 300 \times 560^2}{2}$$

$$k = \frac{14 \times 7}{14 \times 7 + 20}$$

$$= 0.298$$

$$M.R = 88.31 \text{ kN.m}$$

$$M.R = \frac{C}{2} \times b \times x_a \left(d - \frac{x_a}{3}\right)$$

$$j = 1 - \frac{k}{3}$$

$$= 0.9$$

(ii)

$$M.R = F_c \times \left(d - \frac{x_a}{3}\right)$$

$$= \frac{C \times b \times x_a}{2} \left(d - \frac{x_a}{3}\right)$$

$$\frac{b x_a^2}{2} = m A_{st} \left(d - \frac{x_a}{3}\right)$$

$$\frac{300 \times x_a^2}{2} = 14 \times 5 \times \frac{\pi \times 20^2}{4} \left(560 - \frac{x_a}{3}\right)$$

$$150 x_a^2 = 21991.15 x_a - 12.315 \times 10^6$$

$$x_a = 222.46 \text{ mm}$$

$$60 \times 10^6 = \frac{C}{2} \times 300 \times 222.46 \left(560 - \frac{222.46}{3}\right)$$

$$C = 3.7 \text{ N/mm}^2 < 7 \text{ N/mm}^2$$

$$= f_t \times \left(d - \frac{x_a}{3}\right)$$

$$60 \times 10^6 = t \times A_{st} \left(d - \frac{x_a}{3}\right)$$

$$= t \times 5 \times \frac{\pi \times 20^2}{4} \left(560 - \frac{222.46}{3}\right)$$

$$t = 78.61 \text{ N/mm}^2 < 230 \text{ N/mm}^2$$

Notes:

* Structures more than 45m length should be designed with one or more expansion joints.

$$* E_{ce} = \frac{E_c}{1 + \theta} \quad \theta - \text{creep co-effi}$$

Days	θ
7 days	2.2
28 days	1.6
1 years	1.0

* Individual variation in the compressive strength of 3 cubes in the sample should not exceed $\pm 15\%$.

* fck	@ 1 day	16%
fck	@ 3 day	45% 40%
fck	@ 7 day	65%
fck	@ 14 day	90%
fck	@ 28 day	99%

* The rate of increase in compressive strength decreases with increase in time. Therefore we consider 28 days fck of concrete.

* Minimum grade of concrete for various types of structures :

Type of structure	Minimum grade of concrete
1. Lean concrete bases	M ₅ , M _{7.5}
2. P.C.C	M ₁₀
3. R.C.C. (General construction & under normal condition)	Grade M ₂₀
4.) water tank, Dome, folded plates, shell roof.	M ₂₀
5.) structures R.C.C. in sea water	M ₂₀ (P.C.C.) M ₃₀ (R.C.C.)
6.) Post tensioned Pre stressed concrete	M ₃₀
7.) Pre tensioned Pre stressed concrete	M ₄₀

* Types of concrete damper.

* Factors affecting crushing strength

cube :

→ size factor :

As the size of the cube decreases strength increases because of better homogeneity.

For eg: cube of 100mm size will have 5% more strength than 150mm cube

→ shape factor

cylinder of size $h=30\text{cm}$ $d=15\text{cm}$
has strength 80% of as that of cube
of 150mm .

→ Slenderness Ratio:

As slenderness ratio of specimen
increase strength decreases.

* weight batching is preferable to
compared to volume batching.

* Qty of water required per on
bag of cement for M_{15} mix is
32 litres, and for M_{20} mix 30 litres.

* As the size of the cube increases
the strength of concrete reduces
due to chances of more weak spot

DOUBLY REINFORCED BEAM:

If steel bars are provided in compression zone as well as in tension zone of an R.C.C section. In addition to steel bars in the tension zone then that R.C.C section is called as doubly reinforced section.

Doubly Reinforced Beam are adopted under following circumstances:

- * If depth restricted
- * If breadth restricted
- * If external B.M is greater than M.R.
- * If there is reversal of stress
- * If precast pile is used.
- * If there is shortage of height during form work.

→ If tension steel is provided in 2 rows in tension zone only, 25% of M.R. increases (Singly reinforced beam).

→ When steel rods provided in compression zone the equivalent area of concrete is $1.5m A_{sc}$, due to development of inelastic strain caused by creep and

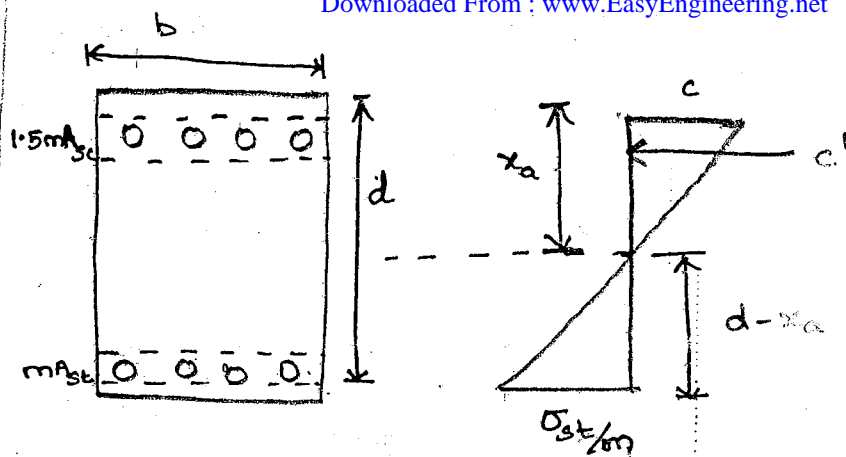
shrinkage induced in the ^{compression} creep zone of the beam.

→ The stress developed in the compression steel = $\frac{1}{2} \frac{BmC'}{I}$.

It means the stress developed in compression steel is mainly governed by concrete in the compression zone.

∴ Use of high grade steel in compression is meaningless and hence doubly reinforced section in working stress is not economical.

→ If hanger bars are provided to supports to stirrups in a singly reinforced beam, then it cannot be called as doubly reinforced section. i.e. to be called as a compression steel the min. Area of hanger bars shall be more than or equal to 0.2% of concrete area.



$$\frac{c}{c'} = \frac{xa}{xa - d'}$$

$$c(xa - d') = c'$$

Actual neutral Axis.

$$(A \times C \cdot G)_C = (A \times C \cdot G)_T$$

$$\left(b \times xa \times \frac{xa}{2} \right) - Asc(xa - d') + 1.5mAsc(xa - d')$$

$$= mAsc(d - xa)$$

$$\boxed{b \frac{xa^2}{2} + (1.5m - 1) Asc(xa - d') = mAsc(d - xa)}$$

critical neutral Axis

$$\boxed{x_c = k \times d} \quad \text{same as singly}$$

reinforced beam.

Force in Concrete Compression Zone (F_c)

$$F_c = F_{c1} - F_{\text{holes}} + F_{c2}$$

$$= \left(\frac{0 + c}{2} \times b \times xa \right) - (c' \times Asc) + (c' \times 1.5mAsc)$$

$$= \frac{cbxa}{2} + [(1.5m - 1) \times [c' \times Asc]]$$

$$\boxed{F_c = \frac{1}{2} cbxa + (1.5m - 1) (c' \times Asc)}$$

By similar triangle property

$$\boxed{c' = \frac{c(xa - d')}{xa}}$$

Force in tension zone (F_{st}):

$$F_{st} = \sigma_{st} \times A_{st}$$

Moment of Resistance Due to Compression

Force:

$$M.R = (F_{c1} \times Z_1) - (F_{holes} \times Z_2) + (F_{steel} \times Z_2)$$

$$= \left[\frac{1}{2} cbx_a (d - \frac{x_a}{3}) \right] - A_{sc} \times c' (d - d')$$

$$+ 1.5m A_{sc} c' (d - d')$$

$$M.R = \left[\frac{1}{2} cbx_a (d - \frac{x_a}{3}) \right] + (1.5m - 1) A_{sc} c' (d - d')$$

1. Design R.C.C section 250×500 mm) subjected to B.M = 110 kN.m. $F_e 45$, $M=5$ $d' = 50$ mm.

$$M.R = Q b d^2$$

$$Q = \frac{Ck_j}{2}$$

$$C = \frac{25}{3} = 8.33 \text{ N/mm}^2$$

$$K = \frac{280}{280 + 3 \times 280}$$

$$j = 1 - \frac{k}{3} = 0.9037$$

$$= \frac{280}{280 + 3 \times 280}$$

$$= \frac{m \sigma_{cbc}}{m \sigma_{cbc} + \sigma_{st}}$$

$$M.R = \left(\frac{8.33 \times 0.288 \times 0.9037}{2} \right)$$

$$K = 0.288$$

$$\times 250 \times 500^2$$

$$= 67.75 \times 10^6 < 110 \text{ kNm}$$

Since Hence external moment is greater than M.R (i.e) $110 > 67.75$ kNm adopt doubly reinforced section.

$$M.R = \frac{cbx_a^3}{2} + (1.5m - 1) A_{sc} (d - d') c'$$

$$x_a = kd$$

$$= 500 \times 0.28$$

$$x_a = 140$$

$$110 \times 10^6 = \left(\frac{8.5 \times 250 \times 140}{2} \right) \times \left(500 - \frac{140}{3} \right)$$

$$+ (1.5 \times 10.98 - 1) \times A_{sc} (500 - 50)$$

$$= 67.43 \times 10^6$$

$$= 148750 + 6736.5 A_{sc} \times 5.46$$

$$A_{sc} = 1157.38 \text{ mm}^2$$

To find A_{st} $f_c = F_c$

$$F_c = F_t$$

$$\frac{cbx_a}{2} + (1.5m - 1) A_{sc} c' = t \times A_{st}$$

$$\left(\frac{8.5 \times 250 \times 140}{2} \right) + \left([1.5 \times 10.98 - 1] \times 1157.38 \times 5.46 \right) = 230 \times A_{st}$$

$$148750 + 97.76 \times 10^3 = 230 \times A_{st}$$

$$A_{st} = 1071.78 \text{ mm}^2$$

$$m = \frac{280}{250}$$

$$= \frac{280}{250}$$

$$m = 10.98$$

$$c' = \frac{c(x_a - d')}{x_a}$$

$$= \frac{8.5(140 - 50)}{140}$$

$$= 5.46$$

24/9/2015

FLANGED BEAMS: (T-beam & L-beam)

T-BEAM:

* T-Beam is a R.C.C beam where beam and slabs are constructed monolithically such that slab is always under compression and hence extra concrete is not required for the beam to behave like compression zone, as a result depth of the beam is reduced to a greater extent.

* T-beam is the most economical R.C.C beam where compression zone is totally or partially not required.

* A simply supported beam having monolithic ~~base~~ slab @ the top is called T-beam.

But if the beam is kept above the slab to get plane soffit then the beam is not a T-beam (i.e) it is an inverted rectangular beam.

* For a continuous beam @ support, hogging B.M develops (Tension @ top fibre) due to which the slab does not contribute compression zone for beam.

Flanged beam

T-beam

and beam are considered as a single structure

Uses:

- * Depth can be reduced
- * Economical section

Hypothetical uniform load = Max. load @ centre of the beam

$$f_c \text{ due to actual compressive stress} = f_c \text{ due to hypothetical stress}$$

$$\text{hypothetical width} = b_{eff} \leq B_f \text{ actual}$$

hypothetical width is effective width of flange



≤ Actual width of the flange

and hence the beam shall be designed as rectangular beam only @ intermediate support;

* In case of portico (canopy) if the beam is kept @ the bottom of the ^{cantilever} slab projecting both the sides the beam shall be designed as rectangular beam.

But if the beam is kept @ the top it shall be designed as T-beam

* In case of T-beam the max. Bending Compressive stress developed in the slab is always @ the centre of web. which gradually decreases towards middle of the slab b/w two webs.

As a result the T-beam cannot be analysed due to variation of stress @ one section in such case a hypothetical width of the flange, shall be considered where uniform stress having maximum value equal to the stress @ the centre of web.

* The compressive force due to actual bending compressive stress must be equal to the ~~the~~ compressive force due to hypothetical uniform stress.

The hypothetical width of the flange is known as effective width

Effective width of T-beam depend upon
for continuous beam $0.4l$.

- l_0
- b_0
- D_f
- c/c spacing of T-beam
- Support condition
- type of load

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of the flange which is always less than or equal to actual width of flange.

* The effective depth width of T-beam depend upon following factors:

→ Effective span of T-beam (l_0)
(Distance b/w zero B.M to zero B.M)

→ In case of Continuous

Beam $l_0 = 0.7 \times l$

→ width of the web. (b_w)

→ Depth of the slab (D_f)
↳ (or) flange

→ C/c spacing of T-beam.

→ Support condition of beam

→ Type of Load

no of beams

$$b_f = \frac{l_o}{6} + bw + \frac{D_f}{6} \quad \text{for T}$$

$$b_f = \frac{l_o}{12} + bw + \frac{D_f}{2} \quad \text{for L}$$

Inverted T beams

$$b_f = \frac{l_o}{\left(\frac{l_o}{6} + 4\right)} + bw$$

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28/9/2015

T-BEAM

Effective width of Flange (b_f):

* For no. of T-beams.

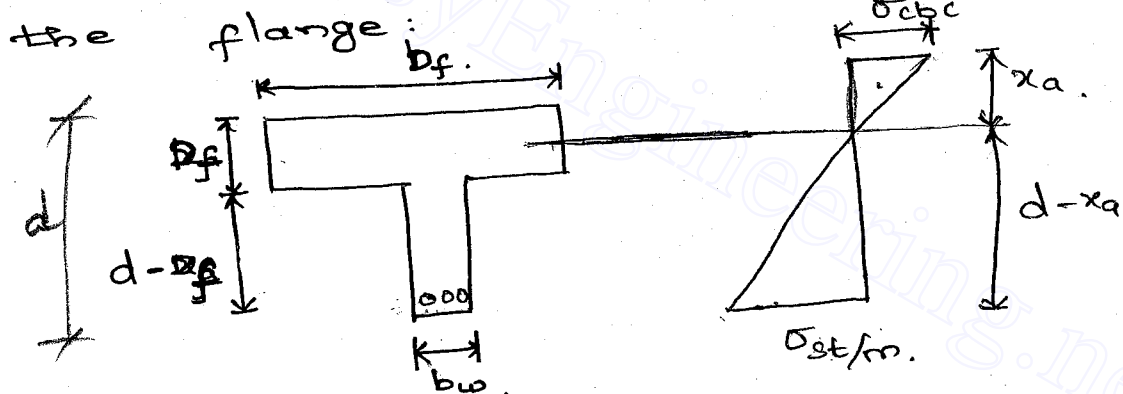
$$b_f = \frac{l_0}{b} + b_w + 6D_f \leq b \text{ (T-beam)}$$

$$b_f = \frac{l_0}{12} + b_w + 3D_f \leq b \text{ (L-beam)}$$

* For isolated T-beam.

$$b_f = \frac{l_0}{\left(\frac{l_0}{b} + 4\right)} + b_w$$

Case (i) If the T-beam N.A lies within



→ Actual Neutral Axis:

$$(Area \times C.G.)_c = (A \times C.G.)_T$$

$$b \times x_a \times \frac{x_a}{2} = m A_{st} (d - x_a)$$

$$\boxed{\frac{b x_a^2}{2} = m A_{st} (d - x_a)}$$

b_{eff} :

→ For no. of T-beams

$$* b_{eff} = \frac{b_0}{6} + b_w + 6d_f \leq b \text{ (T-beam)}$$

$$* b_{eff} = \frac{l_0}{12} + b_w + 6d_f \leq b \text{ (L-beam)}$$

→ For ^{Isolated} no. of Flanged beam

$$* b_{eff} = \frac{l_0}{\left(\frac{l_0 + 4}{b}\right)} + b_w \text{ (T-beam)}$$

Design of T-beam:

Case (i) N.A lies within the flange: (Design of T-beam

* A.N.A:

$$\frac{b_f x_a^2}{2} = m A_{st} (d - x_a)$$

is similar to design of singly reinforced beam

* C.N.A:

$$x_c = kd$$

* F_c :

$$F_c = \frac{1}{2} c b_f x_a$$

* F_t :

$$F_t = t A_{st}$$

* Lever arm: $z = dj$

* M.R due to concrete:

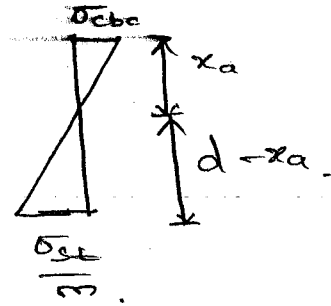
$$M.R = \frac{1}{2} c b_f x_a \left(d - \frac{x_a}{3}\right)$$

* M.R due to tension zone.

$$M.R = t A_{st} \left(d - \frac{x_a}{3}\right)$$

→ Critical Neutral Axis

$$x_c = kd$$



→ Force in Compression zone.

$$F_c = \frac{0 + C}{2} \times b \times x_a$$

$$F_c = \frac{1}{2} C_b x_a$$

→ Force in Tension zone.

$$F_t = \frac{\sigma_{st}}{\gamma} \times A_{st}$$

$$F_t = \sigma_{st} A_{st}$$

→ Lever Arm:

$$z = d_j$$

→ M.R Due To Compression Zone:

$$M.R = \frac{1}{2} C_b x_a \left(d - \frac{x_a}{3} \right)$$

→ M.R Due To Tension zone:

$$M.R = t A_{st} \left(d - \frac{x_a}{3} \right)$$

Note:

* In T-beam if actual neutral axis lies within the flange, the design similar to the design of singly reinforced Beam.

Case (ii) If N.A lies outside the flange.

* A.N.A.

$$b_f D_f \left(x_a - \frac{D_f}{2} \right) + \left(b_w \times \frac{(x_a - D_f)^2}{2} \right) = m A_{st} (d - x_a)$$

* C.N.A

$$x_c = kd$$

* F_c :

$$F_c = F_{c1} + F_{c2}$$

$$F_c = \left(\frac{c + c'}{2} \times D_f b_f \right) + \left[\frac{1}{2} c' b_w (x_a - D_f) \right]$$

$$c' = \frac{c(x_a - D_f)}{x_a}$$

* F_t :

$$F_t = t A_{st}$$

* Lever arm:

$$z_1 = d - \bar{y} = d - \left(\frac{c + 2c'}{c + c'} \times \frac{D_f}{3} \right)$$

$$z_2 = d - D_f - \left(\frac{x_a - D_f}{3} \right)$$

* M.R due to compression zone

$$M.R = \left[\frac{c + c'}{2} \times b_f D_f \times \left(d - \left(\frac{c + 2c'}{c + c'} \times \frac{D_f}{3} \right) \right) \right]$$

$$+ \left[\frac{1}{2} c' b_w (x_a - D_f) \times \left(d - D_f - \left(\frac{x_a - D_f}{3} \right) \right) \right]$$

* M.R due to tension zone

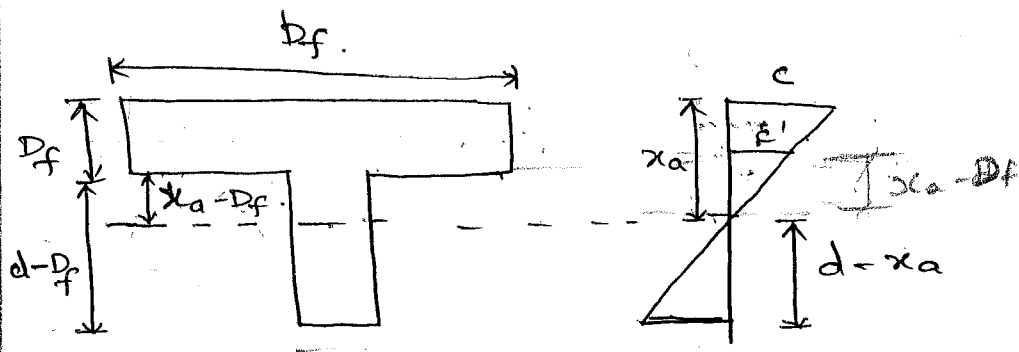
$$M.R = t A_{st} \left(d - \frac{x_a}{3} \right)$$

* MOST Economical depth,
Total cost = cost of concrete + cost of steel.

$$d = \sqrt{\gamma_c \frac{M_{max}}{\sigma_{st} j \times b_w}}$$

$$\gamma_c = 70-80\%$$

Case (ii) If N.A lies outside the flange



→ Actual Neutral Axis:

$$(A_1 \times C \cdot G_1) + (A_2 \times C \cdot G_2) = (A \times C \cdot G_1)_T$$

$$\left[b_f \times D_f \times \left(x_a - \frac{D_f}{2} \right) \right] + \left[b_w \times (x_a - D_f) \times \left(\frac{x_a - D_f}{2} \right) \right] = m A_{st} (d - x_a)$$

$$\left[b_f \times D_f \times \left(x_a - \frac{D_f}{2} \right) \right] + \left[b_w \times \frac{(x_a - D_f)^2}{2} \right] = m A_{st} (d - x_a)$$

→ Critical Neutral Axis:

$$x_c = kd$$

→ Force in compression zone:

$$F_c = F_{c1} + F_{c2}$$

$$F_{c1} = \frac{\text{Stress}}{2} \times \frac{c + c'}{2} \times \text{Area} \times b_f \times D_f$$

$$F_{c2} = \frac{\text{Stress}}{2} \times \frac{0 + c'}{2} \times \text{Area} \times b_w \times (x_a - D_f)$$

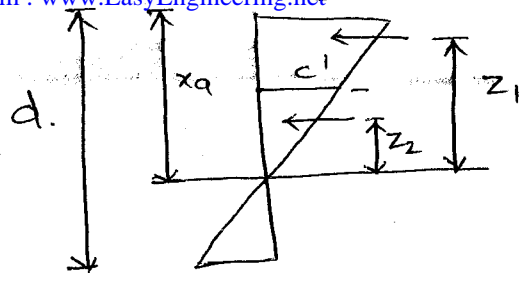
$$F_{c2} = \frac{1}{2} \times c' \times b_w \times (x_a - D_f)$$

$$F_c = \left[\left(\frac{c + c'}{2} \right) \times b_f \times D_f \right] + \frac{1}{2} \times c' \times b_w \times (x_a - D_f)$$

→ Force in Tension zone

$$F_t = t A_{st}$$

→ Lever Arm: www.EasyEngineering.net



$$z_1 = d - \bar{y}$$

$$\bar{y} = \frac{a+2b}{a+b} \times \frac{h}{3}$$

$$\bar{y} = \frac{c+2c'}{c+c'} \times \frac{D_f}{3}$$

$$z_1 = d - \left[\left(\frac{c+2c'}{c+c'} \right) \times \frac{D_f}{3} \right]$$

$$z_2 = d - D_f - \left(\frac{x_a - D_f}{3} \right)$$

→ M.R Due to compression force.:

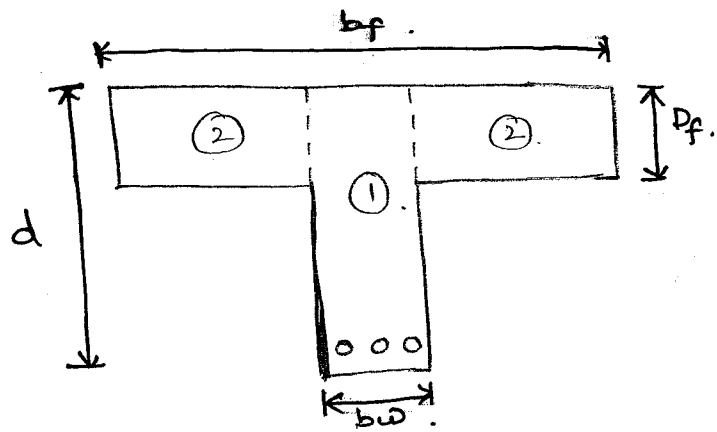
$$M.R = \left[\left(\frac{c+c'}{2} \right) \times b_f \times D_f \times \left(d - \left[\left(\frac{c+2c'}{c+c'} \right) \times \frac{D_f}{3} \right] \right) \right] + \frac{1}{2} c' b_w (x_a - D_f) \times \left(d - D_f - \left(\frac{x_a - D_f}{3} \right) \right)$$

→ M.R Due to tension zone:

$$M.R = \pm A_s t (d - \frac{x_a}{3})$$

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ECONOMICAL DEPTH OF T-BEAM / (COST RATI



Rate concrete = 5500
 = 6500/m
 1 m³ concrete = 180 kg
 1 kg = 65

$$\text{Total cost} = (\gamma_c \times V_c) + (\gamma_s \times V_s)$$

$$= [\gamma_c \times A_c \times L] + [\gamma_s \times A_{st} \times L]$$

$$= \gamma_c \times [(b_w \times d) + [(b_f - b_w) \times D_f]] \times L + \left[\gamma_s \times \frac{M_{max}}{t_j \times d} \times L \right]$$

$$C = \frac{\gamma_c \times L}{L} \left[(b_w \times d) + [(b_f - b_w) \times D_f] + \left(\frac{\gamma_s}{\gamma_c} \times \frac{M_{max}}{t_j \times d} \right) \right]$$

$$\frac{\partial C}{\partial d} = 0$$

$$0 = \gamma_c \times L \left[b_w + \frac{\gamma_s}{\gamma_c} \frac{M_{max}}{t_j} \left(\frac{-1}{d^2} \right) \right]$$

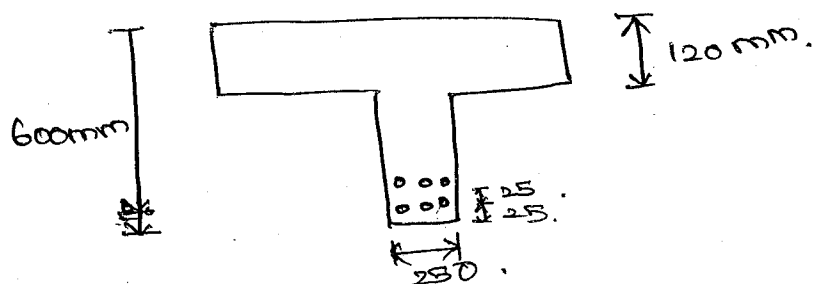
$$0 = b_w - \frac{\gamma_s}{\gamma_c} \frac{M_{max}}{t_j} \left(\frac{1}{d^2} \right)$$

$$d = \sqrt{\frac{\gamma_s}{\gamma_c} \frac{M_{max}}{t_j b_w}}$$

$$\gamma_r = \frac{\gamma_s}{\gamma_c} = 70 - 80\%$$

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1. An isolated T-beam $b = 1100 \text{ mm}$ and $D = 600$
 $D_f =$ Thickness of slab, $D_f = 120 \text{ mm}$, $b_w = 250$
 There are 6 bars of 25mm dia in Tension zone. Determine M.R of T-beam if
 $l_0 = 6 \text{ m}$. use M_{20} and Fe 280.



$$d' = 600 - 25 - \frac{25}{2} - 25$$

$$d' = 537.5$$

$$b_{eff} = \frac{l_0}{\left(\frac{l_0}{b}\right) + 4} + b_w = \left(\frac{6000}{\frac{6000}{1100} + 4}\right) + 250$$

$$b_f = 884.61 \text{ mm}$$

$$b_f = 885 \text{ mm}$$

$$m = \frac{280}{3 \times 6} = \frac{280}{18}$$

$$\frac{b_f x_a^2}{2} = m A_{st} (d - x_a)$$

$$m = 14$$

$$m = 13.33$$

$$\frac{885 \times x_a^2}{2} = 13.33 \times 2945.24 (537.5 - x_a)$$

$$442.5 x_a^2 + 39260.0492 x_a - 21.102 \times 10^6 = 0$$

$$x_a = 178.47 \text{ mm} > 120 \text{ mm}$$

Horizontal spacing. (whichever is great)

→ ϕ bar

→ size of aggregate + 15mm.

Vertical spacing (whichever is great)

→ ϕ bar

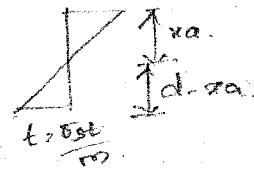
→ $\frac{2}{3}$ size of aggregate

→ 15mm.

$x_c > x_d$

U.R.S

OC



NOTE:

* In U.R.S. Steel yields first

$\therefore t = \sigma_{st}$ and $c \neq \sigma_{bc}$

we have to find out c from the following expression

$$c = \frac{t \times x_a}{(d - x_a)}$$

$$c = \frac{t \times x_a}{(d - x_a)}$$

* In O.R.S concrete yields first

$\therefore c = \sigma_{bc}$ $t \neq \sigma_{st}$

$$t = \frac{c(d - x_c)}{x_a}$$

$$t = \frac{c(d - x_c)}{x_a}$$

$$b_f D_f \left(x_a - \frac{D_f}{2} \right) + \left(b_w \cdot \left(\frac{x_a - D_f}{2} \right)^2 \right) = m A_{st} (d - x_a)$$

$$\left[885 \times 120 \left(x_a - \frac{120}{2} \right) \right] + \left[\frac{250}{2} \left(x_a - 120 \right)^2 \right] = 13.33 \times 2945.24 (537.5 - x_a)$$

$$\left. \begin{aligned} 106200 x_a - 6.372 \times 10^6 + 125 x_a^2 - 30000 x_a \\ + 1.8 \times 10^6 \end{aligned} \right\} = \begin{aligned} 13.33 \\ \times 2945.24 \\ (537.5 - x_a) \end{aligned}$$

$$125 x_a^2 + 115460.0492 x_a - 25.674 \times 10^6 = 0$$

$$x_a = 185.22 \text{ mm}$$

*

$$M \cdot R = \frac{c + c'}{2} \times b_f D_f \left[\left(\frac{c + 2c'}{c + c'} \right) \times \frac{D_f}{3} \right] + \frac{c'}{2} b_w (x_a - D_f) \times \left[d - D_f - \left(\frac{x_a - D_f}{3} \right) \right]$$

$$c' = \frac{c \times (x_a - D_f)}{x_a} = \frac{7 \times (185.22 - 120)}{185.22}$$

$$c' = 2.465 \text{ N/mm}^2$$

$$= \frac{7 + 2.465}{2} \times 885 \times 120 \times \left[537.5 - \left(\frac{7 + 2.465}{7 + 2.465} \right) \times \frac{120}{3} \right]$$

$$+ \frac{2.465}{2} \times 250 \times (185.22 - 120)$$

$$\times \left(537.5 - 120 - \left(\frac{185.22 - 120}{3} \right) \right)$$

$$= (502591.5 \times 487.08) + 7.953 \times 10^6$$

$$M = 252.76 \times 10^6 \text{ N.m}$$

$$x_c = kd. \quad k = \frac{280}{280 + 30\sigma_{st}} = 0.418.$$

$$= 0.418 \times 537.5.$$

$$x_c = 224.626 > x_a.$$

$$\therefore c < \sigma_{cbc} \quad t = \sigma_{st}/\beta$$

$$\frac{c}{\sigma_{st}/m} = \frac{x_a}{d - x_a}$$

$$c = \frac{\cancel{224.626} \times 185.22}{(537.5 - 185.22)} \times \frac{130}{13}$$

$$\boxed{c = 5.257}$$

$$c' = \frac{c(x_a - D_f)}{x_a} = \frac{5.257(185.22 - 120)}{185.22}$$

$$\boxed{c' = 1.85}$$

$$M.R = \frac{c+c'}{2} b_f D_f \left(d - \frac{c+2c'}{c+c'} \times \frac{D_f}{3} \right) + \frac{c'}{2} \times b_w (x_a - D_f) \times \left[d - D_f - \left(\frac{x_a - D_f}{3} \right) \right]$$

$$= \left(\frac{5.257 + 1.85}{2} \right) \times 885 \times 120 \left(537.5 - \left(\frac{5.257 + 2(1.85)}{5.257 + 1.85} \right) \times \frac{120}{3} \right)$$

$$+ \frac{1.85}{2} \times 250 \left(185.22 - 120 - \left(\frac{185.22 - 120}{3} \right) \right)$$

$$= 377381.7 \times 487.08 + 10.03 \times 10^3$$

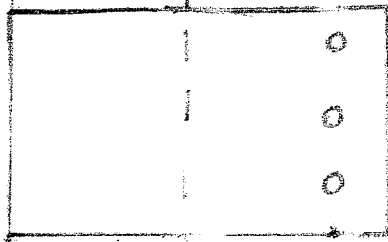
$$\boxed{M.R = 183.82 \times 10^6 \text{ N.m}}$$

LIMIT STATE METHOD

LIMIT STATE OF COLLAPSE

WLM

$$\sigma_{cbc} = \frac{f_{ck}}{3} = 0.33f_{ck}$$



$\frac{\sigma_{st}}{m}$

$$\sigma_{cbc} = \frac{f_{ck}}{3} = 0.33f_{ck}$$

$$\sigma_{st} = \frac{f_y}{1.8} = 0.55f_y$$

U.LM

$$= 0.55f_{ck}$$



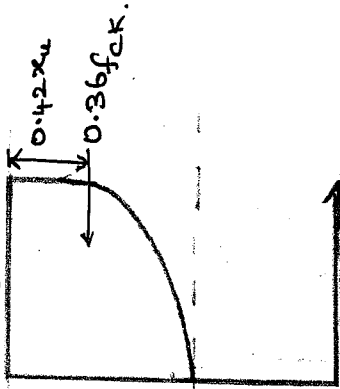
f_y

$$\frac{f_{ck}}{1.8} = 0.55f_{ck}$$

$$f_y = 1.8f_y$$

L.SM

$$0.45f_{ck}$$



$0.87f_y$

$$\frac{R_f}{1.15} = 0.87f_y$$

ULM

* ULM was developed in 1950.

Type of Load	Live Load	Dead Load
F.O.S.	2.2	1.5
Material	steel	concrete
Permissible stress	100% (f_y)	55% ($\frac{f_{ck}}{1.8}$)

* Shape of stress block diagram - Rectangle.
 * depth of stress block diagram - $0.43d$

* ULM was specified in IS 456:1964

* ULM Safe-collapse condition: Unsafe - Serviceability Condition

* WSM. Safe - serviceable condition Unsafe-collapse condition.

LSM

* It is a rationalized method of ULM.

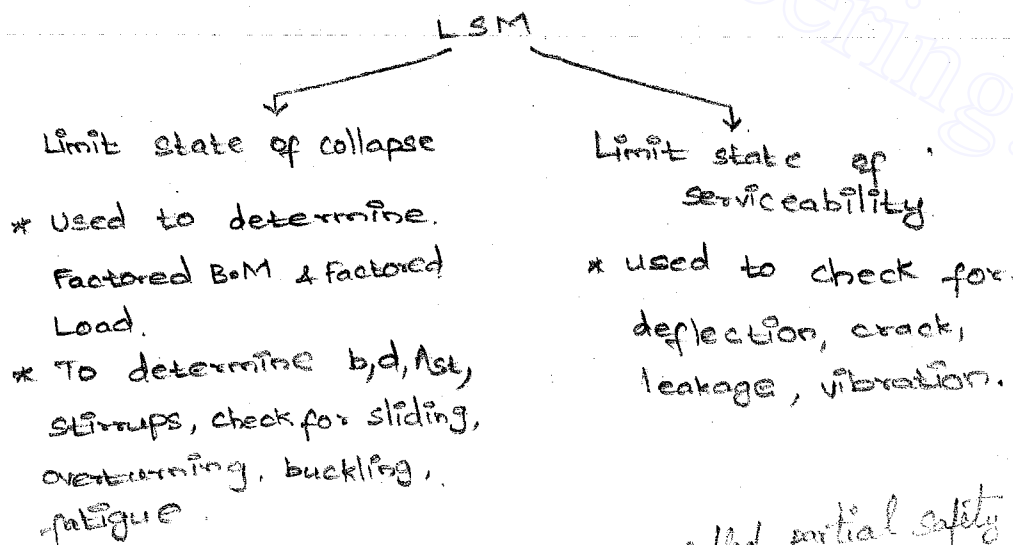
* LSM was developed in 1970

* Adopted in IS 456:1978 and ^{revised code} IS 456:2000 and used for R.C.C structures.

* In LSM B.M. is increased by 50%.

* M.R. is increased by 200%.

*



* LSM uses multiple safety factor ^{called partial safety factor} (i.e) F.O.S. separately for load and material.

Material	steel	concrete
Partial safety factor	1.15	1.5

- * LSM is a rationalized method of ULM.
- * ULM was developed in 1950, where Load factor for Live load was 2.2 and Dead Load 1.5.
- * In ULM the permissible stress for steel was 100% f_y for concrete 55% f_{ck} and the stress block was rectangle.
Depth = 0.43d.
- * ULM was specified in IS 456:1964.
- * ULM was safe for collapse condition but unsafe for serviceability condition.
- * WSM is safe for service condition but it is unsafe for collapse condition.
- * LSM ~~can~~ has been developed to take care of safety @ collapse and serviceability condition.
- * LSM was developed in 1970. and adopts by IS 456 in 1978.
- * Now IS 456:2000 is widely used for R.C.C structure.
- * In LSM the extra bending moment is increased by 50% but M.R of the section increases to 200%.
- * There are 2 methods of LSM.
 - Limit state collapse
 - Limit state of serviceability.

* Partial safety factor is dependent on Load combination:
(collapse limit condition)

$$\rightarrow 1.5 D \cdot L + 1.5 L \cdot L$$

$$\rightarrow 1.5 D \cdot L + 1.5 W \cdot L$$

$$\rightarrow 1.5 D \cdot L + 1.5 E \cdot L$$

$$\rightarrow 0.9 D \cdot L + 1.5 W \cdot L \quad (\text{Reversal of stresses, and also for overburning check})$$

$$\rightarrow 1.2 D \cdot L + 1.2 L \cdot L + 1.2 W \cdot L$$

$$\rightarrow 1.2 D \cdot L + 1.2 L \cdot L + 1.2 E \cdot L$$

$$\rightarrow \cancel{1.2 D \cdot L + 1.2 L \cdot L + 1.2 E \cdot L + 1.2 W \cdot L} \quad (\text{wind and Earthquake load do not occur at the same time, as per IS 456:2000})$$

* Partial safety factor of limit state of serviceability.

(a) $D \cdot L + L \cdot L$

(b) $D \cdot L + W \cdot L$

(c) $D \cdot L + E \cdot L$

(d) $D \cdot L + 0.8 L \cdot L + 0.8 W \cdot L$

(e) $D \cdot L + 0.8 L \cdot L + 0.8 E \cdot L$

* Max. permissible strain in concrete in ben
 $\epsilon_{cbc} \neq 0.0035$ (flexural compression)

* $\epsilon_{cbc} \neq 0.002$ (axial compression)

* $\epsilon_{st} \neq 0.002 + \frac{0.8 f_y}{E_s}$ (or) $0.002 + \frac{f_y}{\gamma_{mb} E_s}$

* Limit state of collapse is used to design

as R.C.C structure based on collapse Load. (Factored B.M & Factored Load (i.e) to

* determine breadth, depth, A_{st} , stirrups, Check for sliding, check for overturning, buckling, fatigue).

* Limit state of serviceability is used to check deflection, crack, leakage, vibration.

* LSM uses multiple safety factor (i.e) separately for load and for material. This safety factor is ^{also} called as partial safety factor.

* The partial safety factor for concrete is 1.5 and for steel 1.15.

* The partial safety factor load is depends on load combination.

→ (a) $1.5 DL + 1.5 LL$ (IL)

(b) $1.5 DL + 1.5 WL$

(c) $1.5 DL + 1.5 EL$

(d) $0.9 DL + 1.5 WL$ -

(e) $1.2 DL + 1.2 L + 1.2 \overline{L}$

(f) $1.2 DL + 1.2 L + 1.2 E.L$

~~(g) $1.2 DL + 1.2 L + 1.2 WL + 1.2 EL$~~

↙ wind and Earthquake load do not

occur at the same time as per IS 456: 2000.

Reversal of stresses
(or)
Overturning check.

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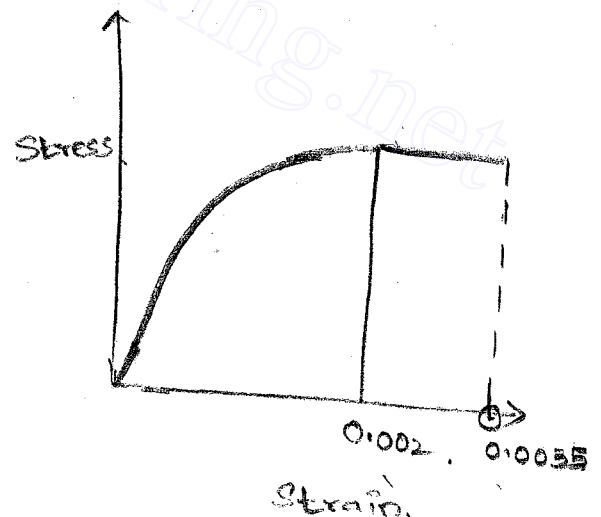
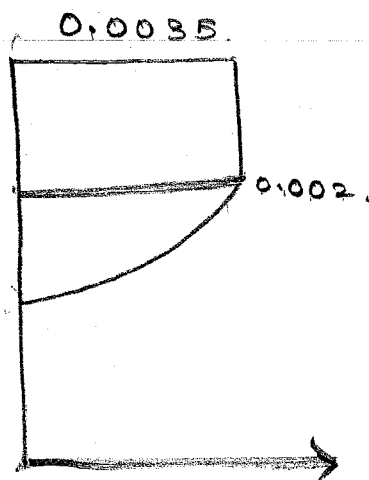
Downloaded From : www.EasyEngineering.net
* The partial factor of safety of Limit state of serviceability.

- a) $1.0 DL + 1.0 L \cdot L$
- (b) $1.0 DL + 1.0 W \cdot L$
- (c) $1.0 DL + 1.0 E \cdot L$
- (d) $1.0 DL + 0.8 L \cdot L + 0.8 W \cdot L$
- (e) $1.0 DL + 0.8 L \cdot L + 0.8 E \cdot L$

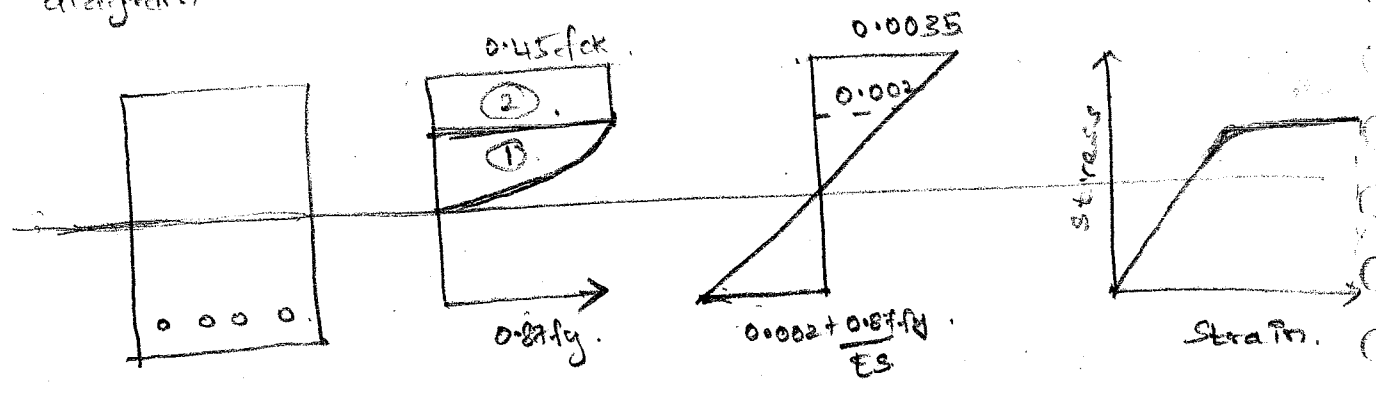
* The maximum permissible strain in concrete in bending compression (flexural compression) $\epsilon_{bc} \neq \underline{0.0035}$.

* The maximum permissible strain in concrete in axial compression. $\neq \underline{0.002}$

* The strain in tension steel, shall not be less than $0.002 + \frac{0.87f_y}{E_s}$.



Limit State Method Stress Block & strain block diagrams



* Characteristic strength of concrete in L.S.M } = 0.85 compressive strength of cylinder
 = 0.85 x (0.8 x cube strength of concrete)
 = 0.67 cube strength of concrete

* Design strength of concrete for L.S.M } = $\frac{0.67 f_{ck}}{1.5} = 0.45 f_{ck}$

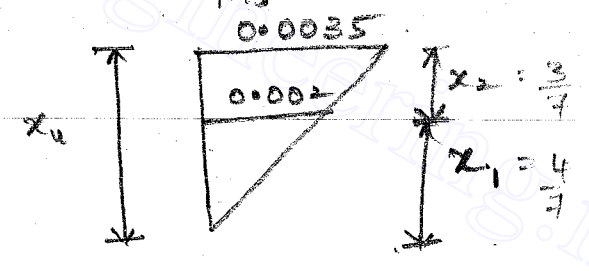
* Design strength of steel for L.S.M = $\frac{f_y}{1.15} = 0.87 f_y$

* Average stress

$$x_1 = \frac{4}{7} x_u$$

$$x_2 = \frac{3}{7} x_u$$

$$f_{avg} = 0.36 f_{ck}$$

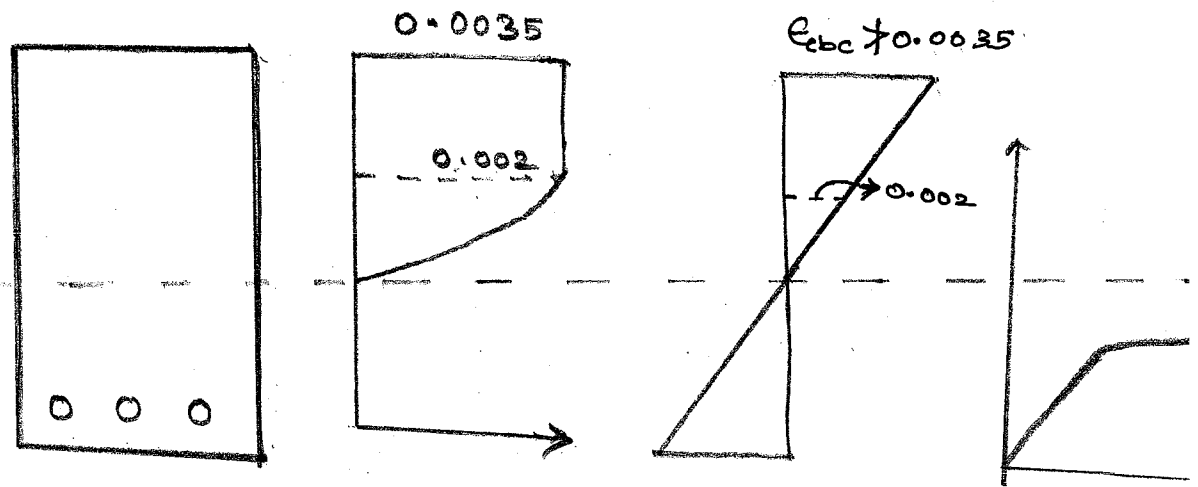


* Expression for critical neutral axis. (from strain diagram)

$$\frac{E_{cb} \epsilon_{cb}}{E_{cb} \epsilon_{cb} + E_{st} \epsilon_{st}} x_{u,max} = \frac{E_{cb} \epsilon_{cb}}{E_{cb} \epsilon_{cb} + E_{st} \epsilon_{st}} x_d = f_d$$

Grade of steel	$\frac{x_{u,max}}{d}$
Fe 250	0.53
Fe 415	0.48
Fe 500	0.46

Limit state Method Stress Block & strain block diagram.



* The characteristic strength of concrete for limit state = 0.85 times of compressive strength of cylinder.

cylinder strength = 0.8 times of cube strength

∴ ~~f_{ck}~~ characteristic strength of concrete

$$\text{for L.S.M} = 0.8 \times (0.85 f_{ck})$$

$$= \underline{0.67 f_{ck}}$$

$$\left. \begin{array}{l} \text{The design strength} \\ \text{of concrete for LSM} \end{array} \right\} = \frac{0.67 f_{ck}}{\gamma_{mc}}$$

$$= \frac{0.67 f_{ck}}{1.5}$$

$$\left. \begin{array}{l} \text{Design strength of concrete for} \\ \text{LSM} \end{array} \right\} = 0.45 f_{ck}$$

* The design strength of steel according to LSM } = $\frac{\text{Characteristic strength of steel}}{\text{partial safety factor}}$

$$= \frac{f_y}{1.15}$$

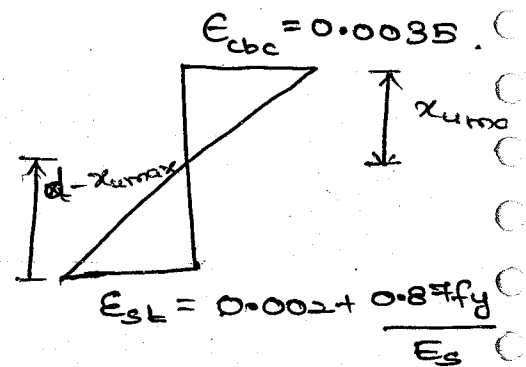
The design strength of steel according to LSM } = $0.87 f_y = \frac{f_y}{1.15}$

EXPRESSION FOR CRITICAL NEUTRAL AXIS:

It is based on strain diagram which is linear one.

By 1st triangle property.

$$\frac{E_{cbc}}{E_{st}} = \frac{x_{u\max}}{d - x_{u\max}}$$



$$E_{cbc}(d - x_{u\max}) = E_{st} x_{u\max}$$

$$E_{cbc} d = (E_{cbc} x_{u\max}) + (E_{st} x_{u\max})$$

$$x_{u\max} (E_{cbc} + E_{st}) = E_{cbc} d$$

$$x_{u\max} = \left(\frac{E_{cbc}}{E_{cbc} + E_{st}} \right) \times d$$

$$x_{u\max} = k \times d$$

Grade of steel

$$\frac{x_{u\max}}{d}$$

class 10.

Fe 250

$$0.53$$

Fe 415

$$0.48$$

Fe 500

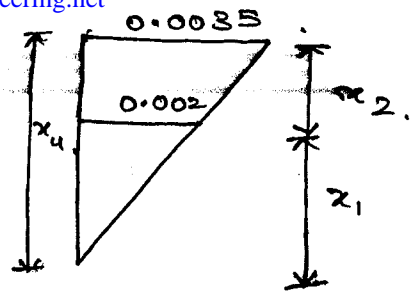
$$0.46$$

By 11th triangle property.

$$\frac{0.0035}{x_u} = \frac{0.002}{x_1}$$

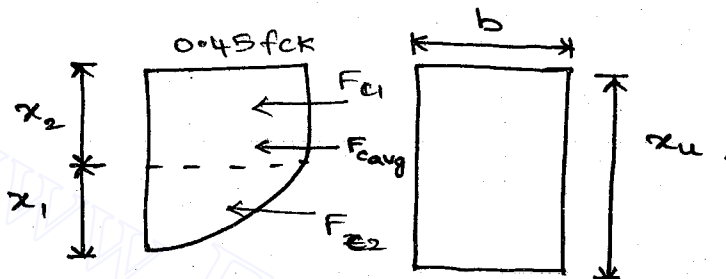
$$x_1 = \frac{0.002}{0.0035} \times x_u$$

$$x_1 = \frac{4}{7} x_u$$



$$x_2 = \frac{3}{7} x_u$$

Average stress:



$$F_{c \text{ avg}} = F_{c1} + F_{c2}$$

$$f_{c \text{ avg}} \times b \times x_u = (f_{c1} \times A_1) + (f_{c2} \times A_2)$$

$$= \left(\frac{2}{3} \times 0.45 f_{ck} \times b \times x_1 \right) + \left(0.45 f_{ck} \times b \times x_2 \right)$$

$$= b \left[\frac{2 \times 0.45}{3} f_{ck} \times \frac{4x_u}{7} + 0.45 f_{ck} \times \frac{3x_u}{7} \right]$$

$$f_{c \text{ avg}} \times b \times x_u = b \times x_u \left[\frac{3.6}{21} f_{ck} + \frac{1.02}{7} f_{ck} \right]$$

$$f_{c \text{ avg}} = 0.36 f_{ck}$$

$$f_{c \text{ avg}} = 0.36 f_{ck}$$

Note:

To increase the bond strength economically, we have to provide more no. of smaller dia bars.

Grade of steel	Strain to extreme fibres
Fe 250	0.0030
Fe 415	0.0038
Fe 500	0.0041

SPACING OF REINFORCEMENT;

* Minimum spacing b/w 2 bars shall be greater of the following.

1.) ϕ bar (If ϕ of bars are of equal ϕ).

2.) ϕ of largest bar.

3.) $5\text{mm} + \frac{\text{nominal size of aggregate}}{K}$.

→ If needle vibrator is used to compact concrete,

1.) $\frac{2}{3} \times \text{nominal max. size of aggregate}$.

* Minimum vertical distance : (whichever is greater)

1.) 15 mm (or) $\frac{2}{3} \times \text{max. size of aggregate}$

(or) Max. size of bar.

* Maximum distance b/w bars in tension depending on cracking of concrete.

1.) Max. crack width in mild and aggressive environment

2.) 0.3 mm and 0.1 mm

* side face Reinforcement.

→ If depth of the web in a beam exceeds 750 mm, side face reinforcement should be provided along the two faces.

→ Total Area of side face reinforcement } = 0.1% of web area.

on both faces. and spacing of reinforcement should not exceed 300mm c/c.

* Deep beams.

$$\frac{l}{d} \neq 2 \quad \text{Simply supported}$$

$$\frac{l}{d} \neq 2.5 \quad \text{Continuous beam.}$$

1.) Minimum percentage of reinforcement =

Fe 415

$$P_t = \frac{0.85 \times 100}{f_y} = \frac{0.85 \times 100}{415} = 0.2\%$$

Fe 250

$$P_t = \frac{0.85 \times 100}{250} = 0.34\%$$

Fe 500

$$P_t = \frac{0.85 \times 100}{500} = 0.17\%$$

2) Assume straight line instead of parabola for stress strain curve of concrete. as follows:

F.O.S = 1. Rectangular beam.

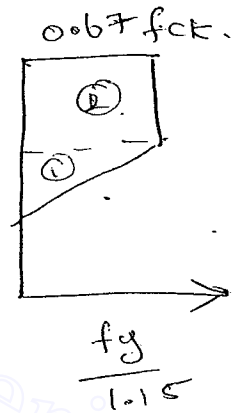
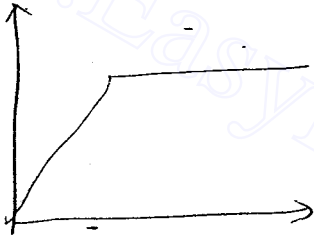
$b = 300\text{mm}, d = 500\text{mm}$

with 8 bars of 16mm ϕ M20, Fe415.

Depth of N.A is from the compressive fiber is _____.

Depth of the N.A. as obtained by Is 456 200. and the difference obtained above is _____.

case(i)



$F_c = F_t$

$F_{c1} + F_{c2} = 0.67 f_{ck} \times 3 \times \frac{\pi}{4} \times 16^2$

$(0.67 f_{ck} \times \frac{1}{2} \times b \times \frac{4x_u}{7}) + [0.67 f_{ck} \times b \times \frac{3x_u}{7}] = 217.78$

$[\frac{1}{2} \times \frac{4x_u}{7}] + [\frac{3x_u}{7}] = \frac{217.78}{0.67 \times 20 \times 300}$

$x_u = 54.174 \times \frac{7}{5}$

$x_u = 75.84 \text{ mm}$

Case (ii) : x_u

$$x_u = 99.3$$

$$F_c = F_{gt}$$

$$0.57x \cdot F_{c1} + F_{c2} = F_{gt}$$

$$\left[0.45 f_{ck} \times \frac{2}{3} \times b \times \frac{4}{7} x_u \right] + \left[0.45 f_{ck} \times b \times \frac{3}{7} x_u \right]$$

$$= 0.87 f_y A_{st}$$

$$\left[0.45 \times 20 \times \frac{2}{3} \times 300 \times \frac{4}{7} x_u \right] + \left[0.45 \times 20 \times 300 \times \frac{3}{7} x_u \right]$$

$$= 0.87 \times 45 \times \frac{\pi}{4} \times 16^2 \times 3$$

$$1028.57 x_u + 1157.14 x_u = 217.780 \times 10^3$$

$$x_u = 99.64 \text{ mm}$$

$$\text{Difference} = 99.64 - 75.84$$

$$= \underline{\underline{23.8 \text{ mm}}}$$

2.) In design of beam for L.S of collapse and flexure as per IS 456:2000. Strain in concrete is ~~limited to~~ limited to 0.0025. (Instead of 0.0003) For this situation consider a rectangular beam $b = 250\text{mm}$ $d = 350\text{mm}$. $A_{st} = 15\text{mm}^2$

Use $f_e \frac{250}{475}$, M_{30} .

(a) Depth of NFA for balanced failure. 156 mm

(b) @ L.S of collapse in flexure the Force acting on compression zone is —

Solution:

$$\frac{x_i}{x_u} = \frac{0.0025}{0.0025}$$

$$x_i = \frac{4}{5} x_u$$

$$x_2 = \frac{1}{5} x_u$$

$$F_c = F_t$$

$$\left[0.45 f_{ck} \cdot b \cdot x_u \cdot \frac{x_u}{3} \right] + 0.45 f_{ck} \cdot b \cdot x_u \cdot \frac{x_u}{3} = 0.87 f_y \cdot A_{st}$$

$$0.45 \times 30 \times$$

$$x_{u\max} = \left(\frac{E_c b c}{E_c c + \frac{E_{st}}{3}} \right) \times d$$

$$= \left[\frac{0.0025}{0.0025 + \left(\frac{0.87 \times 250}{2 \times 10^5} \right) + 0.002} \right] \times 350$$

$$x_{u\max} = 156.59 \text{ mm}$$

$$F_c = 0.45 f_{ck} \times \frac{2}{3} b x_1 + 0.45 \times b x_2 \times f_{ck}$$

$$= \left[0.45 \times 30 \times \frac{2}{3} \times 250 \times \frac{4}{5} \times u \right] + \left[\frac{30 \times}{1} \times 0.45 \times 250 \times \frac{u}{5} \right]$$

$$= \left[0.45 \times 30 \times \frac{2}{3} \times 250 \times \frac{4}{5} \times 156.59 \right] + \left[\frac{30 \times}{1} \times 0.45 \times 250 \times \frac{156.59}{5} \right]$$

$$= 281880 + (105.705 \times 10^3)$$

$$= 387.585 \text{ kN}$$

$$f_t = 0.87 f_y A_{st}$$

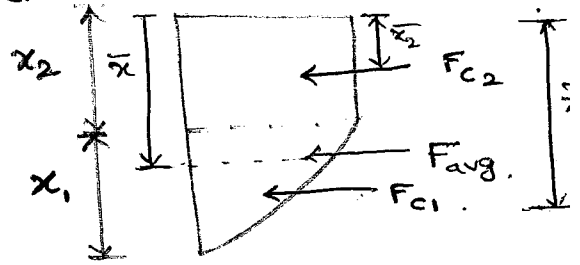
$$= 0.87 \times \frac{250}{15} \times 1500$$

$$= 326.25 \text{ kN}$$

LEVER ARM:

According to, Varignon's Principle.

$$F_{avg} \times \bar{x} = F_{c1} \times \bar{x}_1 + F_{c2} \times \bar{x}_2$$



$$F_{avg} = 0.36 f_{ck} \times b \times x_u$$

$$F_{c1} = \frac{2}{3} \times 0.45 f_{ck} \times b \times x_u$$

$$\bar{x}_1 = \frac{3x_u}{7} + \frac{3}{8} \times \frac{4}{7} x_u = \frac{9}{14} x_u$$

$$F_{c2} = 0.45 f_{ck} \times b \times x_u$$

$$\bar{x}_2 = \frac{1}{2} \times \frac{3}{7} x_u = \frac{3}{14} x_u$$

$$0.36 f_{ck} \times b \times x_u \times \bar{x} = \left[\frac{2}{3} \times 0.45 \times f_{ck} \times b \times x_u \times \frac{9}{14} x_u \right]$$

$$+ \left[0.45 f_{ck} \times b \times x_u \times \frac{3}{14} x_u \right]$$

$$0.36 f_{ck} \times b \times x_u \times \bar{x} = \frac{f_{ck}}{b} \times x_u^2 \times [0.1102 + 0.041]$$

$$\bar{x} = \frac{x_u}{0.36} [0.1102 + 0.041]$$

$$= x_u \frac{0.1515}{0.36}$$

$$\bar{x} = 0.42 x_u$$

$$\text{Lever arm} = d - \bar{x}$$

$$\text{Lever arm} = d - 0.42 x_u$$

$$F_c = F_t.$$

$$0.36 f_{ck} \times b \times x_{u_{max}} = 0.87 f_y A_{st}.$$

For Balanced section:

$$0.36 f_{ck} \times b \times x_{u_{max}} = 0.87 f_y A_{st}.$$

$$\frac{A_{st}}{bd} \times 100 = \frac{0.36 f_{ck} x_{u_{max}}}{0.87 f_y d} \times 100$$

$$P_{t \text{ lim}} = \frac{0.36 f_{ck}}{0.87 f_y} \times 100 \times \frac{x_{u_{max}}}{d}$$

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ANALYSIS OF SINGLY REINFORCED SECTION BY LIMIT STATE METHOD.

(i) To Find x_u

compressive force, $F_c = 0.36 f_{ck} b x_u$.

Tensile force, $F_t = 0.87 f_y A_{st}$.

$$F_c = F_t$$

$$0.36 f_{ck} b x_u = 0.87 f_y A_{st}$$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$$\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b d}$$

(ii) Limiting Neutral Axis : $(x_{u \text{ lim (or) } } x_{u \text{ max}})$

For		$\frac{x_{u \text{ max}}}{d}$
	Fe 250	0.5
	Fe 415	0.48
	Fe 500	0.46

(iii) Lever arm

$$z = d - 0.42 x_u$$

(iv) Moment of Resistance.

$$M \cdot R = 0.36 f_{ck} b x_u (d - 0.42 x_u)$$

For Balanced section

$$x_u = x_{u \text{ lim}}$$

$$M \cdot R_b = 0.36 f_{ck} b \times \frac{d}{d} \times x_{u \text{ max}} d \left(1 - 0.42 \frac{x_{u \text{ max}}}{d}\right)$$

$$M \cdot R = 0.36 f_{ck} b d^2 \frac{x_{u\max}}{d} \left[1 - 0.42 \frac{x_{u\max}}{d} \right]$$

$$M_{ulim} = 0.36 f_{ck} b d^2 \frac{x_{u\max}}{d} \left[1 - 0.42 \frac{x_{u\max}}{d} \right]$$

Moment of Resistance of Under Reinforced Section :

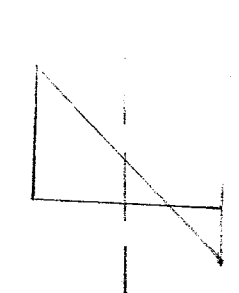
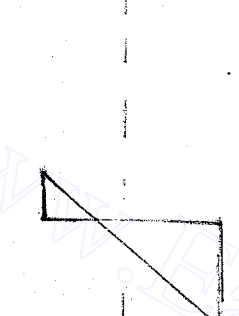
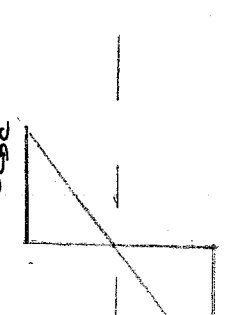
$$M \cdot R = 0.87 f_y A_{st} (d - 0.42 x_u)$$

$$= 0.87 f_y A_{st} \left[d - \frac{0.4 \times 0.87 \times f_y A_{st}}{0.36 f_{ck} b} \right]$$

$$= 0.87 f_y A_{st} d \left[1 - \frac{f_y A_{st}}{0.36 f_{ck} b d} \right]$$

$$M \cdot R = 0.87 f_y A_{st} d \left[1 - \frac{f_y A_{st}}{f_{ck} b d} \right]$$

Appendure G
in 1:1

Balanced Section.	Under Reinforced Section	Over Reinforced section.
 <p>compression.</p>		
<p>* Concrete and steel both attain max permissible strain simultaneously</p> <p>* $x_u = x_{u,max}$</p> <p>* $P_{actual} = P_t \text{ lim}$</p> <p>* $M_{u,lim} = 0.36 f_{ck} b x_{u,max} [d - 0.42 x_{u,max}]$</p> <p>$M_{u,lim} = 0.87 f_y A_{st} [d - 0.42 x_{u,max}]$</p> <p>* $M_{u,lim} = M_u$</p>	<p>* Strain in steel $>$ strain in concrete</p> <p>* Strain in steel $>$ Est</p> <p>* $x_u < x_{u,max}$</p> <p>* $P_{actual} < P_t \text{ lim}$.</p> <p>* $M_u = 0.87 f_y A_{st} d [1 - \frac{f_y A_{st}}{f_{ck} b d}]$</p> <p>* $M_u < M_{u,lim}$</p>	<p>* In this section steel does not attain its max permissible strain.</p> <p>* Actual stress $<$ Est. in steel.</p> <p>* $x_u > x_{u,max}$.</p> <p>* $P_t \text{ actual} > P_t \text{ lim}$.</p> <p>* $M_u < M_{u,lim}$</p> <p>* $M_u = f_{sc} A_{st} (d - 0.42 x_u)$</p> <p>* $M_u > M_{u,lim}$</p> <p>* As per IS 456 2000 over reinforced section is not at all allowed hence it is designed as Balanced section (or) Doubly reinforced section.</p>

1. A singly reinforced concrete beam has
 $b = 150\text{mm}$ and $d = 330\text{mm}$. $f_{ck} = 20\text{ N/mm}^2$
 $f_y = 415\text{ MPa}$. Adopts the stress block diagram
for concrete as given in IS 456:2000
and take $x_{ulim} = 0.48d$. What is the
 M_{lim} in $\text{KN}\cdot\text{m}$. Limiting Area of tension
steel.

Solution:

$$M.R = 0.36 f_{ck} b d^2 \frac{x_{max}}{d} \left[1 - 0.42 \frac{x_{max}}{d} \right]$$

$$= 0.36 \times 20 \times 150 \times (330)^2 \times 0.48 \left[1 - 0.42 \times 0.48 \right]$$

$$M_{lim} = 45.07 \text{ KN}\cdot\text{m}$$

$$M_{lim} = 0.87 f_y A_{st} d$$

$$\frac{A_{st}}{bd} = \left[\frac{0.36 f_{ck} \frac{x_{max}}{d}}{0.87 f_y} \right]$$

$$\frac{A_{st}}{150 \times 300} = \frac{0.36 \times 20 \times 0.48}{0.87 \times 415}$$

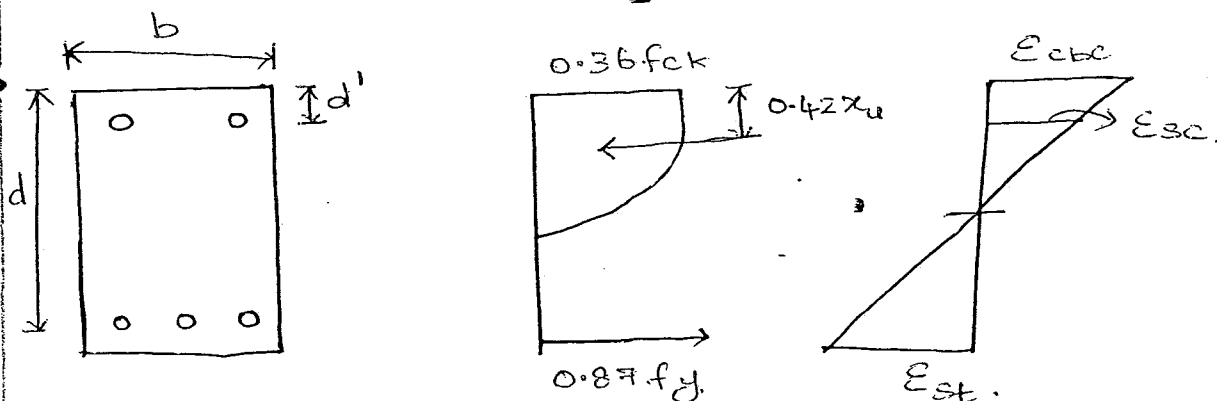
$$A_{st} = 473.81$$

$$A_{st} = 475 \text{ mm}^2$$

Concrete Grade	Mix Proportion	Mix Method	f_{ck} @ 28 days	$\sigma_{cbc} = \frac{f_{ck}}{\gamma_c}$	$\sigma_{cc} = \frac{f_{ck}}{\gamma_c}$	T_{bd} (Fe 250)	m ($\frac{280}{f_{ck}}$)	Purpose.
M5	1:5:10	Nominal mix	5	-	-	-	-	* Lean concreting - Foundation
M7.5	1:4:8	Nominal	7.5	-	-	-	-	* Lean concreting
M10	1:3:6	Nominal	10	3	2.5	-	-	* Mass concreting (ordinary concreting)
M15	1:2:4	Nominal	15	5	4	0.6	19 (18-67)	* Mass concreting (As per IS 456:2000)
M20	1:1.5:3	Nominal	20	7	5	0.8	13	* ordinary concreting (As per IS 456:1978)
M25	1:1:2	Design mix	25	8.5	6	0.9	11	* Ordinary R.C.C, Mild Exposure.
M30	-	Design mix	30	10	8	1	9.5	* water Tank, Foundation, Moderate Exposure.
M35	-	Design mix	35	11.5	9	1.1	8	* Minimum grade of Post tensioned concrete, Severe. Exposure.
M40	-	Design mix	40	13	10	1.2	7	* Aggressive sub soil, Very Severe exposure
M45	-	Design mix	45	14.5	11	1.3	-	* Pre tensioned concrete, Extreme exposure
M50	-	Design mix	50	16	12	1.4	-	* Pre tensioned

- Analysis and design of tension and Compression members, beam-column and column bases.
- Connections:
 - * simple and eccentric
 - * Beam - column connections
 - * plate girders and trusses.
- Plastic analysis of beams and frames.

DOUBLY REINFORCED BEAM (L.S.M):



By similar triangle property.

$$\frac{\epsilon_{sc}}{\epsilon_{cbc}} = \frac{(x_u - d')}{x_u}$$

$$\epsilon_{sc} = \frac{\epsilon_{cbc} (x_u - d')}{x_u}$$

$$\epsilon_{sc} \times E = f_{sc}$$

$$f_{sc} > 0.87 f_y$$

$$F_c = 0.36 f_{ck} b x_u + f_{sc} A_{sc}$$

$$F_t = 0.87 f_y A_{st}$$

$$M.R = 0.36 f_{ck} b x_u (d - 0.42 x_u)$$

$$+ f_{sc} A_{sc} (d - d')$$

x_u :

$$F_c = F_t$$

$$0.36 x_u b f_{ck} + f_{sc} A_{sc} = 0.87 f_y A_{st}$$

Take

$$x_u = 0.7 - 0.8 (x_{u\max})$$

$$x_u = 0.75 x_{u\max}$$

STEPS TO SOLVE DOUBLY REINFORCED

BEAM:

1. Assume, $x_u = 0.7$ to $0.8 x_{u \max}$

2. Find out E_{sc} .

3. \rightarrow Find out f_{sc} where

4. $f_{sc} = E_{sc} \times E_s$ - for mild steel only

\rightarrow For deformed bar find E_{sc}

and find f_{sc} from given graph.

4.) Find out the actual value of x_u by equating $F_c = F_t$.

$$0.36 f_{ck} b x_u + f_{sc} A_{sc} = 0.87 f_y A_{st}$$

5.) After finding value of x_u from above equation compare it with assumed value of x_u .

If both are equal then no need to take next trial.

But if not take some more trial

6.) Find out $x_{u \max}$.

$x_{u \max}$	f_e
0.53d	250
0.48d	415
0.46d	500

7.) Compare x_u and $x_{u \max}$ and decide the type of section.

8.) Find $M_u \lim$,

9.) If $M_{extermal} > M_{ulim}$ then the

section must be designed as
balanced doubly reinforced section.
where, $x_u = x_{u,max}$ and

M.R is given by the formula,

$$M_u = 0.36 f_{ck} b x_{u,max} (d - 0.42 x_{u,max}) + f_{sc} A_{sc} (d - d')$$

$$= 0.36 f_{ck} b x d x \frac{x_{u,max}}{d} (d - 0.42 \frac{x_{u,max}}{d}) + f_{sc} A_{sc} (d - d')$$

$$M_u = M_{u,lim} + f_{sc} A_{sc} (d - d')$$

$$A_{sc} = \frac{M_u - M_{u,lim}}{f_{sc} (d - d')}$$

10.) Find out A_{st} from

$$F_c = F_t$$

$$0.36 x_u b f_{ck} = 0.87 f_y A_{st1}$$

$$f_{sc} A_{sc} = 0.87 f_y A_{st2}$$

$$M_u = 0.87 f_y A_{st1} (d - 0.42 x_{u,max})$$

$$A_{st} = A_{st1} + A_{st2}$$

1. size of an R.C.C beam is restricted

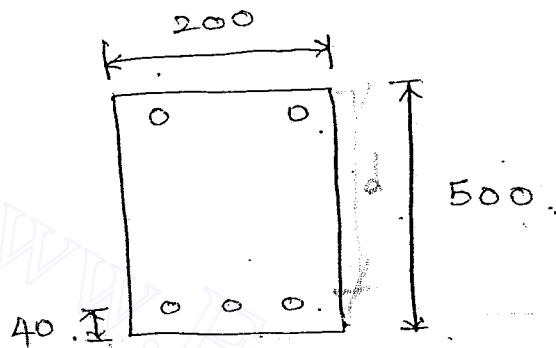
to $b \times 250 \times 500 \text{ mm}$. It carries

I.L = 25 kN/m $l = 6 \text{ m}$. Determine

the reinforcement. Use M_{20} , $F_e 415$.

use following table given.

stress.	$0.8f_{cd}$	$0.83f_{yd}$	0.9	0.975	0.975	f_{yd}
Strain.	0.00144	0.00163	0.00192	0.0024	0.00216	0.003



$$d = 500 - 40$$

$$= 460 \text{ mm}$$

$$M_{u \text{ lim}} = 0.36 f_{ck} b^2 \frac{x_{u \text{ max}}}{d} \left(1 - 0.42 \frac{x_{u \text{ max}}}{d} \right)$$

$$= 0.36 \times 20 \times 250^2 \times 0.48 \left(1 - 0.42 \frac{0.48}{0.48} \right)$$

$$M_{u \text{ lim}} = 145.96 \text{ kN}\cdot\text{m}$$

$$M_u = \frac{w_u l^2}{8}$$

$$= \frac{42.18 \times 6^2}{8}$$

$$w_u = 1.5 (D \cdot L + I \cdot L)$$

$$= 1.5 (0.25 \times 0.5 \times 25 + (25))$$

$$w_u = 42.18 \text{ kN}$$

$$M_u = 189.84 \text{ kN}\cdot\text{m}$$

$$M_u > M_{u \text{ lim}}$$

Design as doubly reinforced beam.

$$\epsilon_{sc} = \frac{\epsilon_{cbc} (x_{u\max} - d')}{x_{u\max}}$$

$$= \frac{0.0035 ((0.48 \times 460) - 40)}{(0.48 \times 460)}$$

$$\epsilon_{sc} = 0.002865$$

$$f_{sc} = 0.976 f_{yd}$$

$$= 0.976 \times 0.87 \times 415$$

$$f_{sc} = 352.39 \text{ N/mm}^2$$

$$M_u - M_{u\lim} = f_{sc} A_{sc} (d - d')$$

$$A_{sc} = \frac{(189.84 - 145.96) \times 10^6}{352.39 \times (460 - 40)}$$

$$A_{sc} = 296.47 \text{ mm}^2$$

$$F_c = F_t$$

$$(0.36 f_{ck} b x_u) + f_{sc} A_{sc} = 0.87 f_y A_{st}$$

$$(0.36 \times 20 \times 250 \times 0.48 \times 460) + (352.39 \times 296.47) = 0.87 \times 415 \times A_{st}$$

$$A_{st} = 1390 \text{ mm}^2$$

$$M_{u\lim} = 0.87 f_y A_{st1} (d - 0.42 x_{u\max})$$

$$145.96 \times 10^6 = 0.87 \times 415 \times A_{st1} (460 - 0.42 (0.48 \times 460))$$

$$A_{st1} = 1100.75 \text{ mm}^2$$

$$f_{sc} A_{sc} = 0.87 f_y A_{st2}$$

$$352.39 \times 296.47 = 0.87 \times 415 \times A_{st2}$$

$$A_{st2} = 289.35 \text{ mm}^2$$

2. $200 \times 450 \text{ mm}$ is reinforced with
 4 # 20 mm in Tension 3 # 12 mm in
 compression Determine $M_u \text{ lim}$
 use M_{20} and $f_{ck} = 25$.

$$x_u = 0.75 \times 0.53 \times 410$$

$$x_u = 162.975 \text{ mm}$$

$$M_u = 0.36 f_{ck} b x_u$$

$$E_{sc} = \frac{E_{cfc} (x_u - d')}{x_u}$$

$$= \frac{0.0035 (162.975 - 40)}{162.975}$$

$$f_{sc} = E_{sc} \times E_s$$

$$= 2.6409 \times 10^{-3} \times 2 \times 10^5$$

$$f_{sc} = 528.19 \text{ N/mm}^2$$

$$M_u = 0.36 f_{ck} b x_u (d - 0.42 x_u) + f_{sc} A_{sc} (d - d')$$

$$= 0.36 \times 20 \times 200 \times 162.975 [410 - 0.42(162.975)]$$

$$+ 528.19 \times 3 \times \frac{\pi \times 12^2}{4} \times (410 - 40)$$

$$M_u = 146.46 \text{ kN.m}$$

3. Determine M_{ultim} of R.C.C of size 250×500 mm overall is reinforced with 3 # 20 mm in compression and 4 # 20 mm in Tension. $d' = 40$ mm. Use M_{20} and Fe415.

$$\begin{aligned} x_u &= 0.75 x_{u,max} \rightarrow 0.48d \\ &= 0.75 \times 0.48 \times 460 \rightarrow d \\ x_u &= 165.6. \end{aligned}$$

$$\begin{aligned} \epsilon_{sc} \quad \epsilon_{sc} &= \frac{\epsilon_{cbc} (x_u - d')}{x_u} \\ &= \frac{0.0035 (165.6 - 40)}{165.6} \\ &= 0.00265. \end{aligned}$$

$$f_{sc} = 0.967 f_{yd}$$

$$= 0.967 \times 0.87 \times 415.$$

$$f_{sc} = 349.187 \text{ N/mm}^2 \leftarrow 361.05$$

$$M_{ultim} = 0.36 f_{ck} b x_u (d - 0.42 x_u)$$

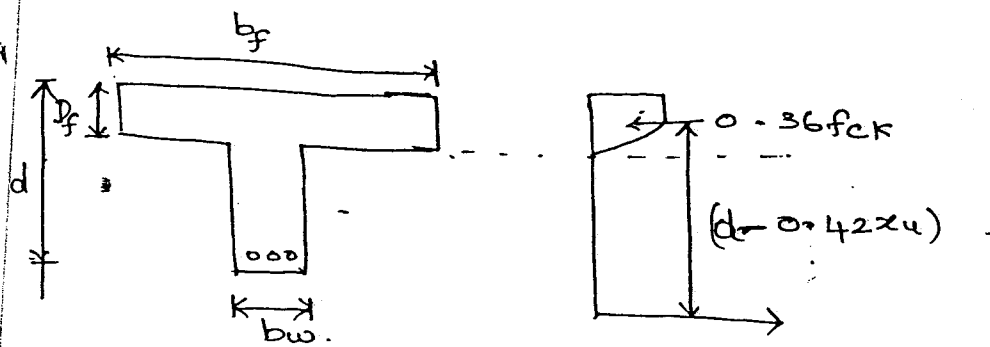
$$+ f_{sc} A_{sc} (d - d')$$

$$= 0.36 \times 20 \times 250 \times 165.6 (460 - 0.42 \times 165.6)$$

$$+ 349.187 \times \frac{\pi}{4} \times 3 \times 20^2 \times (460 - 40)$$

$$M_{ultim} = 254.6 \text{ kNm}$$

T-BEAM (L.S.M)



Case (i)

(i) N.A lies with flange (Design as Rectangular Beam):

$$F_c = F_t$$

$$0.36f_{ck} b_f x_u = 0.87f_y A_{st}$$

$$x_u = \frac{0.87f_y A_{st}}{0.36f_{ck} b_f}$$

M.R of Beam:

$$M.R = 0.36f_{ck} b x_u (d - 0.42x_u)$$

$$M.R = 0.87f_y A_{st} (d - 0.42x_u)$$

Case (ii): N.A lies outside the flange.

$$\frac{I_f}{d} \leq 0.2$$

$$F_c = F_{c1} + F_{c2}$$

$$F_c = 0.36f_{ck} b_w x_w + 0.45f_{ck} (b_f - b_w) \frac{D_f}{2}$$

$$M.R = 0.36f_{ck} b_w x_w (d - 0.42x_w) + 0.45f_{ck} (b_f - b_w) \frac{D_f}{2} (d - \frac{D_f}{2})$$

$$I_f \frac{D_f}{d} > 0.2. \quad \text{Put } D_f = y_f.$$

$$y_f = 0.15 x_u + 0.65 \bar{D}_f.$$

$$M_o R = 0.36 f_{ck} b_w x_u (d - 0.42 x_u) + 0.45 f_{ck} (b_f - b_w) y_f \left(d - \frac{y_f}{2} \right)$$

1. A large hall of inner dimension $6m \times 30m$ is provided with no. of ^{R.C.C.} beams monolithic constructed with slab along shorter span thickness of brick wall 30cm thickness of slab = 150mm $b_w = 250mm$, overall depth is 550mm. 4 # 20mm bars in tension zone. Determine.
- (i) Effective width of flange.
 - (ii) Depth of A.N.A
 - (iii) Type of section.
 - (iv) Ultimate Moment of resistance.
 - (v) Safe Super imposed load.

Use M_{20} and $Fe 250$. Spacing of beam is 3m c/c.

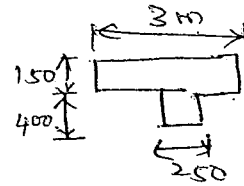
Solution: $d = D - 40 = 550 - 40 = 510$

(i) L_{eff}

$$L_{eff} = 6m + \frac{0.3}{2} + \frac{0.3}{2} = 6.3m.$$

$$L_{eff} = 6m + 0.51 = 6.51m.$$

$$L_{eff} = 6.3m$$

(ii) b_f :

$$b_f = \frac{l_o}{6} + b_w + 6D_f$$

$$= \left(\frac{6300}{6} \right) + 250 + 6(150)$$

$$\boxed{b_f = 2200 \text{ mm}} \quad \angle \quad 3000 \text{ mm}$$

(iii) x_u :

$$F_c = F_t$$

$$0.36 f_{ck} b_f x_u = 0.87 f_y A_s t$$

$$x_u = \frac{(0.87 \times 250 \times 4 \times \pi \times 20^2)}{0.36 \times 20 \times 2200}$$

$$= 17.25 \text{ mm} \quad \angle \quad 150 \text{ mm}$$

(iv) $M.R$

$$M.R = 0.36 f_{ck} b_f x_{u_{max}} (d - 0.42 x_{u_{max}})$$

$$= 0.36 \times 20 \times 2200 \times 0.53 \times 510$$

$$\left(510 - (0.42 \times 0.53 \times 510) \right)$$

$$M_{u_{lim}} = 1.697 \times 10^3 \text{ KN}\cdot\text{m}$$

(v)

$$M_u = \frac{\omega l^2}{8}$$

$$1.697 \times 10^3 = \frac{\omega \times (6.3)^2}{8}$$

$$\omega = 342.15 \times 10^3 \text{ KN}$$

$$\omega = D.L + I.L$$

$$342.15 \times 10^3 = (0.15 \times 3 \times$$

(iv)

$$M_R = 0.36 f_{ck} b_f x_u (d - 0.42 x_u)$$

$$= 0.36 \times 20 \times 2200 \times 17.25 (d - (0.42 \times 17.25))$$

$$M_R = 137.37 \text{ kN}\cdot\text{m}$$

(v)

$$M_R = \frac{\omega_u l_{\text{eff}}^2}{8}$$

$$\omega_u = \frac{137.37 \times 8}{6.3^2}$$

$$\omega_u = 27.68 \text{ kN}$$

$$\omega = \frac{27.68}{1.5} = 18.46 \text{ kN}$$

$$\omega_u = D \cdot L + L \cdot L$$

$$18.46 = \left[(0.15 \times 3) + (0.4 \times 0.23) \right] \times 25$$

$$+ L \cdot L$$

$$18.46 = 13.75 + L \cdot L$$

$$L \cdot L = 4.71 \text{ kN/m}$$

$$L \cdot L = \frac{4.71}{3 \times 1}$$

$$L \cdot L = 1.57 \text{ kN/m}^2$$

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2. Determine ultimate M_R of T-beam.

$$b_f = 750 \text{ mm} \quad D_f = 120 \text{ mm} \quad D = 550 \text{ mm}$$

$$d = 500 \text{ mm} \quad A_{st} = 5 \# 25 \text{ mm } \phi$$

$$b_w = 300 \text{ mm} \quad \text{Use } M_{20} \text{ Fe } 415$$

Solution:

Assume $x_u < D_f$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b_f}$$

$$= \frac{0.87 \times 415 \times 5 \times \frac{\pi \times 25^2}{4}}{0.36 \times 20 \times 750}$$

$$= 164.1 \text{ mm} < 150 \text{ mm}$$

$$\frac{D_f}{d} = \frac{120}{500} = 0.24 > 0.2$$

$$D_f = y_f \quad y_f = 0.15 x_u + 0.65 D_f$$

$$0.36 f_{ck} b_w x_u + 0.45 f_{ck} (b_f - b_w) D_f = 0.87 f_y A_{st}$$

$$0.36 \times 20 \times 300 \times x_u + 0.45 \times 20 \times (750 - 300) \times (0.15 x_u + 0.65 \times 120) = 0.87 \times 415 \times 5 \times \frac{\pi \times 25^2}{4}$$

$$2160 x_u + 4050 (0.15 x_u + 78)$$

$$2767.5 x_u + 315900 = 394875 \quad \Rightarrow \quad x_u = 206.052 \text{ mm}$$

$$x_u = 206.052 \text{ mm}$$

$$y_f = 0.15 (206.052) + 0.65 (120)$$

$$= 104.63 \text{ mm} < D_f$$

$$M.R = 0.36 f_{ck} b_w x_u (d - 0.42 x_u)$$

$$+ 0.45 f_{ck} (b_f - b_w) x_{uf} \left(d - \frac{x_{uf}}{2} \right)$$

$$= 0.36 \times 20 \times 300 \times 206.052 (500 - 0.42 (206.052))$$

$$+ 0.45 \times 20 (750 - 300) \times \frac{104.63}{2}$$

$$\times (500 - 104.63)$$

$$M.R = 351.46 \text{ KN}\cdot\text{m}$$

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STEEL STRUCTURES

→ Analysis and design of tension and compression members, beam-column and column bases.

→ Connections:

* simple and eccentric

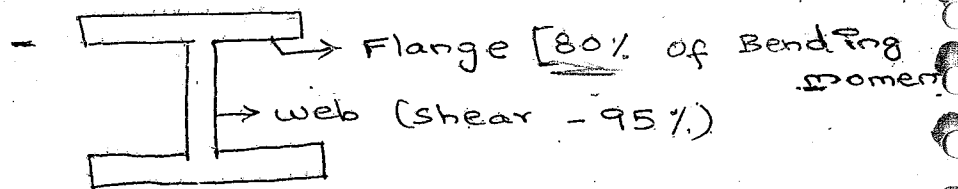
* Beam - column connections

* plate girders and trusses.

→ Plastic analysis of beams and frames.

STEEL STRUCTURES

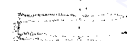

- * $\gamma_{\text{con}} = 2500 \text{ kg/m}^3$ (R.C.C)
- * $\gamma_{\text{steel}} = 7850 \text{ kg/m}^3$
- * Structural steel $\mu = 0.3$
- * Rigidity modulus of steel = $0.769 \times 10^5 \text{ MPa}$.
- * Specific gravity of steel = 7.85.



* Drainage cover plate → checkers plate

- * Steel structure: Made up of structural steel

Section Available:

- * Flate plate. 
- * Angle section 
- * channel section.
- * I - section
- * T - section.

- * All structural steel consists of mild steel - 250

Truss - All joints are pinned.

- * I - section is very strong section.

There is no torsion moment produced

In I - section.

STEEL STRUCTURES:

* Steel structures is made up of structural steel and structural steel are made up of, hot rolled section.

* Structural steel is of mild steel and of grade Fe 250.

* Mild steel is used because it is highly ductile and hence can be moulded into shapes.

Advantages of Steel structures:

* Strength of depth ratio, is high.

* Prefabricated

* ~~Reusable~~ Reusable

* High scrap value.

* Can be dismantled easily.

Disadvantage of Steel structures:

* Corrosion is high.

* Vibration and noise.

* Fire resistance is low.

* Fatigue failure.

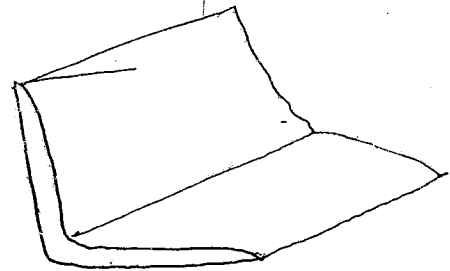
Type Of Steel Section:

* Flat Plate:

- * Designated by thickness \times breadth.
- * Used for walkways, plate girder.
- * Thickness shall not be less than 8 mm when maintenance is not possible.
- * Thickness shall not be less than 6 mm when maintenance is possible.
- * Thickness shall not be less than 4 mm in case of secondary member.

* Angle Section:

- * It is designated by length \times breadth \times thickness.



- * Ex : ISA 90 x 90 x 6 mm - Equal angle.

ISA - Indian standard angle.

- * $100 \times 90 \times 6$ where, 100 \rightarrow longer leg
90 - shorter leg
6 - thickness.

(generally longer legs are connected legs)

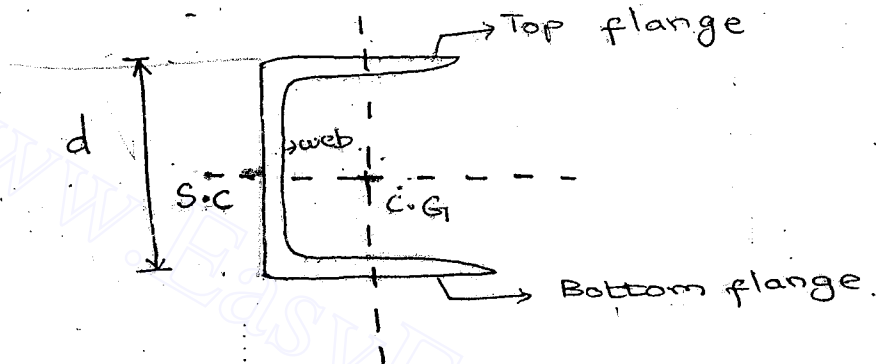
- * Maximum available angle section is

ISA - 200 x 200 x 25

- * Uses:

- * Purlins, Bottom chord, lacings.

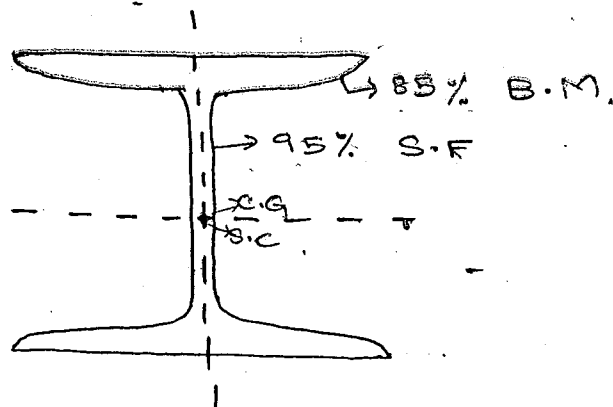
- * Designated by ISJC, ISLC, ISMC, ISGC, MCP (Medium channel with parallel flange).
- * Shear centre lies outside the section
- * It is subjected to the torsional moment when it is loaded.
- * Used as purlins, secondary beams (or) where light loads occurs and as bottom chord.
- * Maximum available section is ISMC-400



I-section:

- * Designated by ISJB, ISLB, ISMB, ISWB, and ISHB, SC

- ISJB - Indian standard Junior Beam
- ISLB - Indian standard light Beam
- ISMB - Indian standard Medium Beam
- ISWB - Indian standard wide flange Beam
- ISHB - Indian standard Heavy Beam
- SC - Indian column section



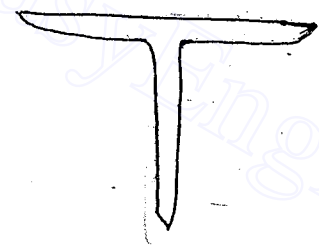
* Used as beams and columns for heavy load maximum available I-section is ISMB-⁶⁰⁰

- when $\frac{d}{t} < 85$ (No stiffener is used)
- $85 < \frac{d}{t} < 200$ (use stiffeners)
- $\frac{d}{t} = [85 - 200]$ (vertical stiffener)
- $\frac{d}{t} > 200$ (Horizontal stiffener along with vertical stiffener)

* Flanges are responsible for 85% of BMC
* web ^{is} ~~are~~ responsible for 95% of Shear force

Uses: for beams and columns.

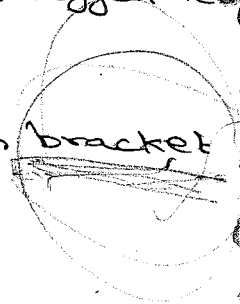
T-section:



- * Designated by
 - ISNT - Indian standard Normal Tee.
 - ISH T - Indian standard heavy weight Tee
 - ISMT - Indian standard Medium weight Tee
 - ISLT - Indian standard light weight Tee
 - ISDT - Indian standard Deep legged Tee

* Designated by Dx b x t.

* Used in water tank and in bracket connections.

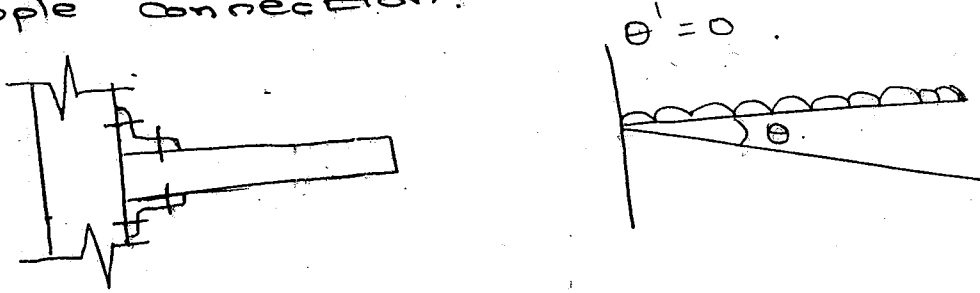


Connections:

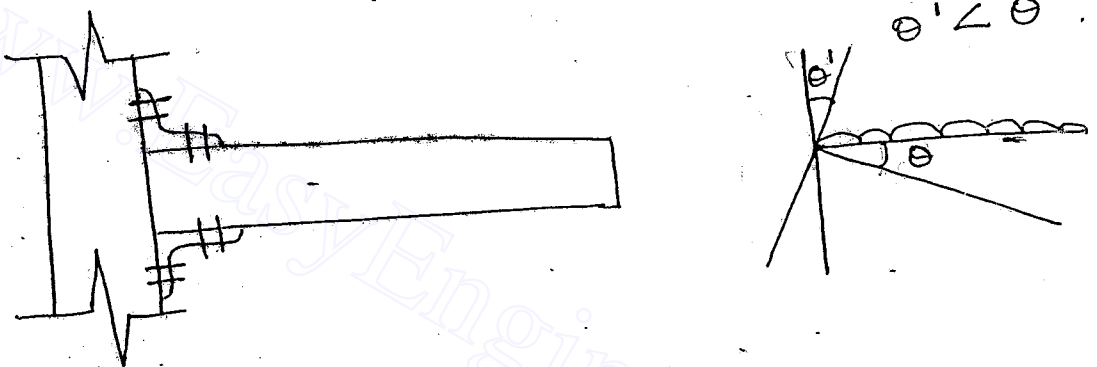
There are 3 types of connection.

- * Simple Connection.
- * Semi-rigid connection.
- * Rigid connection.

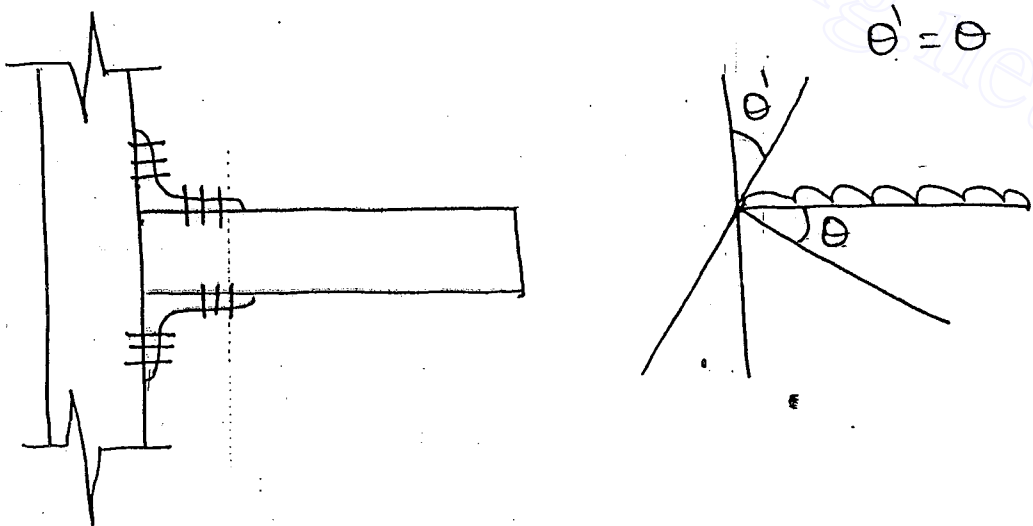
Simple connection:



Semi-rigid connection:



Rigid connection:



STEEL

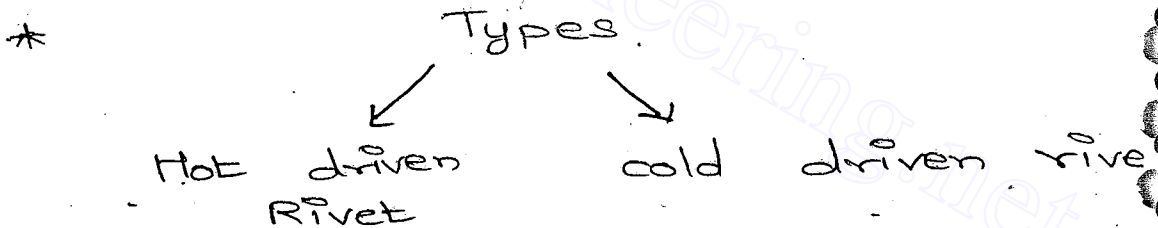
Connection:

* It is an important part in steel structure because unlike concrete structure it is not casted monolithically.

* Initially connections were made using rivets but now they are obsolete.

* Riveted Connections:

* Riveted connection has a disadvantage of making erection stresses and high temperature is required for erection (Hot driven Rivet)

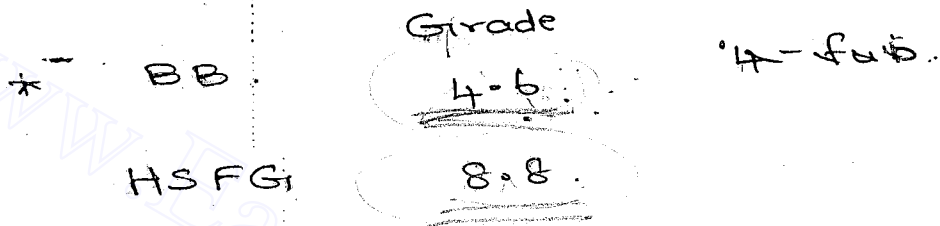
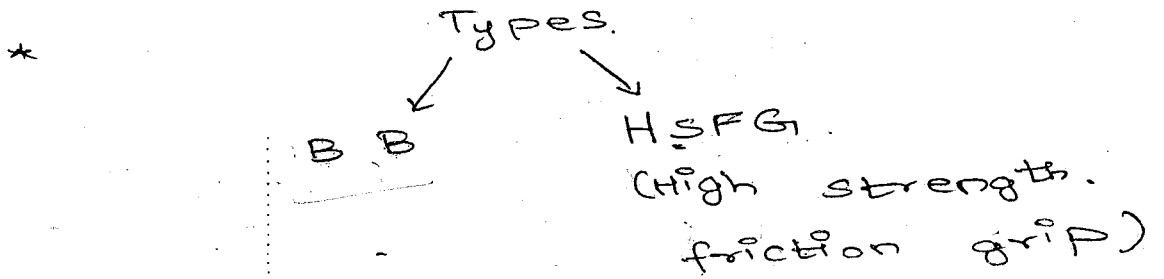


* In hot driven rivet steel is heated up to melting point and then rivet is inserted in the joint.

* cold driven rivet involves pressure injection or pressure erection.

* Gross area of plate is get affected in riveted connection.
Hence net area affects the load carrying capacity of plate

Bolted connection:



* 4 * $\frac{1}{100th}$ of ultimate stress

i.e $f_{ub} = 4 \times 100 = \underline{400 \text{ MPa}}$

0.6 - the yield stress.

i.e $f_{yb} = 0.6 \times 400 = \underline{240 \text{ MPa}}$

* HSFG Bolt: (8.8) \rightarrow Grade

$f_{ub} = 8 \times 100 = \underline{800 \text{ MPa}}$

$f_{yb} = 0.8 \times 800 = \underline{640 \text{ MPa}}$

$\text{MPa} = \text{N/mm}^2 \Rightarrow \frac{\text{N}}{\text{mm}^2} \times 1000 \Rightarrow \frac{\text{N}}{\text{m}^2}$

Advantages of Bolted connection:

- * Easy erection
- * It can be dismantled easily.
- * It can be reused.
- * No skilled labour required.

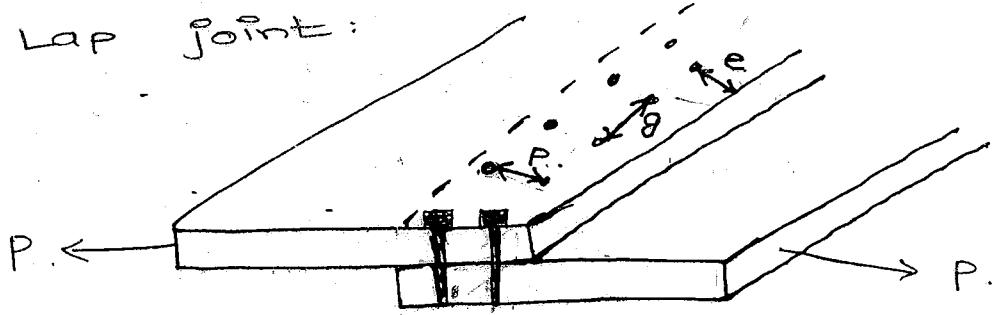
Disadvantage:

- * Bolt connection get loosen due to vibration.
- * Gross area of plate is affected.
- * Extra weight is added to the structure.

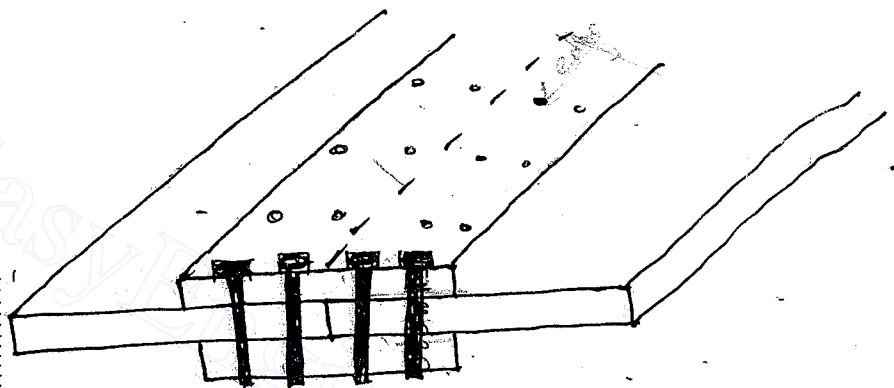
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Type of joints:

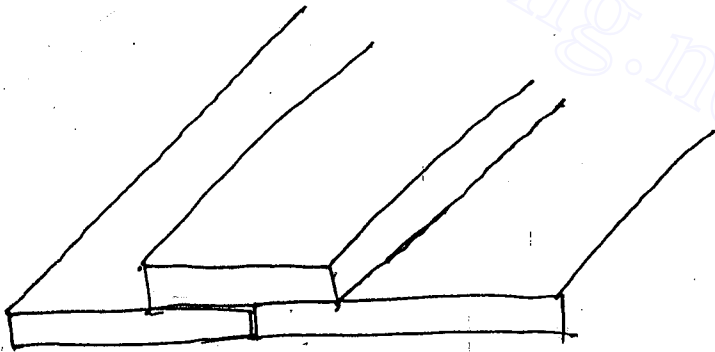
* Lap joint:



* ^{Double.} Single cover butt joint.



* Single cover butt joint.



Types of failure of bolt.

* Shearing failure bolt

→ single shear

→ Double shear

* Bearing failure of bolt

→

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18/8/15

D.S.S.

Pitch Distance:

It is the c/c distance ^{between} 2 consecutive bolts measured along parallel to direction of forces (or) stresses in a member.

In a wide plate pitch may also be defined as c/c distance ^{between} of bolt along measured along length of member (or) connection.

When bolts are placed in a staggered manner the pitch calculated is called staggered pitch.

Minimum pitch : $2.5d$

where d - nominal dia of bolt.

Max pitch $\rightarrow 12t$ (or) 200mm for Comp

" $\rightarrow 16t$ (or) 200 for tension

$\rightarrow 32t$ (or) 200 for locking or stick (not exposed to weather)

$\rightarrow 16t$ (or) 200 for locking or stick (exposed to weather)

In case of tee flat plates, angles, channels or I-section

for Comp member Max distance $\rightarrow 600$ mm

for tension member Max distance $\rightarrow 1000$ mm

- * Maximum pitch \rightarrow $12t$ (or) 200 mm whichever is less for compression member.
- * Maximum pitch \rightarrow $16t$ (or) 200 mm whichever is less for tension member..
- * $32t$ (or) 300mm whichever is less for tacking (or) stitching of plates (when the plates are not exposed to weather).
- * $16t$ (or) 200mm whichever is less for tacking (or) stitching of plates (when the plates are exposed to weather).
- * The distance b/w the centre of any two consecutive bolts should not exceed $32t$ (or) 300mm whichever less.
- * In case of two flat plates, angles, channels (or) Tee-sections.
 Maximum pitch of tacking bolts.
 (In which tack (or) stick bolts are to be provided along the length to connect each of them)
 - \rightarrow Not exceeding 600mm for compression member.
 - \rightarrow Not exceeding 1000mm for tension member.

Gauge Distance: (2)

* It is the distance b/w adjacent to the bolt line (or) the c/c distance b/w two consecutive bolts measured along the width of the member (or) the connection.

Maximum gauge should not be more than $(100 + 4t)$ (or) 200mm whichever is less.

End Distance:

It is the distance from centre of bolt hole to the nearest edge of member (or) cover plate in the direction of stress (or) force.

Edge Distance:

It is the distance from centre of bolt hole to the nearest edge of member (or) cover plate at right angle to the direction of stress.

* Minimum edge distance = $1.7 d_H$

d_H = dia of bolt hole for sheared (or) hand flame cut edges.

* Minimum edge distance = $1.5 d_H$.

In case of rolled, machine flame cut edge

Maximum Edge distance:

* Maximum edge distance to nearest edge of bolt hole to an edge unstiffened part should not exceed. $\rightarrow 12t + \xi$

where, $\xi = \sqrt{\frac{250}{f_y}}$

t - thickness of thinner outside plate

* $40 \text{ mm} + 4E$

where,

t = thickness of thinner outside plate
(for corrosive environment)

ϕ of bolt holes (d_o):

$d_o = \text{Nominal dia of bolt} + 1 \text{ mm}$

for bolt ϕ (12mm & 14mm)

$d_o = \phi \text{ of bolt} + 2 \text{ mm}$

for bolt ($\phi < 24 \text{ mm}$) \Rightarrow (16-24mm)

$d_o = \phi \text{ of bolt} + 3 \text{ mm}$

for bolt ($\phi > 24 \text{ mm}$)

Failure of Bolted Connection:

The failure of connections with bearing bolts in shear involves either bolt failure (or) the failure of the connected plates.

Types:

- * Shearing failure of bolt.
- * Bearing failure of bolt.
- * Tension failure of bolt.
- * Bearing failure of plate.
- * Tearing failure of plate.
- * Block shear failure.

Design Strength Of Bearing Type Bolts

Against Factor Shear Force:

(a) Design Shear Strength Of Bolts (V_{dsb})

Bolt subjected to factor shear force

$$V_{dsb} = \frac{V_{nsb}}{\gamma_{mb}}$$

$$\gamma_{mb} = 1.25$$

where,

V_{nsb} → Nominal shear strength of bolt.

γ_{mb} → Partial factor of safety of material for bolts.

$$V_{nsb} = \frac{f_{ub}}{\sqrt{3}} (n_p A_{nb} + n_s A_{sb})$$

$$V_{dsb} = \frac{f_u}{\sqrt{3}} (n_p A_{nb} + n_s A_{sb})$$

where,

f_{ub} - ultimate tensile strength of bolt

n_p - no. of shear plane with threads

intercepting the shear plane.

n_s → no. of shear plane without threads intercepting the shear plane.

$$A_{sb} \rightarrow \frac{\pi}{4} d^2$$

$$A_{nb} \rightarrow \frac{\pi}{4} d^2 \times 0.78 \quad (\text{or}) \quad 0.78 \times A_{sb}$$

A_{nb} \rightarrow NET tensile area @ thread

A_{sb} \rightarrow nominal plain shank area of the bolt

γ_{mo} - Yield stress

γ_{mi} - ultimate stress.

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DESIGN OF STEEL STRUCTURE

$$V_{dsb} = \frac{(f_u / \sqrt{3})}{\gamma_{mb}} \left[n_n A_{nb} + n_s A_{sb} \right] \times \beta_{lj} \times \beta_{lg} \times \beta_{kg}$$

β_{lj} - Reduction factor for long joints.

* In long joints distance b/w first and last joint exceeding 15d in the direction of load, the nominal shear capacity of bolt can be reduced by factor β_{lj}

$$\beta_{lj} = 1.075 \left(\frac{l_j}{200d} \right)$$

where l_j - Length of joint

$$\left[\beta_{lj} \right] \quad (0.75 \leq \beta_{lj} \leq 1.0)$$

β_{lg} - Reduction factor for long grip length

* when grip length of bolt increases (if it exceeds 5d (5 times nominal dia)) the bolt subjected to greater B.M. due to shear force acting on its shank

$$\beta_{lg} = \frac{8d}{3d + l_g}$$

l_g - grip length shall not be greater than 8d.

β_{pkg} - Reduction factor for packing plate

* when packing plate is more than 6mm thick the shank of bolt is subjected to bending which affects the nominal shear capacity of the bolt:

$$\beta_{pkg} = 1.0 - 0.0125 t_{pkg}$$

t_{pkg} - thickness of thicker ^{packing} plate.

DESIGN BEARING STRENGTH OF BOLT AND

PLATE: (V_{dpb})

$$V_{dpb} = \frac{V_{npb}}{\gamma_{mb}}$$

where

$$V_{npb} = 2.5 k_b \times d \times t \times f_u$$

$$k_b = \frac{e}{3d_0} \text{ (or) } \left(\frac{P}{3d_0} - 0.5 \right) \text{ (or) } \frac{f_{ub}}{f_u} \text{ (or) } 1$$

whichever is less.

V_{npb} - Nominal bearing strength of both.

* e , and P are the edge and pitch distance of the fastener along the direction length of bearing.

f_{ub} → ultimate tensile stress of the bolt

f_u → smaller of the ultimate tensile stress of bolt (or) plate

d - dia of bolt in mm.

t = thickness of connected plates experience bearing stress in the same direction.

DESIGN TENSILE STRENGTH OF BOLT: (T_{db})

The nominal tensile strength of bolt in tension is given by.

$$T_{nb} = 0.9 f_{ub} A_{nb} \leq \frac{f_{yb} A_{sb}}{\gamma_{m0}} \frac{\gamma_{mb}}{\gamma_{m0}}$$

$$T_{db} = \frac{T_{nb}}{\gamma_{mb}}$$

where

$$T_{db} = \frac{0.9 f_{ub} A_{nb}}{\gamma_{mb}} \leq \frac{f_{yb} A_{sb}}{\gamma_{m0}}$$

DESIGN BOLT STRENGTH (OR) DESIGN BOLT VALUE (V_{db}).

* It is the least value of design strength of bolt in shear, bearing, tension (if exist)

FORMULA :

DESIGN TENSILE STRENGTH OF A PLATE

Tensile strength of plate is given by

$$T_{pb} = \frac{0.9 f_u A_n}{\gamma_m}$$

Net Area for chain bolting

$$A_n = (b - n d_o) \times t$$

Net Area for staggered bolting.

$$A_n = \left(b - n d_o + \frac{s_1^2}{4g} + \frac{s_2^2}{4g} + \dots \right) \times t$$

DESIGN STRENGTH OF CONNECTION: (V_d):

The strength of a bolted connection is the minimum design strength of bolt in shear, bearing and tension (if exist) and minimum design strength of connected member against gross cross section yielding (or) net section rupture.

Efficiency of The Joint (or) Percentage strength of Joint: (η)

Efficiency of bolted joint also called Percentage strength of joint is the ratio of design strength of joint to the design strength of main member, expressed as percentage.

$$\eta = \frac{\text{Design strength of joint}}{\text{Design strength of main member}} \times 100$$



- 1) calculate the strength of 16mm dia bolt of grade 4.6 for a lap joint. The main plates to be joint are 10mm thick of Fe 410 grade. Assume pitch and edge distance of a bolt is 40mm and 30mm. and thread of the bolt is intercepting the shear plane.

Solution:

Give Data:

$$f_{ub} = 400 \text{ N/mm}^2 \quad f_{yb} = 240 \text{ N/mm}^2$$

$$\phi = 16 \text{ mm}$$

$$d_o = 16 + 2 = 18 \text{ mm}$$

$$P = 40 \text{ mm}$$

$$d_o = 16 + 2 = 18 \text{ mm}$$

$$f_{up} = 410$$

$$t = 10 \text{ mm}$$

Lap joint.

Shear strength:

$$V_{dsb} = \frac{f_{ub}}{\sqrt{3} \times 1.25} \times (n_b \times 0.78 \times A_{sb})$$

$$= \frac{400}{\sqrt{3} \times 1.25} \times (1 \times 0.78 \times \frac{\pi \times 16^2}{4})$$

$$V_{dsb} = 28.97 \text{ KN}$$

Bearing strength:

$$V_{dpb} = \frac{2.5 \times k_b \times d \times t \times f_u}{1.25}$$

$$k_b = \frac{30}{3 \times 18} \left(\frac{40}{3 \times 18} - 0.25 \right) \left(\frac{400}{410} \right) \leq 1$$

$$= 0.555, 0.49, 0.9736, 1$$

$$k_b = 0.49$$

$$= \frac{2.5 \times 0.49 \times 16 \times 10 \times 400}{1.25}$$

$$V_{dpb} = 78.4 \text{ KN} = 62.72 \text{ KN}$$

Strength of bolt is 28.97 kN. (2)

$$\text{No. of bolt} = \frac{\text{Total load}}{\text{Strength of Bolt}} \times \text{one bolt value.}$$

2. 2 plates each 300mm x 16mm are to be joined using 20mm ϕ bolts, B.B to form a lap connection. The connection is supposed to transfer a service load of 375 kN. Calculate no. of bolt required for connection with minimum pitch and edge dist. Assume the thread of bolt does not intercept the shear plane.

Given:

$$t = 16 \text{ mm} \quad d = 20 \text{ mm} \quad d_0 = 20 + 2 = 22 \text{ mm}$$

$$f_{ub} = 400 \text{ N/mm}^2 \quad f_{yb} = 240 \text{ N/mm}^2$$

$$P_u = 1.5 \times 375 = 562.5 \text{ kN} \quad P = \frac{2.5 \times d}{4} = 50 \text{ mm}$$

$$e = 1.5 \times d_0 = 33 \text{ mm}$$

$$V_{dsb} = \left(\frac{400}{\sqrt{3} \times 1.25} \right) \times \left(1 \times \frac{\pi \times 20^2}{4} \right)$$

$$V_{dsb} = 58.04 \text{ kN}$$

$$V_{dpb} = \frac{2.5 K_b \times d \times t \times f_u}{1.25}$$

$$K_b = 0.5, 0.507, 0.976,$$

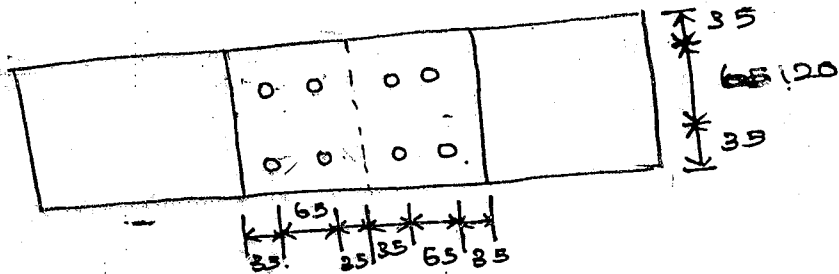
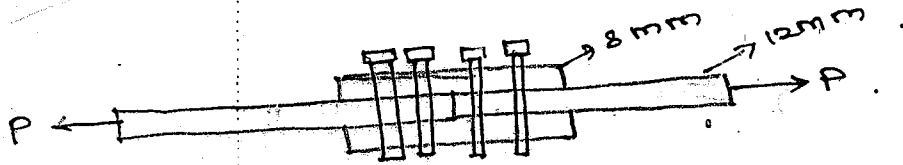
$$= \frac{2.5 \times 0.5}{1.25} \times 20 \times 16 \times 400$$

$$V_{dpb} = 128 \text{ kN}$$

Strength of bolt = 58.04 kN

$$\text{No. of bolt} = \frac{562.5}{58.04} = \underline{\underline{10 \text{ bolts}}}$$

3. Compute the design strengths and η of a bearing type of connection as shown in figure. The bolts are 4th grade and 20mm in dia. Plates are of grade Fe 410.



Solution:

Strength of bolt.

$$\begin{aligned}
 V_{dsb} &= \frac{f_u}{\sqrt{3}} (n_s A_{nb}) \quad \text{one/bolt} \\
 \text{total} &= \left[\frac{400}{\sqrt{3} \times 1.25} \left(0.78 \times \pi \times \frac{20^2}{4} \times 2 \right) \times 4 \right] \\
 &= 362.17 \text{ KN.}
 \end{aligned}$$

$$\begin{aligned}
 V_{dpb} &= \frac{2.5 k_b d t f_u}{\gamma_{m1}} \quad k_b = 0.53, 0.73, 0.97 \\
 &= \left(\frac{2.5 \times 0.53 \times 20 \times 20 \times 400}{1.25} \right) \times 4 \\
 &= 407.04 \text{ KN.}
 \end{aligned}$$

$$\begin{aligned}
 T_{dp} &= \frac{0.9 f_{ub} A_{nb}}{\gamma_{m1}} \\
 &= \frac{0.9 \times 410 \times \left(\frac{190}{1.25} \times \left[\frac{2 \times \pi \times 20^2}{4} \right] \right)}{1.25} \\
 &= \frac{0.9 \times 410 \times (190 - 2(22)) \times 12}{1.25} = 517.19 \text{ KN.}
 \end{aligned}$$

STRUCTURAL ANALYSIS

- Analysis of statically determinate trusses, arches, beams, cables and frames
- Displacement in statically determinate structures.
- Analysis of statically indeterminate structures by force/energy method
- Analysis by displacement method (slope deflection and moment distribution method)
- Influence lines for determinate and indeterminate structures.
- Basic concept of matrix method of structural analysis.

→ Analysis and design of tension and Compression members, beam-column and column bases.

→ Connections:

* Simple and eccentric

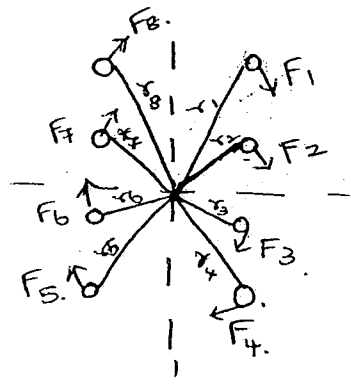
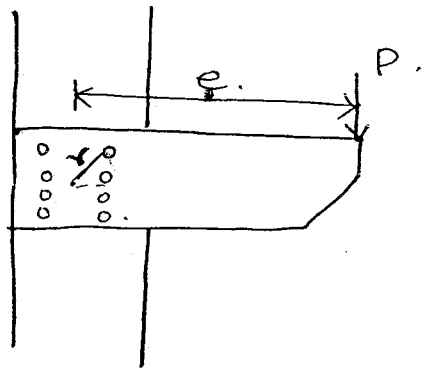
* Beam - column connections

* plate girders and trusses

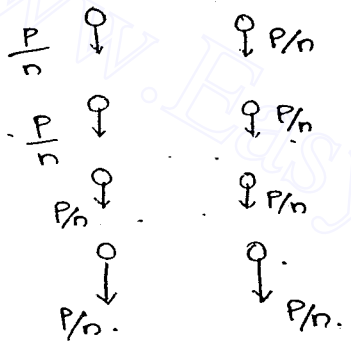
→ Plastic analysis of beams and frames.

Pr. Set 1

INPLANE MOMENT:



1) Direct shear :



$$\text{Direct stress} = \frac{\text{Load}}{\text{No. of bolts.}}$$

2) Moment :

$$F \propto r$$

$$F_1 \propto r_1$$

$$\vdots$$

$$F_1 = K r_1$$

$$F_2 = K r_2$$

$$\vdots$$

$$\sum F_x r = K \sum r_i^2$$

$$M = K \sum r_i^2$$

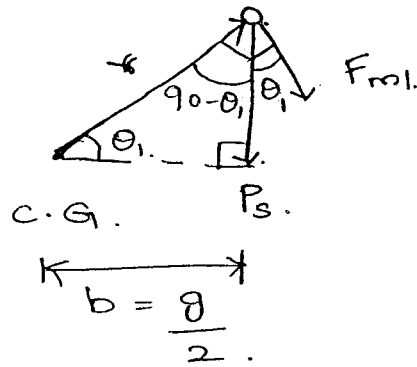
$$K = \frac{M}{\sum r_i^2} = \frac{Pe}{\sum r_i^2}$$

$$F_1 = K r_1$$

$$F_1 = \frac{Pe}{\sum r_i^2} \times r_1$$

$$F_1 r_1 = K r_1 r_1 \quad (\text{Multiplying "r" on both side})$$

$$\sum F_x r = K \sum r_i^2$$



$$R = \sqrt{P_s^2 + F_{m1}^2 + 2P_s F_{m1} \cos \theta_1}$$

$$\cos \theta_1 = \frac{b}{r_1}$$

L.S.M

$$R \leq V_{db}$$

working stress method.

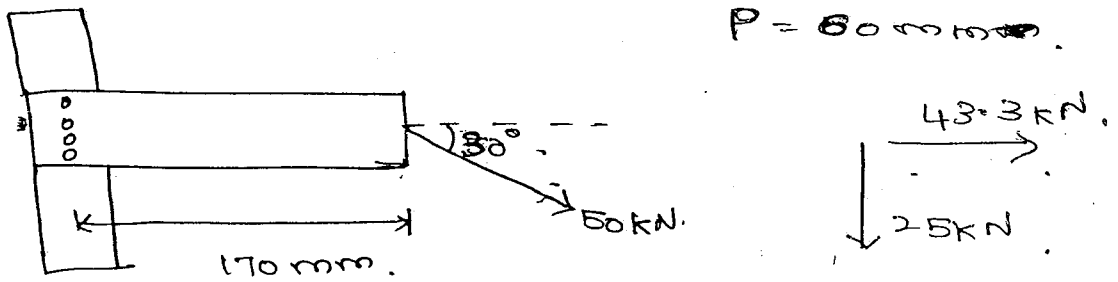
$$T_{yf} > T_{yf \text{ cal}}$$

$$T_{yf} > \frac{R_{max.}}{\left(\frac{\pi}{4} \times d^2\right) \times 0.78}$$

Max shear stress

$r \rightarrow$ distance from C.G. to ^{each} bolt

1) Determine the max force developed in a critical bolt and find out Max. shear stress.



$$\text{Direct stress} = \frac{25 \times 10^3 \text{ N}}{4} = 6.25 \text{ kN}$$

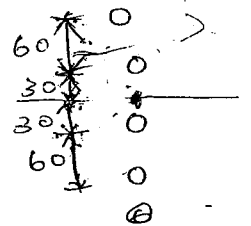
P_{S1}

$$P_{S2} = \frac{43.3 \times 10^3 \text{ N}}{4} = 10.83 \text{ kN}$$

Torsional shear:

$$F_t = \frac{P \times e_1 \times r_1}{\sum r^2}$$

$$= \frac{25 \times 10^3 \times 170}{(90^2 + 30^2 + 30^2 + 90^2)} \times 90$$

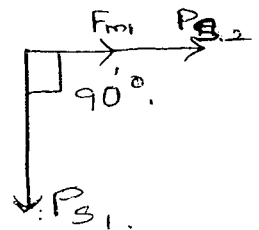


$$F_t = 21.25 \times 10^3 \text{ kN}$$

$$R = \sqrt{(P_{S1})^2 + (F_{t1} + P_{S2})^2 + 2 F_{t1} \times P_{S2} \times \cos \theta_1}$$

$$\cos \theta_1 = 90^\circ$$

$$\cos \theta_1 = \cos 90^\circ = 0$$



$$= \sqrt{P_{S1}^2 + (F_{t1} + P_{S2})^2}$$

$$= \sqrt{(6.25)^2 + (10.83 + 21.25)^2}$$

$P_{S1} \quad P_{S2} \quad F_{t1}$

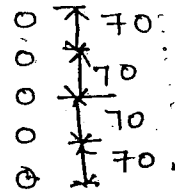
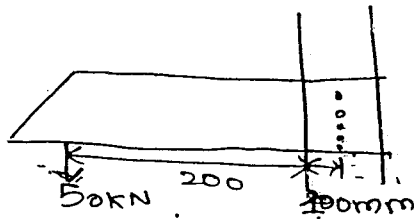
B.S.M

$$R = 32.68 \text{ KN}$$

$$\text{Max. shear stress} = \frac{32.68 \times 10^3}{\left(\frac{\pi \times 20^2}{4}\right) \times 0.78}$$

$$\text{Max. shear stress} = 133.37 \text{ N/mm}^2$$

2.)



$$\text{Direct shear} = \frac{50 \times 10^3}{5} = 10 \text{ KN}$$

$$\text{Torsional shear } \tau_{m1} = \frac{P \times e \times r_1}{\Sigma r^2}$$

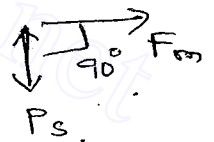
$$= \frac{50 \times 10^3 \times 200}{\left[2(70^2) + 2(140^2)\right]} \times 140$$

$$= 42.86 \text{ KN}$$

$$R_{\text{max}} = \sqrt{F_m^2 + P_s^2}$$

$$R_{\text{max}} = \sqrt{(42.86)^2 + (10)^2}$$

$$R_{\text{max}} = 44 \text{ KN}$$



$$R_{max} \leq \sqrt{db} \rightarrow 0.462 f_u$$

\sqrt{db} Least of $\sqrt{d_{sb}}$, $\sqrt{d_{pb}}$.

$$\sqrt{d_{sb}} = \frac{f_{ub}}{\sqrt{3}} \left(n_s A_{sb} + n_o A_{ob} \right)$$

$$= \frac{400}{\sqrt{3} \times 1.25} \left(1 \times 0.78 \times \frac{\pi \times d^2}{4} \right)$$

$$5.58 \times 10^3 = \frac{400}{\sqrt{3} \times 1.25} \times \frac{0.78 \times \pi \times d^2}{4}$$

$$5.58 \times 10^3 \leq 113.18 d^2$$

$$d = \sqrt{\frac{5.58 \times 10^3}{113.18}}$$

$$= 7.02$$

$$d = 8 \text{ mm}$$

$$\sqrt{d_{pb}} = \frac{2.5 \times K_b \times d \times t \cdot f_u}{\sqrt{3} \times 1.25}$$

$$5.58 \times 10^3 = \frac{2.5 \times 1 \times d \times 10 \times 400}{1.25}$$

$$d = \frac{5.58 \times 10^3}{8000}$$

$$d = 0.697 \text{ mm}$$

Dia of bolt is 8 mm

OUT PLANE MOMENTUM WELDING.

Types:

- (i) Forge welding
- (ii) Thermit welding
- (iii) oxy acetylene welding
- (iv) Electric arc welding

According to IS 816-1969 and IS 800-2007 the min. size of fillet wet weld is dependent on thickness of thicker member

- (a) If the thickness of thicker member is upto 10mm Min size of weld 3mm
- (b) If thickness is 10-20mm then size of fillet weld 5mm.
- (c) If t is 20-32mm then size of weld is 6mm.
- (d) If $t > 32mm$ min size of weld is 8mm in first run subjected to minimum 10mm.

Minimum size of triangle of fillet weld is called size of the fillet weld.

* value of throat thickness is the \perp^r distance from the corner of the weld to the hypotenuse of the triangle of the weld.

* The value of throat thickness is dependent on fusion angle.

* case (a): In case of flat plate or member.

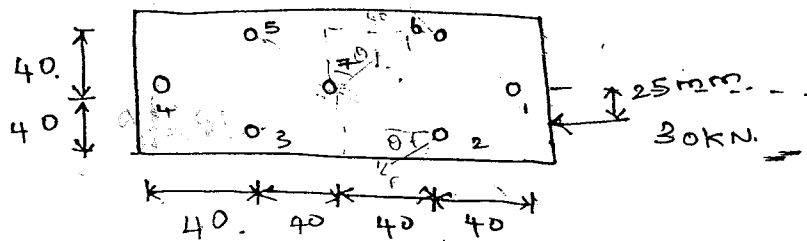
$$S_{max} = t_{plate} - 1.5 \text{ mm.}$$

* case (b) In case of rounded corner (Angle section)

$$S_{max} = \frac{3}{4} \times t_{\text{round corner}}$$

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3.)



Determine suitable ϕ of bolt of grade $A_0^{3.6}$ Thickness of plate = 10 mm.

$$P_s = \frac{30 \times 10^3}{7} = \underline{4.29 \text{ kN}}$$

$$r = \sqrt{40^2 + 40^2} = 56.57$$

$$F_{m1} = \frac{(30 \times 10^3 \times 25)}{\sum r^2} \times r$$

$$= \frac{30 \times 10^3 \times 25 \times 80}{(80^2 + 56.57^2 + 56.57^2 + 80^2 + 56.57^2 + 56.57^2)}$$

$$\theta = \tan^{-1}\left(\frac{40}{40}\right)$$

$$F_{m1} = \underline{2.34 \text{ kN}}$$

$$F_{m2} = \frac{30 \times 10^3 \times 25 \times 56.57}{(80^2 + 56.57^2 + 56.57^2 + 80^2 + 56.57^2 + 56.57^2)}$$

$$F_{m2} = 1.66 \text{ kN} = \cos \theta = 0.707$$

$$R_1 = \sqrt{F_{m1}^2 + P_s^2}$$

$$R_2 = \sqrt{F_{m2}^2 + P_s^2 + 2 F_{m2} P_s \cos \theta}$$

$$= \sqrt{2.34^2 + 4.29^2}$$

$$= \sqrt{1.66^2 + (4.29)^2 + 2(1.66)(4.29) \cos 45^\circ}$$

$$R_1 = \underline{4.89 \text{ kN}}$$

$$\cos 45^\circ$$

$$\theta_1 = \tan^{-1}\left(\frac{40}{40}\right) = 45^\circ$$

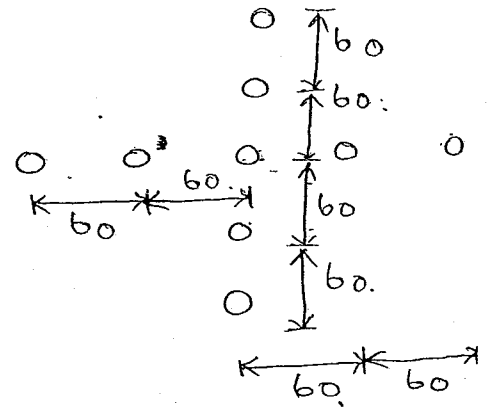
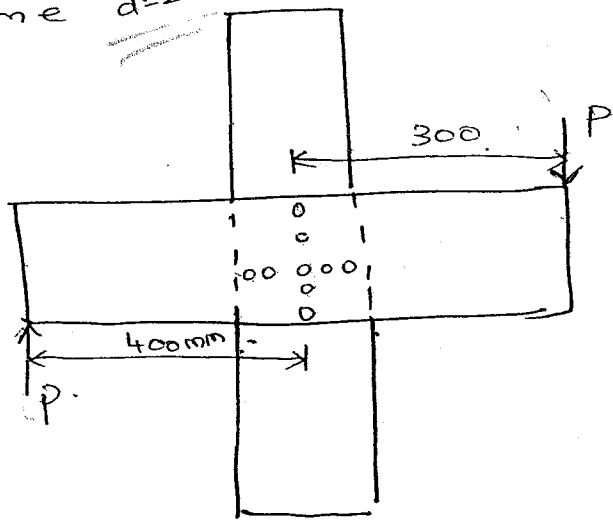
$$R_2 = \underline{5.59 \text{ kN}}$$

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of the system.

Assume $d = 20\text{mm}$.

$$P = Q = 60\text{mm}$$



Direct shear:

$$P_s = \frac{-P + P}{0.9} = 0.$$

$$P_s = 0$$

Torsional shear:

$$= \frac{[(P \times e_1) + (P \times e_2)] \times r_1}{\sum (r^2)}$$

$$= \frac{[(P \times 300) + (P \times 400)] \times 120}{4(120^2) + 4(60^2)}$$

$$R_{max} = 1.17 P.$$

$$R_{max} \leq V d b$$

$$1.17 P \leq \frac{45.3 \times 10^3}{3}$$

$$P \leq \frac{45.3 \times 10^3}{1.17}$$

$$P \leq 38.72 \text{ KN}$$

$$P = 38 \text{ KN}$$

BOLT
IN PLANE MOMENT :

Draw formula.

Direct Shear

$$P_s = \frac{P}{N}$$

Shear due to Torsion

$$F_m = \frac{P_e}{\sum r^2} \times r_{max}$$

$$b = b_{min}$$

$$\eta = \frac{\text{Strength of bolted joint}}{\text{Strength of solid joint}}$$

Resultant $R = \sqrt{P_s^2 + F_m^2 + 2 P_s F_m \cos \theta_{max}}$

$$\cos \theta_{max} = \frac{b}{r_{max}}$$

According to LSM
 $R \leq \sqrt{db}$

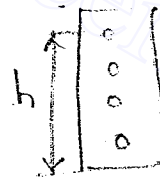
According to WSM

$$T_{yf} > T_y \text{ calculated}$$

$$T_{yf} > \frac{R_{max}}{\left(\frac{\pi d^3}{4} \times 0.78\right)}$$

Out Plane Moment

Case I $V_{sb} = \frac{P}{N}$ Direct Shear



Case II Axial Stress

$$T_b = \frac{P_1}{N} \quad \text{or} \quad T_b = \frac{M_1 \times y_1}{2 \sum y^2}$$

$$M_1 = \frac{P \times e \times y_1}{\sum y^2}$$

$$N.A = \frac{h}{4}$$

$T_{bf} > T_b$ (if) Age ...

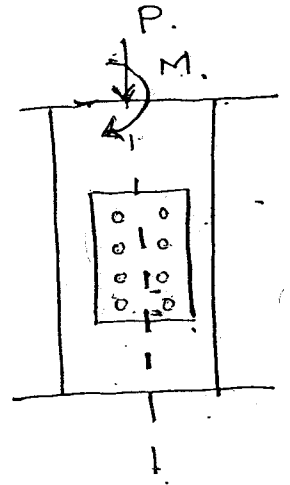
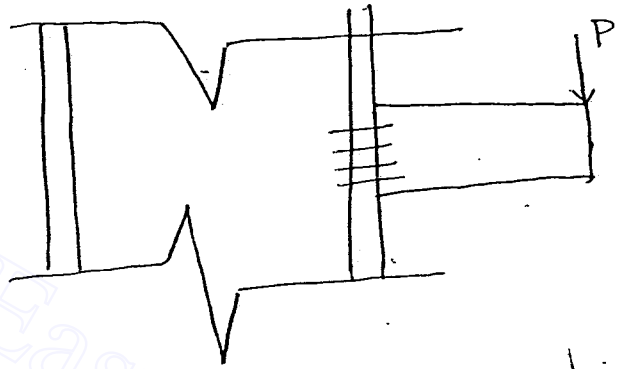
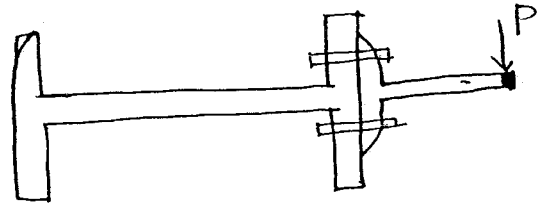
When ...

When ...

When ...

OUT PLANE MOMENT

* Connection subjected to shear and tension



Case 1: Direct stress

$$P_s = \frac{P}{N} = \frac{P}{n+n}$$

N - Total No. of bolts

n - No. of bolts in one row

Case 2: Axial stress

$$T_b = \frac{P_t}{N}$$

$$T_{db} = \frac{f_y A_g}{\gamma_{m0}} \quad (\text{or}) \quad \frac{0.9 f_u A_n}{\gamma_{m1}}$$

Interaction Equation

According to IS 800: 2007 Pg: 76.

$$\left(\frac{V_{dsb}}{V_{db}} \right)^2 + \left[\frac{T_b}{T_{db}} \right]^2 \leq 1.0$$

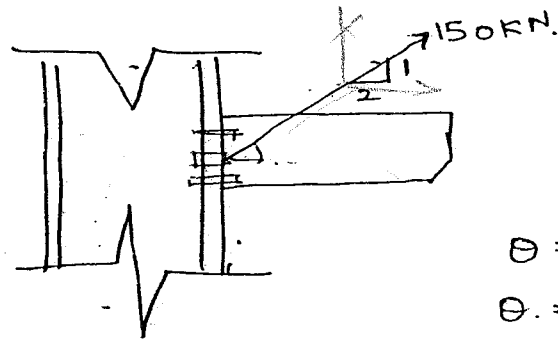
According to IS 800: 1984.

$$\left(\frac{T_{y, cal}}{T_{yf}} \right) + \left(\frac{\sigma_{t, cal}}{\sigma_{tf}} \right) \leq 1.4$$

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1) Determine interaction value of a given bolt connection. 20mm ϕ bolt and grade 4.6
-pitch 60mm



$$\theta = \tan^{-1}\left(\frac{1}{2}\right)$$

$$\theta = 26.56^\circ$$

Interaction equation,

$$\left(\frac{V_{sb}}{V_{db}}\right)^2 + \left(\frac{T_b}{T_{db}}\right)^2 \leq 1$$

$$V_{sb} = \frac{150 \sin 26.56^\circ}{6} = 11.18 \text{ kN}$$

$$T_b = \frac{150 \cos 26.56^\circ}{6} = 22.36 \text{ kN}$$

V_{db}

$$V_{db} = 0.462 f_{ub} \times \frac{\pi}{4} \times d^2 \times 0.78$$

$$= 0.462 \times 400 \times \frac{\pi}{4} \times 20^2 \times 0.78$$

$$V_{db} = 45.28 \text{ kN}$$

T_{db}

$$T_{db} = \frac{0.9 f_u A_{nb}}{\gamma_{m1}} = \frac{0.9 \times 400 \times \frac{\pi}{4} \times 20 \times 0.78}{1.25}$$

$$= 70.57 \text{ kN}$$

$$T_{db} = \frac{f_y A_g}{\gamma_{mo}} = \frac{240 \times \frac{\pi}{4} \times 20^2}{1.1} = 68.54 \text{ kN.}$$

$$\left[\frac{11.18}{45.28} \right]^2 + \left[\frac{22.36}{68.54} \right]^2 \leq 1$$

$$0.167 \leq 1$$

Hence safe.

2) Determine the interaction ratio for the

Load = 100 kN
gauge = 40 mm.

$e = 200 \text{ mm}$

$p = 80 \text{ mm}$

[valid for
In plane &
out plane]

$$n = \sqrt{\frac{6M}{R \cdot p \cdot m}}$$

M - Moment
R - Rivet value
p - ~~80~~ pitch
m - No. of rows

$$n = \sqrt{\frac{6 \times 100 \times 10^3 \times 200}{45.28 \times 10^3 \times 80 \times 2}}$$

$$n = 4$$

$$b = 80 + 80 + 80 + 40$$

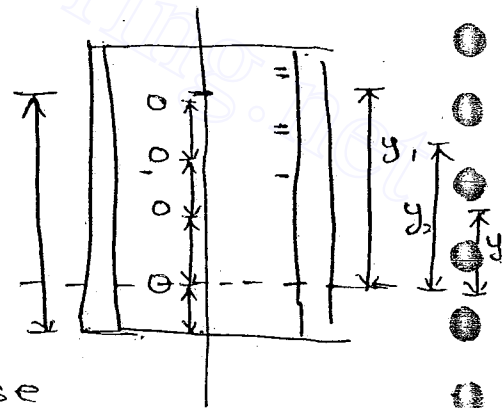
$$b = 280 \text{ mm.}$$

Depth of N.A from base

$$= \frac{b}{7}$$

$$= \frac{280}{7}$$

$$= 40 \text{ mm.}$$



$$y_1 = 240 \text{ mm} \quad y_2 = 160 \text{ mm} \quad y_3 = 80 \text{ mm}$$

$$\tau_b = \frac{M_t \times y_1}{2(y_1^2 + y_2^2 + y_3^2)}$$

$$M_t = \frac{M}{1 + \frac{2b}{21} \frac{\sum y^2}{\sum y^2}}$$

$$M = P \times e = 100 \times 10^3 \times 200 = 20 \times 10^6 \text{ N.m}$$

$$\sum y^2 = 2 [240^2 + 160^2 + 80^2]$$

$$= 179200 \text{ mm}^2$$

$$\sum y = 2(240 + 160 + 80)$$

$$= 960 \text{ mm}$$

$$M_t = \frac{20 \times 10^6}{1 + \frac{2 \times 280}{21} \left(\frac{960}{179200} \right)}$$

$$M_t = 17.5 \times 10^6 \text{ N.m}$$

$$\tau_b = \frac{M_t \times y_1}{2(y_1^2 + y_2^2 + y_3^2)}$$

$$= \frac{17.5 \times 10^6 \times 240}{2(80^2 + 160^2 + 240^2)}$$

$$\tau_b = 23.4375 \text{ KN}$$

$$V_{dsb} = 45.28 \text{ KN}$$

$$\tau_{dsb} = 68.51 \text{ KN}$$

$$\left(\frac{12.5}{45.28} \right)^2 + \left(\frac{23.44}{68.51} \right)^2 \leq 1$$

$$0.193 \leq 1 \quad \text{Hence safe.}$$

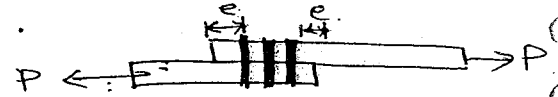
unwin Formula

= Dia of bolt

$$\phi = 6\sqrt{t_{\min}}$$

 t_{\min} - Minimum thickness of plate.

- 3) Determine no. of bolt. 2 bolts $100 \times 8\text{mm}$ and $100 \times 10\text{mm}$ is connected by lap joint with single row of bolt along the length. use 20mm ϕ and grade 4.6 (M16) $P = 60\text{mm}$ $e = 30\text{mm}$.



$$\text{No. of bolt} = \frac{\text{Load}}{\text{Bolt value}}$$

Load = Load carrying capacity of plate

$$T_{dp} = \frac{0.9 f_u \times A_n}{\gamma_{m1}}$$

$$= \frac{0.9 \times 410}{1.25} \times [100 - (20+2)] \times 8$$

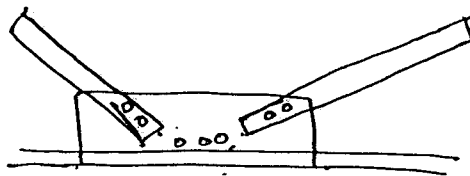
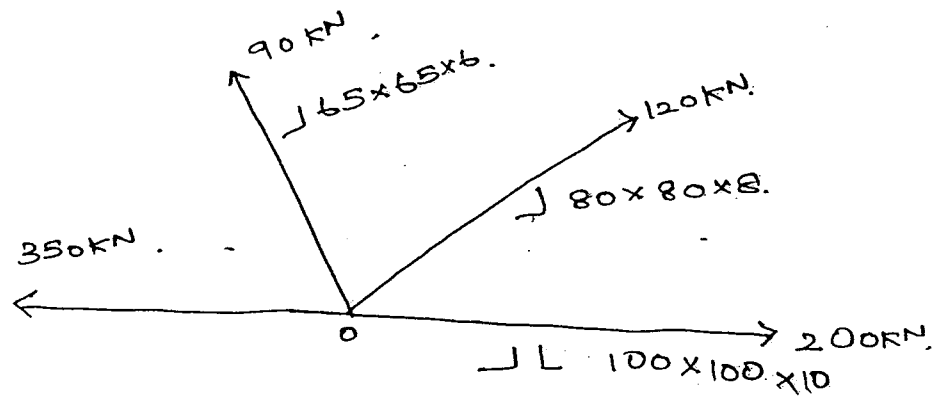
$$= 184.20 \text{ kN}$$

$$\text{No. of bolt} = \frac{184.20 \times 10^3}{45.28 \times 10^3}$$

$$= 4.068$$

$$= 4 \text{ bolt}$$

5. Design the joint as shown in figure.
 Take $P = 60\text{mm}$, edge = 30mm . use 16mm bolt. M 4.6. use 10mm thick gusset plate.



Design of Gusset plate

$$\text{Net Load} = 350 - 200 \\ = 150 \text{ KN}$$

Bolt value.

$$V_{dsb} = \frac{0.462 \times f_{ub} \times A_n \times 2}{\gamma_{mb}} \\ = \frac{0.462 \times 400 \times \frac{\pi}{4} \times 16^2 \times 2 \times 0.78}{(1.25)} \\ = 46.37 \text{ KN}$$

$$k_b = \frac{e}{3d_0} \leq \frac{P}{3d_0} \leq \frac{t_{ub}}{f_u}$$

$$V_{dpb} = 2.5 k_b d t \frac{f_u}{1.25} \\ = \frac{2.5 \times 1 \times 410 \times 16 \times 10}{1.25} \\ = 164 \text{ KN}$$

1. Determine Efficiency of the Lap joint connecting two plates 100×10 and 100×8 mm use 16 mm ϕ bolt and M4.6, use 5 bolts.

Solution:

$$\eta = \frac{\text{strength of bolted joint}}{\text{strength of solid plate}}$$

(i) Design strength of bolt.

Design strength of bolt.

$$\frac{f_u}{\sqrt{3}} \times n_b$$

$$V_{dsb} = 0.462 f_u (n_b A_{nb})$$

$$= 0.462 \times 400 \times 1 \times \left(\frac{\pi \times 16^2}{4} \right) \times 0.78$$

$$V_{dsb} = \frac{28.98}{37.16} \text{ kN}$$

Strength

- (ii) Design bearing strength of bolt.

$$V_{dpb} = 2.5 k_b d \times t \times \frac{f_u}{\gamma_{mb}}$$

$$e = 24 \text{ mm} \quad p = 40 \text{ mm}$$

$$k_b = \frac{e}{3d_0}, \frac{p}{3d_0} - 0.25, \frac{f_{ub}}{f_u}, 1$$

$$= \frac{24}{3 \times 18}, \frac{40}{3 \times 18} - 0.25, \frac{400}{40}, 1$$

$$= 0.44, 0.49, 0.975, 1$$

$$V_{dsb} = 45.056 \text{ kN}$$

$$\begin{aligned} V_{deb} &= \frac{0.9 f_{ub} A_{nb}}{\gamma_{mb}} \\ &= \frac{0.9 \times 400 \times \pi \times 16^2}{1.25} \\ &= 57.9 \text{ kN.} \end{aligned}$$

(iv) Tension in plate.

$$\begin{aligned} P_t (\text{plate}) &= \frac{0.9 f_u A_{nb}}{\gamma_{mb}} \\ &= \frac{0.9 \times 410 \cdot (100 - 18) \times 8}{1.25} \\ &= 193.65 \text{ kN.} \end{aligned}$$

$$\begin{aligned} \text{Bolt value} &= 5 \times 28.98 \\ &= 144.9 \text{ kN.} \end{aligned}$$

Strength of solid plate.

$$\begin{aligned} &= \frac{0.9 f_u \times A_g}{\gamma_{mb}} \\ &= \frac{0.9 \times 410 \times 100 \times 8}{1.25} \\ &= 236.16 \text{ kN.} \end{aligned}$$

$$\eta = \frac{144.9}{236.6} \times 100$$

$$\eta = 61.35 \%$$

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DESIGN OF FILLET WELD.

* Size of fillet weld

It is distance from root to toe of the fillet weld.

* $s_{min} \neq 3mm$.

*.

Thickness Of Thicker Part		Minimum size.
Over	Up to.	
-	10	3
10	20	5
20	32	6
32	50	8 (1st run), & 10mm

Effective Throat Thickness:

It is 1st distance from right angle corner of fillet weld to the hypotenuse

Minimum throat thickness for fillet weld $\neq 3mm$.

$$t_e = k \times \text{Size of weld.}$$

k = constant depending upon fusion angle

Angle	k.
60 - 90	0.7
91 - 100	0.65
101 - 106	0.6
107 - 113	0.55
114 - 120	0.5

END RETURN:

* The fillet weld terminating @ the end or side of the member should be returned around the corner whenever practicable for a distance not less than 2s.

* End return are made twice the size of the weld to relieve high stress concentration @ the ends

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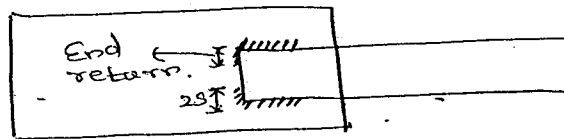
Welding :

- * The effective length of fillet weld
= Total length of weld - 2(weld size)

$$l_{\text{eff}} = l_{\text{total}} - 2S$$

- * The minimum end return of the weld is 2 times the weld size.

$$\text{End return} = 2S$$



- * The minimum effective length of fillet weld
= 4S

for Fillet weld $l_{\text{eff}} = 4S$

- * The minimum effective length of fillet weld intermittent welding shall be 4S (or) 40mm whichever is greater. (Discontinuous welding)
- * The clear spacing of intermittent weld shall not exceed $16t_{\text{min}}$ (or) 200mm whichever is less. In case of tension members.
- = shall not less than $12t_{\text{min}}$ (or) 200mm
- = whichever is less for compression members.

- * The maximum permissible stress in fillet weld is 108MPa - WSM.

$$0.462 f_u \quad - \quad \text{LSM} \quad (\text{shop weld})$$

$$0.385 f_u \quad - \quad \text{LSM} \quad (\text{Field weld}).$$

- * The maximum permissible tensile or compressive stress in welding in fillet weld is 165MPa.

- * The effective thickness of butt weld is $\frac{5}{8} t_{\text{min}}$ - single v-joint.

* $l_{eff} = l_{total} - 2S$

* End return : $2S$

* Weld type
Fillet

l_{eff} min. 4S

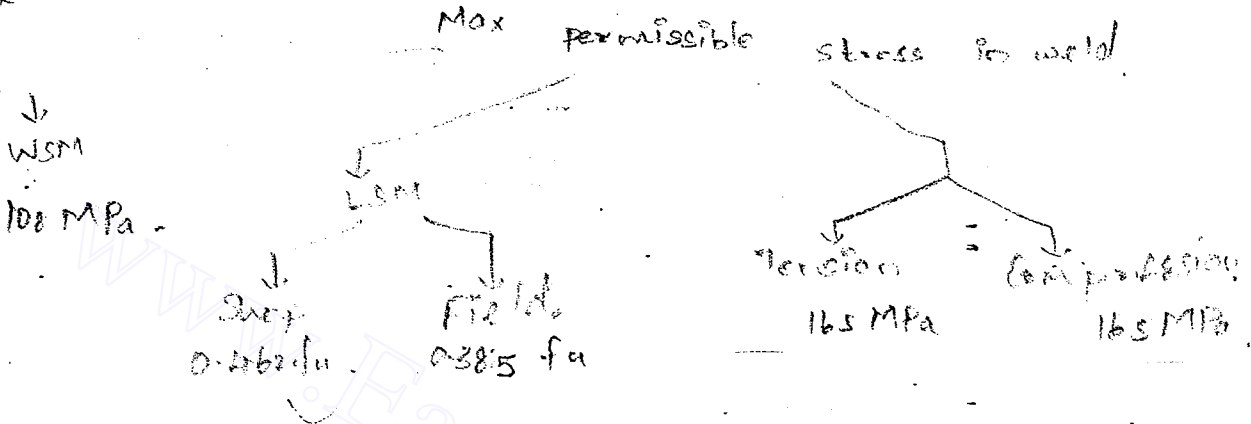
spacing

* Intermittent weld

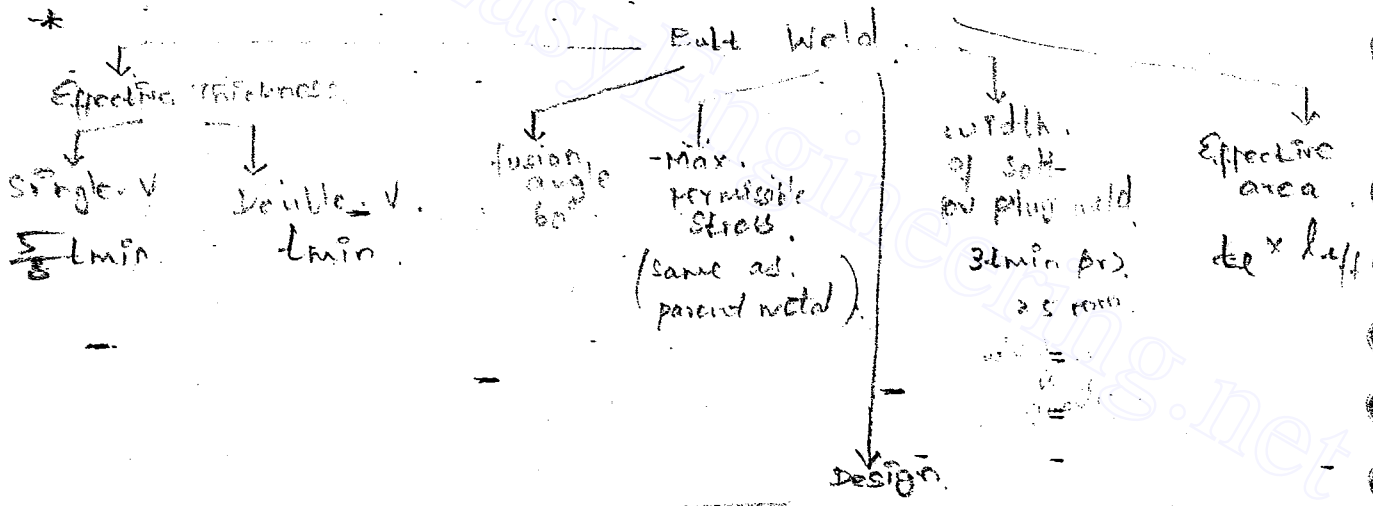
4S for 40mm, whichever is greater.

Comp. zone
12 min (or) 200mm
16 min (or) 200mm

*.



*.



axial

$\tau_{ax} = \frac{f_y l_{eff}}{l_{min}}$

Shear

$\tau_{dw} = \frac{f_y l_{eff}}{l_{min}}$

$f_y = \frac{f_y}{\sqrt{3}} \text{ (or) } \frac{f_y}{\sqrt{3}}$

- * The angle for v-butt weld should be 60°
- * The max. permissible stress of butt weld is same as that of parent metal.
- * The width of the slot (or) plug shall not be less than $3t_{min}$ (or) 25mm whichever is greater.
- * Effective area = $\left(\frac{\text{Effective throat}}{\text{thickness}} \right) \times \left(\text{Effective length} \right)$

* Design strength of butt weld:

a) Design axial strength:

The design strength of butt weld in tension or compression is governed by yield stress.

Where

$$T_{dw} = \frac{f_y \cdot L_{eff} \cdot t_e}{\gamma_{mw}}$$

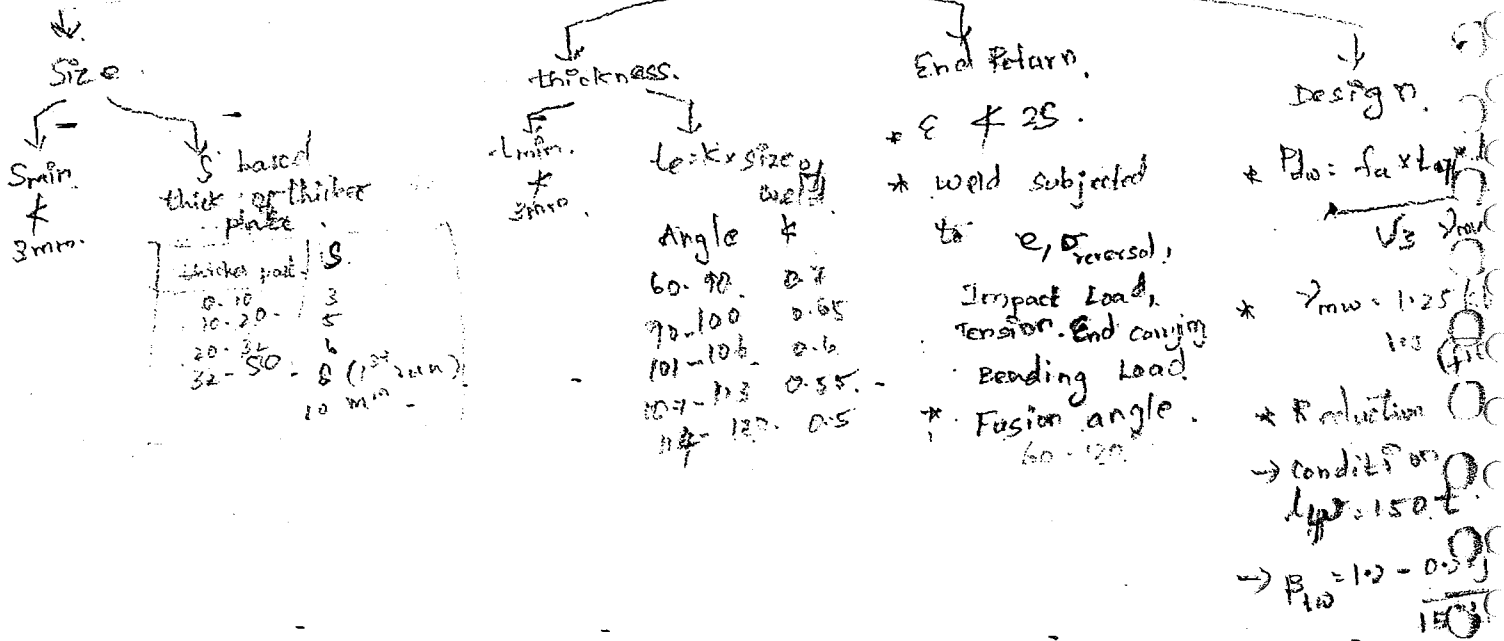
f_y = smaller of yield stress of weld (or) parent metal

(b) Design shear strength of butt weld:

$$V_{dw} = \frac{f_{yw} \cdot L_{eff} \cdot t_e}{\gamma_{mw}}$$

f_{yw} is smaller of

$$\frac{f_{yw}}{\sqrt{3}} \quad \text{(or)} \quad \frac{f_y}{\sqrt{3}}$$



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* End returns must be provided for welded joints which are subjected to eccentricity. / Stress reversal. (or) Impact load.

* Fillet weld is ineffective when fusion angle is beyond 60° + 120°.

+ This is particularly important in tension end of part carrying bending load.

OVER LAP:

The over lap of plates to be welded.

4 t_{min} (or) 40mm whichever is more.

INTERMEDIATE. (or) Intermittent weld:

It is used when length of fillet weld required to transmitting the force less than the continuous fillet weld.

DESIGN STRENGTH OF FILLET WELD:

P_{dw} = $\frac{f_u t_w \times l_{eff}}{\sqrt{3} \gamma_{mw}}$

f_u - smaller of ultimate strength of ~~weld~~ (or) parent metal.

γ_{mw} - 1.25 shop weld
1.5 field weld

Reduction factor for long joint:

If the max. length of side weld transferring shear along its length exceeds 150 times the throat size, the reduction in weld strength as per long joint. β

The design capacity of the weld is reduced by a factor:

$$\beta_{tw} = 1.02 - \frac{0.2 l_j}{150 t_w}$$

l_j - length of joint in force transfer.

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WELD.

FILLET WELD.

Two 16mm thick plates are joined in a shop single U butt weld, Double U butt weld $l_{eff} = 300\text{mm}$. Determine design strength of welded joint $f_y = 250\text{MPa}$ and $f_u = 410\text{MPa}$ of the weld and steel.

solution:

$$(i) \text{ Single U butt } T_{db} = \frac{f_y \times l_{eff} \times t_e}{\gamma_{mb}}$$

$$= \frac{250 \times 300 \times 10}{1.25}$$

$$= 600 \text{ kN.}$$

$$t_e = \frac{5}{8} t_{min}$$

$$= \frac{5}{8} \times 16$$

$$t_e = 10 \text{ mm}$$

(ii) Double U butt

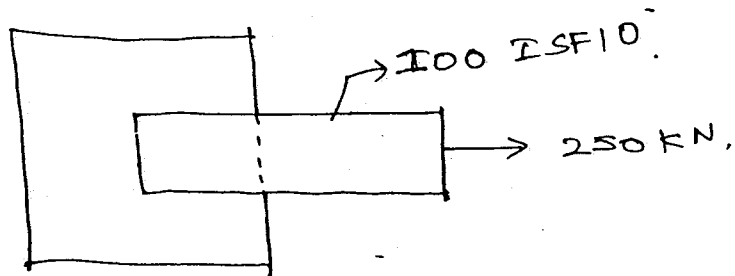
$$T_{db} = \frac{f_y \times l_{eff} \times t_e}{\gamma_{mb}}$$

$$= \frac{250 \times 300 \times 16}{1.25}$$

$$= 960 \text{ kN.}$$

$$t_e = t_{min}$$

2. Determine s, l and overlap b/w plates of the fillet weld lap connection to transmit $P = 250 \text{ kN}$, $f_y = 250 \text{ MPa}$, $f_u = 410 \text{ MPa}$. Assume shop weld ($\gamma_{mb} = 1.25$). b width of plate 100 mm .



$$P_{dw} = \frac{f_u \times l_e \times t_e}{\sqrt{3} \cdot \gamma_{mb}}$$

$$250 \times 10 = \frac{410 \times l_e \times 4 \cdot 2}{\sqrt{3} \times 1.25}$$

$$l_e = 314.32$$

$$l_e = 315.0 \text{ mm}$$

$$t_e = 0.7 \times s$$

$$s_{\min} = 3 \text{ mm}$$

$$s_{\max} = t_{\min} + 5$$

$$s_{\max} = 10 + 5 = 15$$

$$= 8.5 \text{ mm}$$

$$s = 6 \text{ mm}$$

$$t = 0.7 \times 6$$

$$t = 4.2 \text{ mm}$$

$$\text{over lap length} = \frac{315 - 100}{2}$$

$$\text{over lap length} = 107.5 \text{ mm}$$

check:

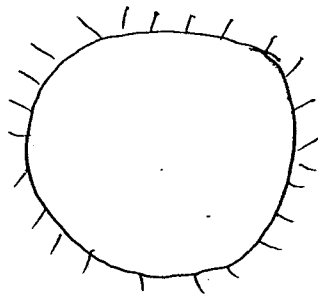
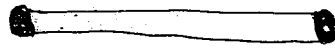
$$\text{Min over lap} = 4 t_{\min} \text{ (or) } 40 \text{ mm whichever is great}$$

$$= 4 \times 10$$

$$\text{Min over lap} = 40 \text{ mm}$$

Hence safe.

3. A circular plate 100 mm ϕ is welded to another plate by means of $S_{\text{weld}} = 6 \text{ mm}$ Fillet weld. Calculate twisting Moment capacity that can be resisted by fillet welded connection. Use ^{steel of} grade 410 and shop welding.



$$l_{\text{eff}} = \pi d$$

$$= \pi \times 100$$

$$l_{\text{eff}} = 100\pi$$

$$t_e = 0.7 S$$

$$= 0.7 \times 6$$

$$t_e = 4.2 \text{ mm}$$

$$P_{dw} = \frac{f_u \times l_{\text{eff}} \times t_e}{\sqrt{3} \times 1.25}$$

$$= \frac{410 \times 100 \times \pi \times 4.2}{\sqrt{3} \times 1.25}$$

$$P_{dw} = 249.869 \text{ kN}$$

$$\text{Twisting Moment} = P_{dw} \times C.G.$$

$$= 249.86 \times 10^3 \times \frac{d}{2}$$

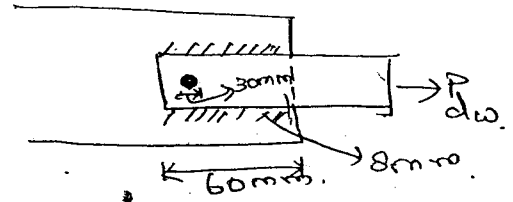
$$= 249.86 \times 10^3 \times \frac{100}{2}$$

$$= 12493 \text{ kNm}$$

$$\text{Twisting Moment} = 12.493 \text{ kN.m}$$

4. Determine service load permitted on connection as shown in the figure. Use grade 410.

solution:



$$P_{dw} = P_{dwf} + P_{dwp}$$

$$= \frac{f_u \times l_{eff}}{\sqrt{3} \times \gamma_{mb}} + \frac{f_u \times A}{\sqrt{3} \times \gamma_{mb}}$$

$$= \frac{410 \times 120 \times 0.7 \times 8}{\sqrt{3} \times 1.5} + \frac{410 \times \pi \times 30^2}{4 \times \sqrt{3} \times 1.5}$$

$$P_{dw} = 217.59 \text{ kN}$$

$$\text{Service load} = \frac{P_{dw}}{1.5} = \underline{\underline{145.06 \text{ kN}}}$$

5. Determine "P" on a butt weld joint

Size 150 x 10mm and 150 x 6mm. $f_y = 250 \text{ MPa}$

Use (i) single V butt weld.

(ii) Double V butt weld. shop weld

$$T_{db} = \frac{f_y \cdot l_{eff} \cdot t_{eff}}{\gamma_{mb}}$$

$$= 273.33$$

$$= \frac{273.33 \times 150 \times 3.75}{1.25}$$

$$T_{db} = 122.99 \text{ kN}$$

$$T_{db} = \frac{273.33 \times 150 \times 6}{1.25}$$

$$T_{db} = 196.8 \text{ kN}$$

$$f_u = 410$$

$$f_y = \frac{410}{1.5}$$

$$= 273.33$$

$$f_y = 250 \text{ MPa}$$

$$t_{eff} = \frac{5}{8} \times 6$$

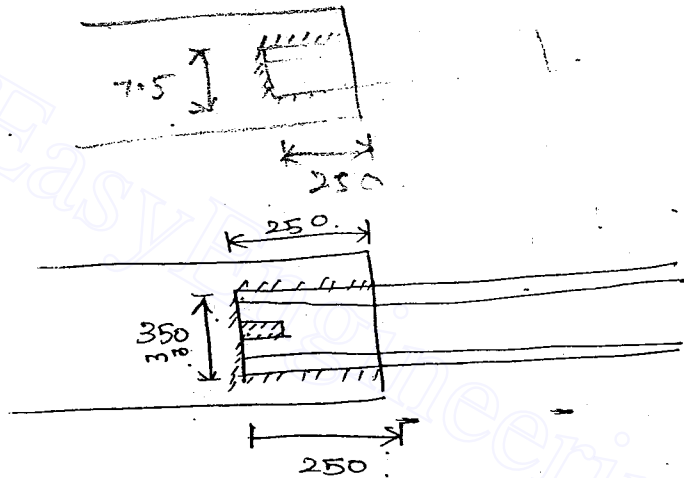
$$= 3.75 \text{ mm}$$

Note :

* welding in case of unequal thickness, slope is provide in 1:5.



- 6) Determine length of slot with gusset plate where c/s area of channel is 3200 mm^2 $t = 7.5 \text{ mm}$ and lap length = 250 mm .
depth of channel 350 mm .



$$P_{dw} = \frac{f_u l_{eff} t_e}{\sqrt{3} \gamma_{mb}}$$

$$P_{dw} = P_{channel}$$

$$P_c = \frac{f_y A}{\gamma_{mo}}$$

$$= \frac{250 \times 3200}{1.1}$$

$$P_c = 727.272 \text{ kN}$$

$$727.272 \times 10^3 = \frac{410 \times l_{eff} \times 402}{\sqrt{3} \times 1.25}$$

$$l_{eff} = 914.39 \text{ mm}$$

$$l_{eff} = 915 \text{ mm}$$

$$f_u = 410 \text{ MPa}$$

$$t_e = 0.7 \times s$$

$$s_{min} = 3 \text{ mm}$$

$$s_{max} = 7.5 - 1.5$$

$$s_{max} = 6 \text{ mm}$$

$$s = 6 \text{ mm}$$

$$t_e = 0.7 \times 6 = 4.2 \text{ mm}$$

$$7207.27$$

$$l_{\text{eff}} = 250 + 250 + 350 + \text{slot length}_{\text{total}}$$

$$915 = 250 + 350 + S_{\text{total}}$$

$$S_{\text{total}} = 65 \text{ mm}$$

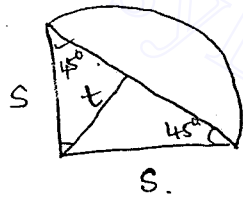
$$\begin{aligned} \text{length in one side of slot} &= \frac{65}{2} \\ &= 32.5 \text{ mm} \end{aligned}$$

$$\text{width of slot} = 25 \text{ mm} < 3t_{\text{min}}$$

$$= 3 \times 7.5$$

$$= 22.5 \text{ mm}$$

Note:



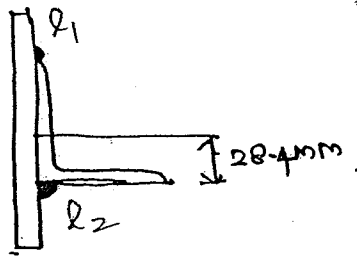
$$\sin 45^\circ = \frac{t_{\text{weld}}}{S}$$

$$t_w = \frac{1}{\sqrt{2}} S$$

$$t_w = 0.7S$$

7. Determine the length of weld @ top & bottom

$s = 6\text{mm}$ ISA $100 \times 100 \times 10$. $P = 250\text{kN}$.



$$P = 0.462 \times f_u \times l_e \times t$$

$$250 \times 10^3 = 0.462 \times 410 \times l_e \times 0.75 \times 6$$

$$l_e = 314.24\text{mm}$$

$$l_1 y_1 = l_2 y_2$$

$$l_1 \times 50 = l_2 \times \frac{28.4}{2}$$

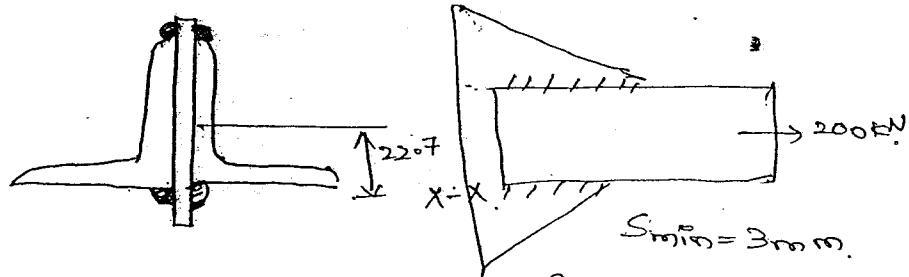
$$l_e = l_1 + l_2$$

$$314.24 = \frac{28.4}{71.6} l_2 + l_2$$

$$l_2 = 225.54\text{mm}$$

$$l_1 = 89.46\text{mm}$$

8. A tie member of truss consists of double angle section $80 \times 80 \times 8$ mm welded on opposite side of 12 mm thick gusset plate. Design a fillet weld to make the joint ax. $P = 200$ kN.



$$l_w = 2(x_1 + x_2)$$

$$S_{\min} = 3 \text{ mm}$$

$$S_{\max} = 8 - 1.5 = 6.5 \text{ mm}$$

$$\begin{aligned} \text{Strength of weld/mm} &= 0.462 \times f_u \times t_e \\ &= 0.462 \times 410 \times 0.7 \times 6 \\ &= 795.584 \text{ N/mm} \end{aligned}$$

$$\text{Strength of weld} = \text{Load}$$

$$2 \times 795.584 (x_1 + x_2) = 200 \times 10^3$$

$$x_1 + x_2 = 125.69 \text{ mm}$$

$$\text{Moment about } x-x$$

$$2 \times 795.584 \times x_1 \times 80 = 200 \times 10^3 \times 22.7$$

$$x_1 = 35.66 \text{ mm}$$

$$x_2 = 90.02 \text{ mm}$$

$$l_w = 2(x_1 + x_2) = 251.36 \text{ mm}$$

Note:

A large size weld requires more than 1 run of welding which means that after first run of welding chipping and clearing will be required for proper bond for successive run. This increases the cost.

* A smaller weld size will be cheaper than larger ~~load~~ one for the same strength.
 Considering volume of the weld

for (eg:) A 300mm length of 5mm size weld will have the same strength as a 150mm long 10mm size fillet weld. (115.5kN)

However the volume of weld metal for a 10mm size will be twice as that of 5mm size.

l	s
300 mm	5mm
150 mm	10mm

Volume of 5mm = $\frac{1}{2} \times 5 \times 5 \times 300 = 3750 \text{ mm}^3$

V of 10mm = $\frac{1}{2} \times 10 \times 10 \times 150 = 7500 \text{ mm}^3$

Equivalent Correlation

In plane moment:

$$F_s = \frac{P}{(b/c) \cdot \gamma}$$

$$Z = \frac{P \cdot e \cdot \sin \alpha}{I_{xx} + I_{yy}}$$

$$e = a + (b - \bar{x})$$

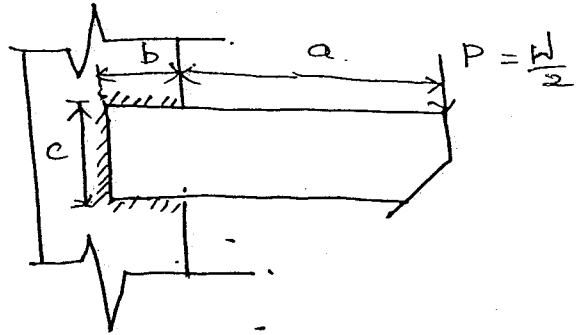
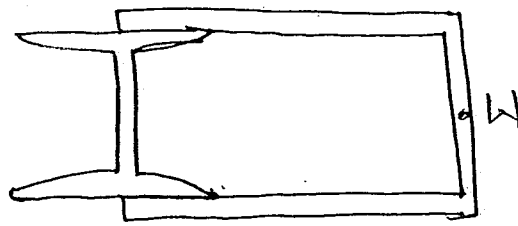
$$\cos \theta = \left(\frac{b - \bar{x}}{\gamma} \right)$$

$$f_r = \sqrt{f_s^2 + Z^2 + 2f_s Z \cos \theta}$$

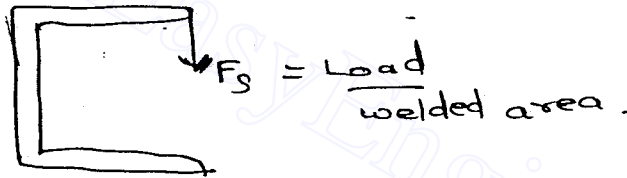
$$f_r \leq 0.452 f_u$$

$$\leq 0.333 f_u$$

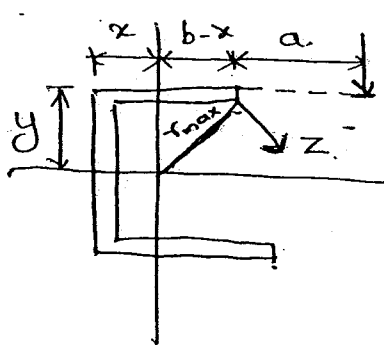
In plane Moment (Torsional shear)



1) Direct Shear.



$$F_s = \frac{P}{(b+c+b) \times t}$$



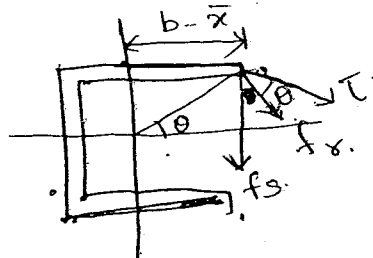
$$\frac{T}{J} = \frac{Z}{R}$$

$$Z = \frac{T}{J} \times R$$

$$\cos \theta = \frac{(b-x)}{r}$$

$$Z = \frac{Pxe}{I_{xx} + I_{yy}} \times r_{max}$$

$$e = a + (b-x)$$



$$f_r = \sqrt{f_s^2 + T^2 + 2f_s T \cos \theta}$$

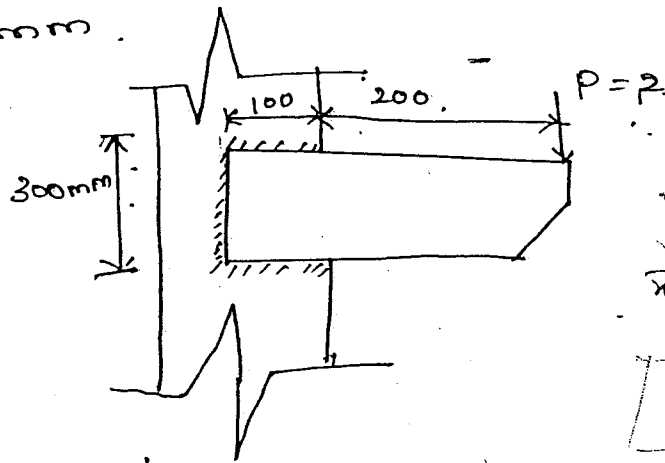
$$f_r \leq \text{Permissible stress}$$

$$\leq 0.462 f_u \text{ (Shop)}$$

$$\leq 0.385 f_u \text{ (Field)}$$

1. Determine Load carrying capacity use.

$$S = 6 \text{ mm}$$



$$I_{xy} = 28.3 \times 10^6 \text{ mm}^4$$

$$I_{yy} = 1.96 \times 10^6 \text{ mm}^4$$

$$\bar{y} = 150 \text{ mm}$$

$$\bar{x} = 200 \text{ mm}$$

$$P = 102.7 \text{ kN}$$

Solution:

$$S = 6 \text{ mm}$$

$$t = 6 \times 0.7 = 4.2 \text{ mm}$$

$$t = 4.2 \text{ mm}$$

$$\text{Total Area of weld} = (300 \times 4.2) + 2(100 \times 4.2)$$

$$= 2100 \text{ mm}^2$$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{A_{\text{total}}}$$

$$= \frac{2(100 \times 4.2 \times 297.9) + 2(300 \times 4.2 \times 150) + (100 \times 4.2 \times 200)}{2100}$$

$$\bar{y} = 150 \text{ mm}$$

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{A_{\text{total}}}$$

$$= \frac{2(100 \times 4.2 \times 50) + (300 \times 4.2 \times 200)}{2100}$$

$$\bar{x} = 200 \text{ mm}$$

$$I_{xx} = 2 \left[\frac{b_1 d_1^3}{12} + A(\bar{y} - y_1)^2 \right] + \frac{b_2 d_2^3}{12} + A(\bar{y} - y)^2$$

$$= 2 \left[\frac{100 \times 4.2^3}{12} + 100 \times 4.2 \times 150^2 \right] + \frac{300 \times 4.2^3}{12} + 300 \times 4.2 \times 150^2$$

$$I_{xx} = 28.35 \times 10^6 \text{ mm}^4$$

$$I_{yy} = \frac{300 \times 4.2^3}{12} + 300 \times 4.2 (200 - 0)^2$$

$$+ 2 \left[\frac{100^3 \times 4.2}{12} + 100 \times 4.2 (50 - 50)^2 \right]$$

$$I_{yy} = 1.96 \times 10^6 \text{ mm}^4$$

$$I_{zz} = 2835 \times 10^6 + 696 \times 10^6$$

$$= 30.31 \times 10^6$$

$$f_s = \frac{P \times 1000}{A_{\text{weld}}} = \frac{P \times 1}{2100} = 4.762 \times 10^{-4} = 0.4762$$

$$\tau = \frac{P \times e}{I_{zz}} \times r_{\text{max}}$$

$$e = 80 + 200$$

$$e = 280 \text{ mm}$$

$$= \frac{P \times 280 \times 170}{30.31 \times 10^6}$$

$$r_{\text{max}} = \sqrt{150^2 + 80^2}$$

$$r_{\text{max}} = 170 \text{ mm}$$

$$\tau = 1.57 \times 10^{-3} P$$

$$= 1.57 P$$

$$\theta = \tan^{-1} \left(\frac{80}{150} \right)$$

$$\theta = 28.072$$

$$f_r = \sqrt{(1.57 \times 10^{-3} P)^2 + (4.762 \times 10^{-4} P)^2}$$

$$+ 2(1.57 \times 10^{-3} \times 4.762 \times 10^{-4} P) \cos 28.072$$

$$f_r = 2.0026 \times 10^{-3} P$$

$$f_r = \frac{f_u}{\sqrt{3 \times 1.25}} = \frac{410}{\sqrt{3 \times 1.25}}$$

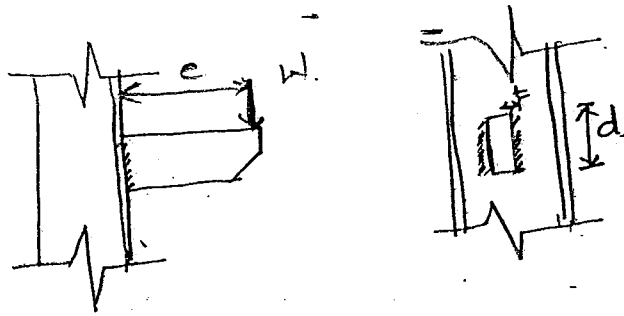
$$= 189.37 \text{ N/mm}^2$$

$$2.0026 \times 10^{-3} P = 189.37$$

$$P =$$

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OUTPLANE MOMENT.



Direct shear stress.

$$q = f_s = \frac{P}{2(dxt)}$$

Bending stress

$$\frac{M}{I} = \frac{f}{y}$$

$$f_a = \frac{M}{I} \times y.$$

$$f_a = \frac{P \times e}{2 \left(\frac{t \times d^3}{12} \right)} \times \left(\frac{d}{2} \right)$$

Resultant stress:

$$f_r = \sqrt{f_s^2 + f_a^2}$$

(According to WSM)

$$f_e = \sqrt{f_a^2 + 3q^2} \leq \frac{(f_u/\sqrt{3})}{\gamma_{mb}}$$

(According to LSM)

* out plane Moment. — Direct stress.

* Direct stress — Bending stress

$$f_s = \frac{P}{2(dt)}$$

* Bending stress

$$f_a = \frac{Pxe}{2\left(\frac{td^3}{12}\right)} \times y$$

* Equivalent stress:

$$f_c = \sqrt{3f_s^2 + f_a^2} \quad \text{LSM} < 0.46f_u \quad \frac{f_u}{\sqrt{3}}$$

$$f_c = \sqrt{f_s^2 + f_a^2} \quad \text{WSM}$$

* Butt weld

thickness of weld $t =$ thickness size of plate
length of weld $d =$ depth of plate.

$$f_c = \sqrt{3f_s^2 + f_a^2} \leq \frac{f_y}{1.1}$$

1. Determine Load carrying Capacity of 300 mm ϕ and 16 mm \times t is connected to a flange of a column with 6 mm fillet weld on both side Load is applied 200 mm from the face of the column.

Given:

$$t = 0.7 \times s$$

$$= 0.7 \times 6$$

$$\boxed{t = 4.2 \text{ mm}}$$

$$q = f_s = \frac{P}{2(dx)t}$$

$$= \frac{P}{2 \times 4.2 \times 300} = 3.97 \times 10^{-4} P.$$

$$\boxed{f_s = 3.97 \times 10^{-4} P}$$

$$f_a = \frac{P \times e}{2 \left(t \times \frac{d^3}{12} \right)} \times \frac{d}{2}$$

$$= \frac{P \times 200 \times \frac{300}{2}}{2 \left(\frac{4.2 \times 300^3}{12} \right)}$$

$$\boxed{f_a = 1.587 \times 10^{-3} P.}$$

$$f_e = \sqrt{3f_s^2 + f_a^2}$$

$$= \sqrt{3(3.97 \times 10^{-4} P)^2 + (1.587 \times 10^{-3} P)^2}$$

$$f_e = 1.7298 \times 10^{-3} P.$$

$$f_e \leq 0.462 f_u$$

$$1.7298 \times 10^{-3} P \leq 0.462 f_u$$

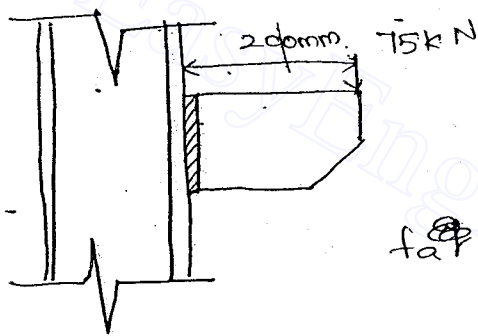
$$P \leq \frac{0.462 \times 410}{1.7298 \times 10^{-3}}$$

$$P \leq 109.5 \text{ kN.}$$

$$\boxed{P = 109 \text{ kN}}$$

2. An I-section is connected to a flange on a col. cantilever - I section @ a $d = 300\text{mm}$. $b_f = 150\text{mm}$ $D = 300\text{mm}$. Length of ϕ weld of each flange 100mm 150mm either side of web. $S = 6\text{mm}$ for flange $S = 5\text{mm}$ for web. What is the load carrying capacity.

3. Determine the max. stress developed in a butt weld connecting flange plate with a flange of a column. $P = 75\text{ kN}$ is applied on the plate @ $e = 200\text{mm}$. Size of plate 300×12 .



$$f_e = \sqrt{3q^2 + f_s^2}$$

$$f_{a\phi} = \frac{P \times e}{\frac{t \times d^3}{12}} \times \frac{y}{2}$$

$$= \frac{75 \times 10^3 \times 200}{\frac{12 \times 300^3}{12}} \times \frac{300}{2}$$

$$f_{a\phi} = 83.33 \text{ N/mm}^2$$

$$q = f_s = \frac{75 \times 10^3}{12 \times 300} = 20.833$$

$$f_e = \sqrt{3(83.33)^2 + (20.833)^2} = \sqrt{3(20.833)^2 + (83.33)^2}$$

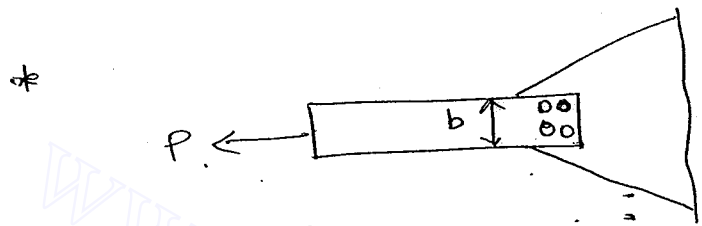
$$f_e = 145.82 \text{ N/mm}^2 \quad f_e = 90.81 \text{ N/mm}^2$$

$$\text{Max stress} = \frac{f_t}{1.1} = \frac{218.240}{1.1} = 218.18 > 145.82 \times 90.81 \text{ N/mm}^2$$

Hence safe.

TENSION MEMBER.

- * Member subjected to axial tension.
- * Eg: Tie of a roof truss.
- * The load carrying capacity of a tension member is of two criteria.
 - yield criteria (Based on A_g)
 - Rupture criteria (Based on A_n)



yielding criteria:

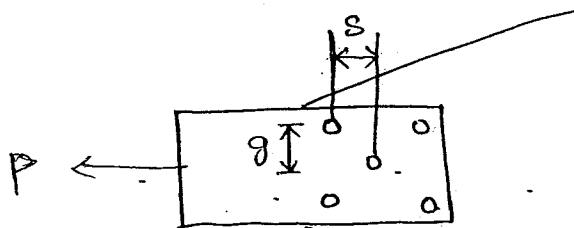
$$P_{at} = \frac{f_y}{1.01} \times A_g$$

Rupture criteria:

$$P_{at} = \frac{0.9f_u}{1.25} \times A_n$$

$$A_n = (b - n d_h) \times t$$

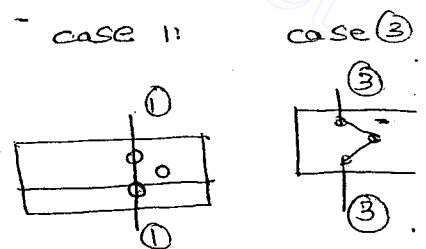
* For staggered bolting:



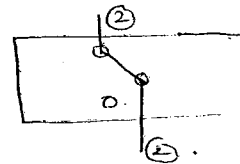
Rupture criteria along changes

$$P_{at} = \frac{0.9f_u}{1.25} \times A_n$$

$$A_n = \left[b - n d_h + \frac{s_1^2}{4g} + \frac{s_2^2}{4g} + \dots \right] \times t$$

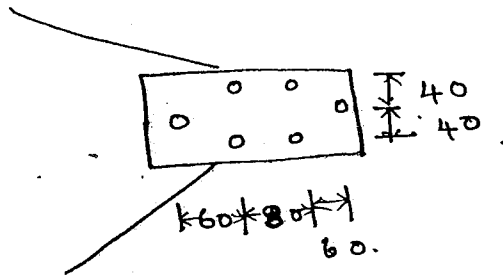


case 2:



1. A tension member is of flat plate $200\text{mm} \times 10\text{mm}$. It is connected to a gusset plate with 6 bolts in 2 rows in chain bolting. $\phi_{\text{bolt}} = 18\text{mm}$ $g = 60\text{mm}$. $p = 80\text{mm}$. Determine load carrying capacity.

(ii) In the above numerical if it is staggered bolting determine P_{at}



Case (i).

Yield: criteria:

$$P_{at} = \frac{f_y}{1.1} \times A_g$$

$$= \frac{240}{1.1} \times (200 \times 10)$$

$$= 436.36 \text{ kN}$$

Rupture criteria:

$$P_{at} = \frac{0.9 f_u}{1.25} \times A_n$$

$$= \frac{0.9 \times 410}{1.25} \left[\frac{[200 - 3(18)] \times 10}{1.25} \right]$$

$$= 430.992 \text{ kN}$$

Load carrying capacity = 430 kN

Case (ii)

Yielding criteria:

$$P = \frac{f_y}{1.1} \times A_g = 436.36 \text{ kN}$$

Rupture criteria:

$$A_{n1} = (200 - 18) \times 10 = 1820 \text{ mm}^2$$

$$A_{n2} = \left(200 - 2(18) + \frac{60^2}{4 \times 40} \right) \times 10 = 1865 \text{ mm}^2$$

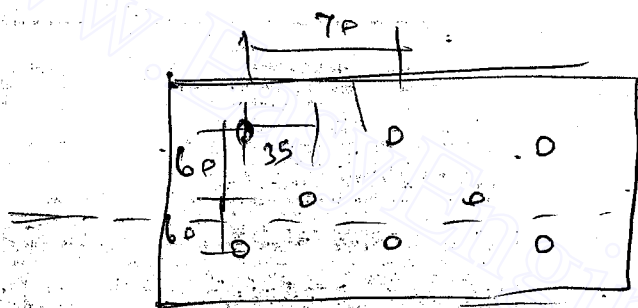
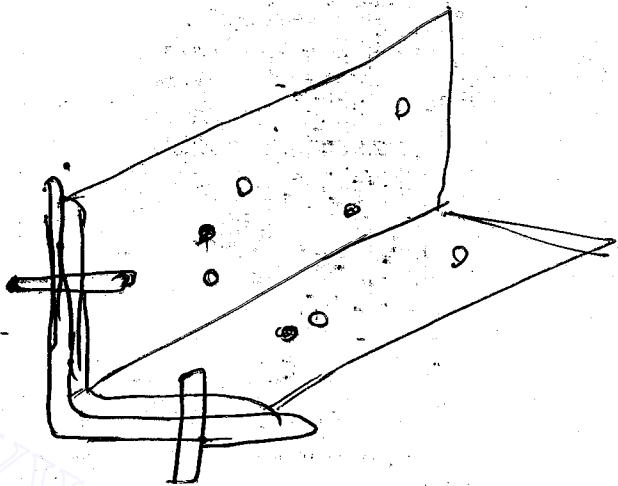
$$A_{n3} = \left[200 - 3(18) + 2 \left(\frac{60^2}{4 \times 40} \right) \right] \times 10 = 1910 \text{ mm}^2$$

$$P = \frac{0.9 f_u}{1.25} \times A_n$$
$$= \frac{0.9 \times 410}{1.25} \times 1820$$

$$P = 537.26 \text{ kN}$$

$$\text{Design Load} = 435 \text{ kN}$$

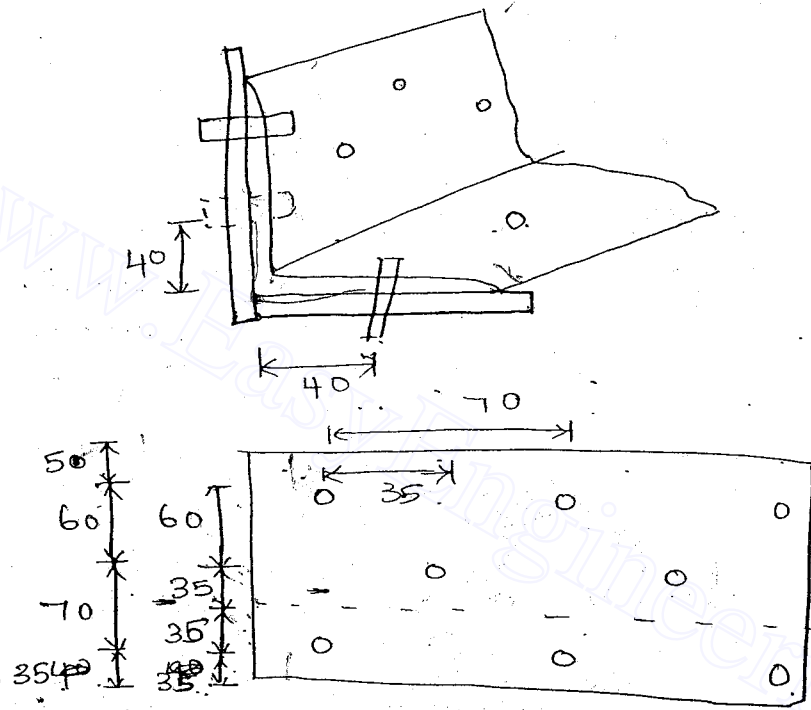
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D.S.S
Tension Member.

1. A tension member consists of an angle, 150x75x10 is connected with gusset plate on both the sides. The longer leg consists of 2 rows of bolt spaced @ $g = 60\text{mm}$. $p = 70\text{mm}$. position of the bolt is 40mm from the corner of the both the leg.

Determine A_{net} and ultimate load. $d = 16$.



$$A_{n1} = 150 - \frac{10}{2} + [215 - 2(18)] \times 10$$

$$= 1790 \text{ mm}^2$$

$$A_{n2} = 215 - 2(18) + \frac{35^2}{4 \times 60}$$

$$= 1841.04 \text{ mm}^2$$

$$A_{n3} = 215 - 3(18) + \frac{35^2}{4 \times 60} + \frac{35^2}{4 \times 70}$$

$$= 1884.79 \text{ mm}^2$$

$$= 1704.79 \text{ mm}^2$$

Yield

$$P_{at} = \frac{f_y \times 215 \times 10^3}{1.1} \left[(150 \times 10) + (75 \times 10) \right]$$

$$= 488.63 \text{ kN}$$

Rupture

$$P_{at} = \frac{0.9 f_u A_{n1}}{1.25}$$

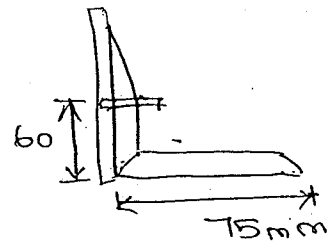
$$= \frac{0.9 \times 410 \times 1104.79}{1.25}$$

$$= 503.25 \text{ kN}$$

2. Single angle $125 \times 75 \times 10$ mm, ϕ bolt = 20 mm
 In single row the gauge is 60 mm from corner. Determine the ultimate load carrying capacity and no. of bolt use. - yielding and rupture criteria.

Solution:

Note: If no. of bolt is not given Assume no. of bolt and proceed atleast actual no. of bolt can be calculated.



- 1.) Yielding criteria.

$$P_{at} = \frac{f_y}{1.1} \times A_g$$

$$= \frac{250}{1.1} \times (125 + 75 - 10) \cdot 10$$

$$P_{at} = 488.64 \text{ kN} \approx 431.82 \text{ kN}$$

- 2.) Net section rupture.

$$P_{at} = \frac{0.9 f_u A_{nc}}{1.25} + \beta \frac{f_y \times A_g}{1.1}$$

$$\beta = 0.9 \times$$

$$\beta = 1.4 - 0.076 \left(\frac{w}{t} \right) \left(\frac{f_y}{f_u} \right) \left(\frac{b_s}{L_c} \right)$$

$$w = 75 \text{ mm} \quad t = 10 \text{ mm}$$

$$b_s = 60 + 75 - 10 = 125 \text{ mm}$$

$$L_c = P_1 + P_2 + P_3 + P_4$$

$$= 4 \times (2.5 \times 20) = 4(25d)$$

$$L_c = 200 \text{ mm}$$

$$\beta \geq 0.7$$

$$\beta \leq \frac{(f_u / \gamma_{m1})}{(f_y / \gamma_{m0})}$$

$$\beta = 1.4 - 0.076 \left(\frac{75}{10} \right) \left(\frac{250}{410} \right) \left(\frac{125}{200} \right)$$

$$\beta = 1.182$$

$$P_{aE} = \frac{0.9 \times 410 \times (125 - 5 - 22) \times 10}{1.25}$$

$$+ \frac{250 \times (75 - 5) \times 10}{1.1} \times 1.182$$

$$P_{aE} = 477.34 \text{ kN}$$

$$\text{Design load} = \frac{430 \text{ kN}}{477.34 \text{ kN}}$$

$$\text{No. of bolts} = \frac{430}{45.6}$$

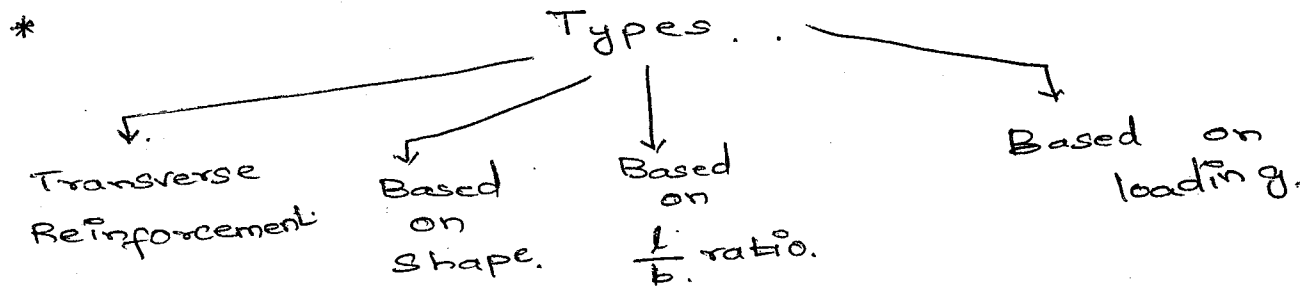
$$= 9.43$$

$$\underline{\underline{10 \text{ bolts}}}$$

19/12/2015

R.C.C.

COLUMN. (COMPRESSION MEMBER).



* Based on $\frac{l}{b}$ ratio.

a) $\frac{l}{b} < 3$. pedestal.

b) $3 < \frac{l}{b} < 12$. short column.

c) $\frac{l}{b} \geq 12$. long (or) slender column.

* Based on shapes:

(a) Rectangle

(b) square

(c) circular.

* Based on transverse reinforcement

(a) Lateral ties

(b) circular ties

(c) Helical (or) hoop reinforcement

* Based on Loading.

(a) Axial Loaded column.

(b) Axial with uniaxial bending

(c) Axial with biaxial bending.

* Minimum no. of bar.

Square & Rectangle. - 4 bars.

Circular. - 6 bars

hexagon - 6 bars

(one @ each corner)

* Min dia of bar for column (to avoid buckling)

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* Load carrying capacity of column.

$$F_{ax} \quad P_u = P_{\text{concrete}} + P_{\text{steel}}$$

For labored transverse reinforcement with lateral ties.

$$P_u = 0.4f_{ck} A_c + 0.67f_y A_{sc}$$

For helical reinforcement,

$$P_u = 1.05 \left[0.4f_{ck} A_c + 0.67f_y A_{sc} \right]$$

* e_{min} . (check for minimum Eccentricity).

$$e_{min} = \frac{l_x}{500} + \frac{b}{30} \quad (\text{or}) \quad 20 \text{ mm.}$$

whichever is greater.

Axial Load condition $e_{min} \leq 0.05b$.

$$e_{min} = \frac{l_y}{500} + \frac{d}{30} \quad (\text{or}) \quad 20 \text{ mm.}$$

Axial load condition $e_{min} \leq 0.05d$.

$$e_{min} = \frac{(\text{unsupported length})}{500} + \left(\frac{\text{least lateral dimension}}{30} \right)$$

Least value of $e_{min} = 20 \text{ mm.}$

* Assumption:

→ The max. axial compressive strain in concrete is taken as 0.002 [$\epsilon_{c,c} = 0.002$]

→ The max. compressive strain @ highly compressed extreme fibre in concrete subjected axial compression and bending and when there is no tension in the section shall be.

$$0.0035 - 0.75 \left[\text{strain in least compressive extreme fibre} \right]$$

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Scha

* Slenderness limits:

→ when both ends are restrained

$$l \neq 60b$$

→ when one end is restrained.

$$l \neq \frac{100b^2}{D}$$

* Cover Concrete.

when size of column $< 200\text{mm}$ clear cover = 25mm

when size of column $\geq 200\text{mm}$ clear cover = 40mm

* Min % of reinforcement = 0.8% of A_g .

* Max % of reinforcement

→ 6% of A_g . (without overlapping)

→ 4% of A_g (with overlapping)

* Min % of reinforcement = 0.15% A_g for pedestal.

* Spacing of longitudinal bars $\neq 300\text{mm}$.

* ϕ of lateral ties = $\frac{1}{4} \times \phi$ largest main bar.

= 6mm (whichever is greater)

* Pitch of lateral ties.

→ $\neq b$. (least lateral dimension)

→ $\neq 16 \phi$ smaller main bar.

→ $\neq 300\text{mm}$.

whichever is smaller.

* Pitch for helical ties.

→ $\neq 25\text{mm}$.

→ $\neq 3 \phi_{\text{helix}}$.

→ $\neq 75\text{mm}$.

→ $\neq \frac{1}{4} D_{\text{core}}$.

(whichever is small)

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* short column with Biaxial Bending (4)

The load contour method given by Bresler in 1960.

$$\left(\frac{M_{ux}}{M_{ux1}} \right)^{\alpha n} + \left(\frac{M_{uy}}{M_{uy1}} \right)^{\alpha n} \leq 1$$

M_{ux}, M_{uy} = External moment

M_{ux1}, M_{uy1} = Moment capacity.

α^n depends on $\left(\frac{P_u}{P_{uz}} \right)$.

* Slender Compression Method.

The additional moment is given by the formula.

$$M_{ax} = \left(\frac{P_D}{2000} \right) \left(\frac{L_{ex}}{D} \right)^2$$

$$M_{ay} = \left(\frac{P_B}{2000} \right) \left(\frac{L_{ey}}{B} \right)^2$$

* A bare moment should be added to eccentric loads.

* why to do we provide lateral ties?

→ To avoid buckling.

→ to hold longitudinal reinforcement

→ to provide confined surface

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* Load carrying Capacity. of column

→ If $e = 0$. (Ideal (or) Purely axially)

$$P_u = 0.45 f_{ck} A_c + 0.75 f_y A_{sc}$$

→ If $e \leq 0.05b$

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

(11% reduction of ideal column)

1. Determine the concentric load carrying capacity of circle column. $D = 500 \text{ mm}$.

8 bars $25 \text{ mm } \phi$. Use M20 and Fe415.

(a) If small eccentricity is applied what is P_u .

(b) If helical reinforcement is provided for above two cases:

Solution:

$$A_g = \frac{\pi}{4} \times 500^2 = 196.35 \times 10^3 \text{ mm}^2$$

$$A_{st} = 3926.99 \text{ mm}^2$$

$$A_c = 196.35 \times 10^3 - 3926.99$$

$$A_c = 192.42 \times 10^3 \text{ mm}^2$$

case (i) $e = 0$.

$$P_u = 0.45 f_{ck} A_c + 0.75 f_y A_{sc}$$

$$= (0.45 \times 20 \times 192.42 \times 10^3)$$

$$+ (0.75 \times 415 \times 3926.99)$$

$$P_u = 2954.06 \text{ kN}$$

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case (ii) $e = 0.05b$.

$$P_u = 0.4f_{ck} A_c + 0.67f_y A_{st}$$

$$= (0.4 \times 20 \times .192 \times 42 \times 10^3) + (0.67 \times 415 \times 3926.9)$$

$$\Rightarrow \boxed{P_u = 2631.26 \text{ KN.}}$$

case (iii) For helical reinforcement.
 $e = 0$.

$$P_u = 1.05 (2954.06)$$

$$\boxed{P_u = 3101.763 \text{ KN.}}$$

$e \leq 0.05b$.

$$P_u = 1.05 (2631.26)$$

$$\boxed{P_u = 2762.82 \text{ KN.}}$$

* longitudinal bars. ϕ must be

48 ϕ ties must be tied by single.
tie and beyond that additional ties
are provide.

Max angle of hook = 135° .

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1. A reinforced concrete column contains longitudinal steel = 1% net c/s Area of the column. Assume $m = 10$. the load carried ~~and~~ by steel and concrete is P_s and P_c . The ratio of $\frac{P_s}{P_c}$, expressed in percentage is _____. (use elastic theory).

Solution:

$$P = \sigma_c A_c + \sigma_{sc} A_{sc}$$

$$A_{sc} = \frac{1}{100} \times A_g$$

$$A_c = \frac{99}{100} A_g$$

$$P_s = \frac{\sigma_{sc} A_g}{100}$$

$$P_c = \frac{\sigma_c 99 A_g}{100}$$

$$\frac{P_s}{P_c} = \frac{\left(\frac{\sigma_{sc} A_g}{100}\right)}{\left(\frac{\sigma_c 99 A_g}{100}\right)}$$

$$\frac{P_s}{P_c} = \frac{\sigma_{sc}}{\sigma_c 99}$$

$$E_{sc} = E_c$$

$$\sigma_{sc} / E_s = \sigma_c / E_c$$

$$\frac{\sigma_{sc}}{\sigma_c} = \frac{E_s}{E_c}$$

$$\frac{P_s}{P_c} = \frac{m}{99}$$

$$\frac{\sigma_{sc}}{\sigma_c} = m$$

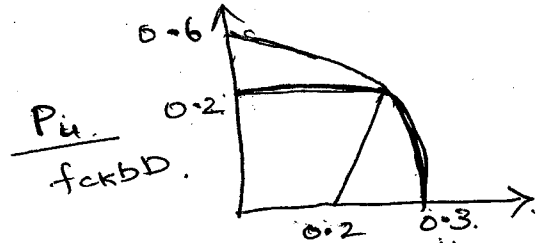
$$= \frac{10}{99}$$

$$\frac{P_s}{P_c} = 10\%$$

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2. R.C.C column of square c/s 400mm size. has its load moment interaction diagram as shown in figure. what is the maximum uniaxial eccentricity if $P_u = 6400 \text{ kN}$. $f_{ck} = 20 \text{ N/mm}^2$.

Solution:



$$\frac{P_u}{f_{ck} b D} = \frac{6400 \times 10^3}{20 \times 400^2}$$

$$\frac{M_u}{f_{ck} b D^2}$$

$$\frac{P_u}{f_{ck} b D} = \frac{6400 \times 10^3}{20 \times 400^2} = 0.2$$

$$\frac{M_u}{f_{ck} b D^2} = 0.3$$

$$M_u = 0.3 \times 20 \times 400^3$$

$$M_u = 384 \text{ kN}\cdot\text{m}$$

$$e = \frac{M_u}{P_u} = \frac{384}{6400} = 0.06$$

$$e = 0.06 \text{ m}$$

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3. Find Asc. for circle $\phi = 400\text{mm}$. (9)

$P_u = 1500\text{kN}$. total length 3.2m . Adopt M_{25} and F_{y415} . Use helical tie bar.

Solution:

$$\frac{l}{D} = \frac{3200}{400} = 8 < 12\text{mm}$$

It is a short column.

$$e_{\min} = \frac{l}{500} + \frac{bd}{30}$$

$$= \frac{3200}{500} + \frac{400}{30}$$

$$e_{\min} = 19.73 \leq 20\text{mm}$$

$$e_{\min} = 20\text{mm}$$

$$0.05b = 0.05 \times 400 = 20\text{mm} = e_{\min}$$

Hence, design as axial column.

$$P_u = 1.05 [0.4 f_{ck} A_c + 0.67 f_y A_{sc}]$$

$$= 1.05 \left[0.4 \times 25 \times \left(\frac{\pi}{4} \times 400^2 - A_{sc} \right) + 0.67 \times 415 \times A_{sc} \right]$$

$$1500 \times 10^3 = 1.05 \left[1.256 \times 10^6 - 10 A_{sc} + 278.05 A_{sc} \right]$$

$$1875000 = 268.05 A_{sc} \times 1.05$$

$$A_{sc} = 612.95\text{mm}^2$$

$$\boxed{A_{sc} = 641\text{mm}^2}$$

$$\text{Min } A_{st} = 0.8\% A_g$$

$$= 0.8 \times \frac{\pi \times 400^2}{4}$$

$$= 1005.3\text{mm}^2$$

~~ϕ req~~

Hence, provide.

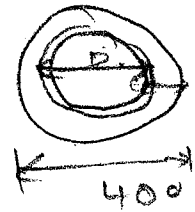
$$A_{sc} = 1005 \text{ mm}^2.$$

ϕ of bar:

$$6 \times \frac{\pi \times d^2}{4} = 1005.$$

$$d = \sqrt{\frac{1005 \times 4}{6 \times \pi}}$$

$$\boxed{d = 16 \text{ mm}}$$



Lateral ties:

$$1.) \frac{\phi_{\text{main bar}}}{4} = \frac{16}{4} = 4 \text{ mm}$$

$$2.) 6 \text{ mm}.$$

Provide 6 mm ϕ bar.

Pitch.

Volume of helical reinforcement
Volume of core.

$$\geq 0.36 \left(\frac{P_g}{A_{\text{Core}}} - 1 \right) \frac{f_{ck}}{f_y}$$

$$+ 3 \phi_{\text{helix}} = 3 \times 6 \text{ mm} = 18 \text{ mm}.$$

$$* 25 \text{ mm}.$$

$$* 25 \text{ mm} \text{ Volume}$$

Dia of core = outer to outer dia of helix.

$$D_{\text{core}} = 400 - 32 - 32.$$

$$\boxed{D_{\text{core}} = 336 \text{ mm}}$$

$$\text{Volume of core} = A_{\text{core}} \times P.$$

$$= \frac{\pi}{4} \times 336^2 \times P.$$

$$= 88.67 \times 10^3 P.$$

$$\text{Volume of helix} = \frac{\pi}{4} \times 8^2 \times l_{\text{tie}}.$$

$$l_{\text{tie}} = \pi \left[D_{\text{core}} - \frac{\phi_{\text{helix}}}{2} - \frac{\phi_{\text{helix}}}{2} \right]$$

$$= \frac{\pi}{4} \times 8^2 \times 328 \times \pi$$

$$= 51.79 \times 10^3.$$

$$\frac{\text{Volume of helix}}{\text{Volume of core}} \geq 0.36 \left(\frac{A_s}{A_{\text{core}}} - 1 \right) \frac{f_{ck}}{f_y}.$$

$$\geq 0.36 \left(\frac{125.66 \times 10^3}{88.67 \times 10^3} - 1 \right) \frac{25}{415}.$$

$$\frac{88.67 \times 10^3 \times P}{51.79 \times 10^3} \geq 9.048 \times 10^{-3}.$$

$$\frac{51.79 \times 10^3}{88.67 \times 10^3 P} \geq 9.048 \times 10^{-3}.$$

$$P \leq 64.55 \text{ mm}.$$

- (i) $> 25 \text{ mm}.$
- (ii) $< 75 \text{ mm}.$
- (iii) $\geq 3 \times 8 = 24 \text{ mm}$

$$(iv) \geq \frac{1}{6} \times D_{\text{core}} = \frac{1}{6} \times 336 = 56 \text{ mm}.$$

Hence provide $8 \text{ mm } \phi$ @ $56 \text{ mm } \phi$ spacing.

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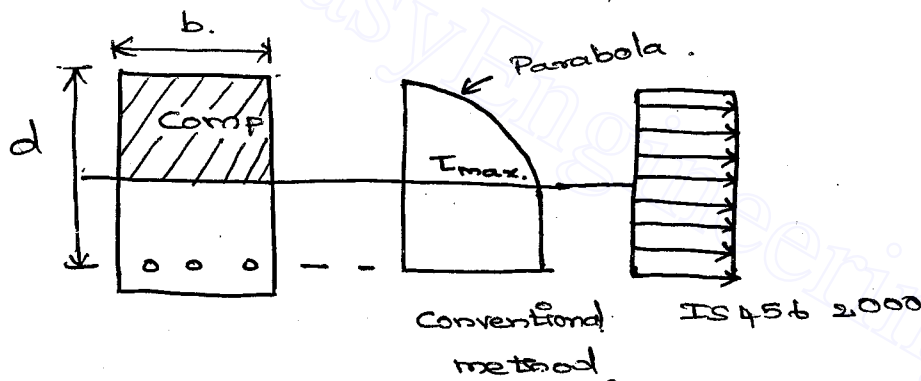
3/1/2015

R.C.C.

SHEAR BOND TORSION.

* The stress distribution in R.C.C based on elastic theory (conventional theory) is parabolic having values @ zero @ max. compressive fibre and max. shear stress @ neutral axis and then it becomes rectangle from N.A to centre of tension steel

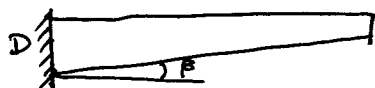
* According to IS 456: 2000 the shear stress distribution is assumed as uniform (rectangle) having value as given below.



$$* \quad T_{max} = \frac{3}{2} T_{avg}$$

$$* \quad T_v = \frac{V_u}{bd} \quad (\text{for rectangular beam}).$$

$$T_v = \frac{V_u \pm \frac{M_u}{d} \tan \beta}{bd} \quad [\text{for beam of varying depths}].$$



$$T_v = \frac{V_u - \frac{M_u}{d} \tan \beta}{bd}$$



$$T_v = \frac{V_u + \frac{M_u}{d} \tan \beta}{bd}$$

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* Shear resistance ~~without~~ ^{by} concrete without stirrups:

- (i) Compressive force. - 20-40% of total shear force
- (ii) Interlocking of aggregate - 33-50%
- (iii) Dowel Action - 15-20%

$$\tau_{cmax} = 0.85 \times 0.75 \cdot \sqrt{f_{ck}} = 0.637 \sqrt{f_{ck}}$$

(Table 20)

Grade of concrete.	M15	M20	M25	M30	M35	M40
τ_{cmax} (LSM)	2.5	2.8	3.1	3.5	3.7	4.0
τ_{cmax} LBM	1.67	1.87	2.06	2.33	2.47	2.67

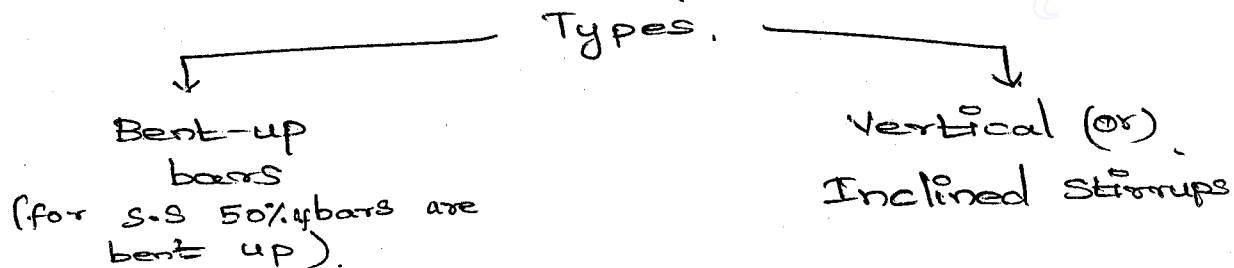
* τ_c depends upon f_{ck} and P_t

* $\tau_v = \tau_c$ No design but nominal stirrups is provided.

* $\tau_v < \tau_c$ "

* $\tau_v > \tau_c$ Shear reinforcement is required.

* Shear reinforcement



$$V_{\text{bent up bars}} = 0.87 f_y A_{sv} \sin \alpha$$

* Shear resistance by combined bent up & stirrups.

$$V_{\text{shear resistance}} = (\tau_v - \tau_c) bd$$

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* Max shear resistance by bentup bar alone.

$$0.87f_y A_{sv} \sin \alpha \leq 50\% (\tau_v - \tau_c) b d$$

* Shear resistance by stirrups.

$$V_{\text{stirrups}} = V_{\text{shear resistance}} - V_{\text{bentup}}$$

* ϕ of stirrups.

6mm, 8, 10, 12 & 16mm.
 Theoretically, } Heavy torsion.

* No. of legs.

(2) 4, 6, 8, 10, 12.
 used for structural element such as beam, column.

* Area of stirrups.

$$A_{sv} = \frac{\pi}{4} \times (\text{dia})^2 \times \text{No. of legs.}$$

Clause 40.4 IS: 456
 * Shear reinforcement shall be provided to carry a shear force equal to.

$$V_{us} = V_u - \tau_c b d$$

The strength of shear reinforcement ~~is~~ V_{us} shall be calculated as below.

$$V_{us} = \frac{0.87f_y A_{sv} d}{S_v} \quad (\text{vertical stirrups})$$

$$V_{us} = \frac{0.87f_y A_{sv} d}{S_v} (\sin \alpha + \cos \alpha) \quad (\text{Inclined stirrups})$$

* For single bars and (or) single group of parallel bars all bent up @ same c/s.

$$V_{us} = 0.87f_y A_{sv} \sin \alpha$$

where S_v - spacing of stirrups (or) bent up bars only along the length of member.

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f_y shall not be greater than 415 MPa. (4)

angle α - angle between the inclined stirrups ^(or) bent of bar and axis of the member.

$$\alpha \neq 45^\circ$$

* The complete development lengths and anchorage shall be determined to have been provided when the bar is bent through an angle of at least 90° round a bar of at least its own diameter and its continued beyond the end of the curve of length for at least 8ϕ .

* when the bar is bent through an angle of 135° and beyond the end of the curve for a length of at least 6ϕ .

* when the bar is bent through an angle of 180° and it is continuous beyond the end of the curve for a length of at least 4ϕ .

* check for maximum spacing of stirrups.

$$S_{rmax} \neq 0.75d. \quad (\text{Normal vertical stirrups})$$

$$\neq d. \quad (\text{Inclined stirrups})$$

$$\neq 300 \text{ mm.}$$

$$\neq \frac{0.87f_y A_{sv}}{0.4b}$$

$$\neq \text{calculated.} \rightarrow S_r = \frac{0.87f_y A_{sv} d}{V_{us}}$$

* The min. percentage of shear reinforcement is d for inclined stirrups.

$$100 \times \frac{A_{sv}}{bd} = \frac{0.87f_y}{0.4b} \cdot \frac{0.4}{0.87f_y} \times 100$$

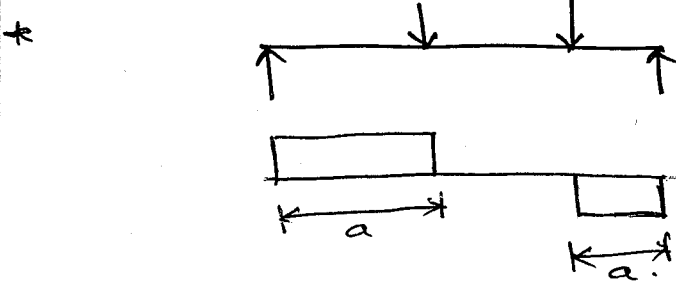
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Grade: P 450.

Fe 215 0.184%

Fe 415 0.11%

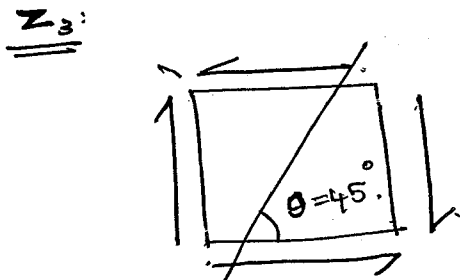
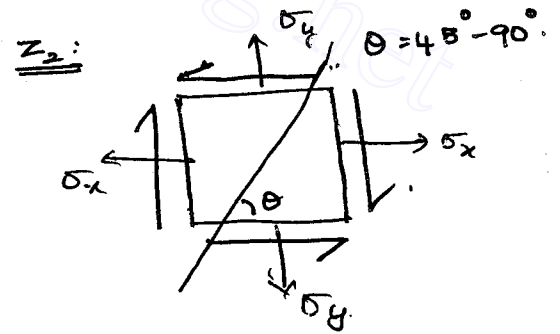
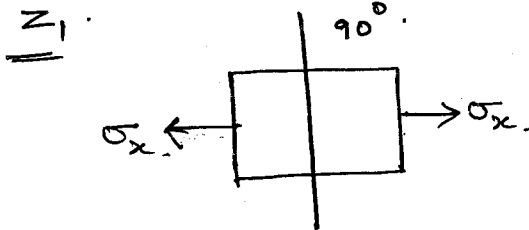
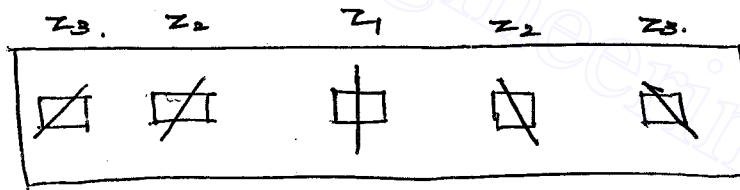
Fe 500 ~~0.09%~~



$$\text{Shear span, } a = \frac{M}{V}$$

* The shear failure pattern is dependent on $\frac{a}{d}$ ratio.

$$\frac{a}{d} = \frac{\text{Shear span}}{\text{Effective depth}}$$



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Zone 1	Flexural zone $\sigma_1 = \sigma_x$.	Flexural Failure
Zone 2.	Combined zone $\sigma_1 = \frac{\sigma_x}{2} + \frac{1}{2} \sqrt{\sigma_x^2 + 4\tau^2}$	Combined Failure.
Zone 3.	Diagonal Tension Crack zone (or) Pure shear. failure zone. where $\sigma_1 = \tau$.	where shear failure

* If $\frac{a}{d} < 1$ (deep beams) - Tension or compression failure.

In this case internal tied arch develops

- * If $\frac{a}{d} \approx 1$ to 2.5 - Shear tension (or) Shear compression Crack.
- * If $\frac{a}{d} \approx 2.5$ to 6 - Diagonal tension Crack.
- * If $\frac{a}{d} > 6$ - Flexural crack.

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1. A beam of rectangular section of 230×400 (effective) is subjected to max shear force 120 kN . Use M_{20} and $F_e 415$ and $F_e 250$. The design shear strength (τ_c) $= 0.48 \text{ MPa}$.

- (a) spacing of 2 legged 8 mm stirrups
 (b) In addition beam is subjected to a torque of $10.9 \text{ kN}\cdot\text{m}$
 Determine the shear force caused by the stirrups.

Solution:

$$S_v = \frac{0.87 f_y A_{sv} \times d}{V_{us}}$$

$$= \frac{0.87 \times 250 \times \frac{\pi}{4} \times 8^2 \times 2 \times 400}{[V_c - (\tau_c b d)]}$$

$$= \frac{8.746 \times 10^6}{[120 \times 10^3 - (0.48 \times 230 \times 400)]}$$

$$S_v = 115.32 \text{ mm}$$

Check:

$$S_v \neq 0.75d = 0.75 \times 400 = 300 \text{ mm}$$

$$\neq 300 \text{ mm}$$

$$S_v \neq \frac{0.87 f_y A_{sv}}{0.4b} = \frac{0.87 \times 250 \times 2 \times \frac{\pi}{4} \times 8^2}{0.4 \times 230}$$

$$= 237.67 \text{ mm}$$

$$S_v = 100 \text{ mm}$$

$$V_s = V_u - T_c b d$$

Since torsion is acting

$$V_u = V_e$$

$$V_s = V_e - T_c b d$$

$$= \left(V_u + \frac{1.6 T_u}{b} \right) - T_c b d$$

$$= \left(120 \times 10^3 + \frac{1.6 \times 10.9 \times 10^6}{230} \right) - \left(\begin{array}{l} 0.48 \\ \times 230 \\ \times 400 \end{array} \right)$$

$$\boxed{V_s = 151.67 \text{ KN}}$$

2. A R.C.C beam is fixed @ one end and free @ other end it is subjected to $V_u = 300 \text{ kN}$ and $l = 3 \text{ m}$.

Reinforcement - 6 # 20 mm ϕ .

C/S @ Fixed $\rightarrow 300 \times 500$.

C/S @ free $\rightarrow 300 \times 300$.

Determine spacing of 2 legged 8 mm stirrups use M_{20} and $F_e 415$ of Effective Cover 40 mm,

$$\tan \beta = \frac{200}{3000}$$

$$= 0.067$$

$$\tau_v = \frac{V_u - \frac{M_u}{d} \tan \beta}{bd}$$

$$\tau_v = \frac{\left(\frac{300}{10^3} \right) - \frac{450 \times 10^6}{460} \times 0.067}{300 \times 460}$$

$$\tau_v = 2.21 \text{ N/mm}^2$$

$$\tau_v = 1.70 \text{ N/mm}^2$$

$$\tau_c = 0.69$$

$$S_v = \frac{0.87 \times 415 \times 2 \times \frac{\pi}{4} \times 8^2 \times 460}{V_u - \tau_c b d}$$

$$= \frac{0.87 \times 415 \times 2 \times \frac{\pi}{4} \times 8^2 \times 460}{(1.7 - 0.69) 460 \times 300}$$

$$= (1.7 - 0.69) 460 \times 300$$

$$S_v = 119.79 \text{ mm}$$

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3. consider a bar of $\phi = d$ embedded in large concrete block as shown in fig. with a pull out force "P" being applied. Let σ_b and σ_{st} be bond and tensile strength of the bar. If the block is held in position and it is assumed that material of the block does not fail which of the following option represents max. value of "P".

Max value of P_s & P_b .

Ans: Max value of $\frac{\pi D^2 \sigma_{st}}{4}$ & $\frac{\pi D L \sigma_b}{4}$.

4. consider 2 beams P & Q, each having 400×750 effective $\tau_{cmax} = 2.51$ MPa for the reinforcement provide and grade of concrete $\tau_c = 0.75$ MPa. The design shear in beam P is 400 kN. Q - 750 kN. considering the IS 456:2000 which of the following statement is true.

$$\underline{P} \quad \tau_v = \frac{400 \times 10^3}{400 \times 750} = 1.33$$

$$\tau_v > \tau_c$$

$$\tau_v < \tau_{cmax}$$

$$V_{us} = (\tau_v - \tau_c) b d$$

$$\underline{V_{us} = 175 \text{ kN}}$$

$$\underline{Q} \quad \tau_v = \frac{750 \times 10^3}{400 \times 750} = 2.5$$

$$\tau_v > \tau_c > \tau_{cmax}$$

Hence the section must be revised.

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5. The ^{beam} width size 250 x 450 (effective) The beam is reinforced by 20mm TOR bars in TOR bar $\gamma V_u = 150 \text{ kN}$. Check the requirement of shear reinforcement and provide if required. use $M_{20}, Fe 415$. use 8mm ϕ stirrups.

$$V_{us} = \frac{0.87 f_y A_{sv} d}{S_v}$$

$$\frac{A_{sv}}{S_v} \geq \frac{0.4 b}{f_y}$$

Above conditioned must be satisfied.

% of steel.	1	1.25	1.5.
T_c N/mm ²	0.62	0.67	0.72.

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17/12/2015

TORSION.

PRINCIPLE STRESSES AND THEORIES OF FAILURE.

* Torque:

Torque is force \times radius $T = F \times r$ and the phenomenon is called torsion which is B.M about its own axis or Z-Z axis.

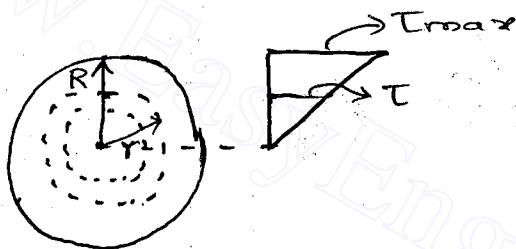
Unit - N.m (or) Joule.

Dimension - ML^2T^{-2}

$\frac{kgm}{s^2}$
 $kg m^2 s^{-2}$
 $ML^2 T^{-2}$

* Torsional Equation:

By similar Δ property.



$$\frac{T_{max}}{T} = \frac{R}{r}$$

$$T = \frac{T_{max} \times r}{R}$$

$$T = F \times r$$

$$dT = dF \times r$$

$$dT = \tau \times dA \times r$$

$$= \tau \times 2\pi r \, dr \times r$$

$$= \frac{T_{max}}{R} \times r \times 2\pi r^2 \, dr$$

$$dT = \frac{2\pi}{R} T_{max} \times r^3 \, dr$$

$$\int dT = \frac{2\pi}{R} T_{max} \int_0^R r^3 \, dr$$

$$T = \frac{2\pi}{R} T_{max} \left(\frac{r^4}{4} \right)_0^R$$

$$= \frac{2\pi}{4} \frac{T_{max}}{R} R^4$$

$$= \frac{\pi}{2} \frac{T_{max}}{R} \left(\frac{D}{2} \right)^4$$

$$= \frac{\pi}{2} \frac{T_{max}}{R} \left(\frac{D^4}{16} \right)$$

2/12/2015

TORSION.

* Torque:

→ It is the B.M about its own axis (or)
z-z axis

→ The phenomenon is called Torsion

→ Torque $T = \text{Force} \times \text{radius}$

* Torsional Equation:

$$\frac{\partial \theta}{\partial x} = \frac{T}{J} = \frac{G \theta}{L}$$

$$\boxed{dT = dF \times r} \quad \boxed{\phi = R \theta}$$

$$\boxed{G = \frac{T}{\phi}}$$

* Polar moment of Inertia

$$J = I_P = I_{xx} + I_{yy}$$

$$J = \frac{\pi D^4}{32} \quad \text{for solid shaft.}$$

$$J = \frac{\pi (D^4 - d^4)}{32} \quad \text{for hollow shaft}$$

$$J = \frac{\pi D^3 t}{4} \quad \text{for thin circular shaft}$$

* Strain Energy Due to Torsion:

$$U = \frac{T^2}{2G} \times \text{Volume.}$$

$$U = \frac{T_{max}^2}{4G} \times \text{Volume}$$

$$U = \frac{T^2 \times L}{2GJ}$$

* Equivalent Torque and Equivalent Bending Moment.

$$T = P \bar{x}$$

$$M = P \bar{y}$$

$$\sigma_{1,2} = \frac{16}{\pi D^3} \left[M \pm \sqrt{M^2 + T^2} \right]$$

$$M_{eq} = \frac{1}{2} \left[M + \sqrt{M^2 + T^2} \right]$$

$$\sigma_1 = \frac{32 M_{eq}}{\pi D^3}$$

$$T_{eq} = \frac{1}{2} \left[\sqrt{M^2 + T^2} \right]$$

$$\tau_{max} = \frac{16}{\pi D^3} \left[\sqrt{M^2 + T^2} \right]$$

$$= \frac{\sigma_1 - \sigma_2}{2}$$

$$T = \frac{T_{max}}{R} \times \left(\frac{\pi D^4}{32} \right)$$

$$= \frac{T_{max}}{R} \times J.$$

$$\boxed{\frac{T}{J} = \frac{T_{max}}{R.}} \quad \text{Torsional equation}$$

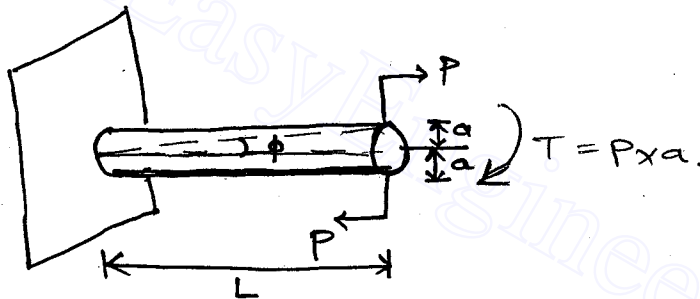
* Polar moment of inertia (I_p (or) J).

for solid circular shaft.

$$\boxed{J = \frac{\pi D^4}{32}}$$

for thin circular shaft.

$$\boxed{J = \frac{\pi D^3 t}{4}}$$



$$\tan \phi = \frac{R\theta}{l}$$

ϕ = shear strain.

θ = $\frac{\text{Arc length}}{\text{Radius.}}$

$$\theta = \frac{l}{R}$$

$$\boxed{l = R\theta}$$

$$G = \frac{\tau}{\phi}$$

$$G = \frac{\tau}{\frac{R\theta}{l}}$$

$$\boxed{\frac{\tau}{R} = \frac{G\theta}{l}}$$

$$\boxed{\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{l}}$$

Torsional Equation.

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* Expression of ^{Strain} Energy Due to Torsion. (3)

$$U = \frac{T^2}{2G} \times \text{volume.}$$

$$du = \frac{T^2}{2G} \times dV.$$

$$T = \frac{T_{\max} \times r}{R}$$

$$= \frac{\left(\frac{T_{\max} \times r}{R}\right)^2}{2G} \times l \times dA.$$

$$= \frac{T_{\max}^2 \times r^2}{2R^2 G} \times l \times 2\pi r dr.$$

$$du = \frac{T_{\max}^2}{2R^2 G} \times 2\pi l \times r^3 dr.$$

$$\int du = \frac{T_{\max}^2}{2R^2 G} \times 2\pi l \times \int_0^R r^3 dr.$$

$$U = \frac{T_{\max}^2}{R^2 G} \times \pi l \times \left(\frac{r^4}{4}\right)_0^R.$$

$$= \frac{T_{\max}^2}{R^2 G} \times \pi l \times \frac{R^4}{4}.$$

$$= \frac{T_{\max}^2}{4G} \times \pi R^2 l.$$

$$U = \frac{T_{\max}^2 \times \text{Volume}}{4G}$$

$$T_{\max} = \frac{T}{J} R.$$

$$U = \frac{\left(\frac{TR}{J}\right)^2 \times A \times l}{4G}.$$

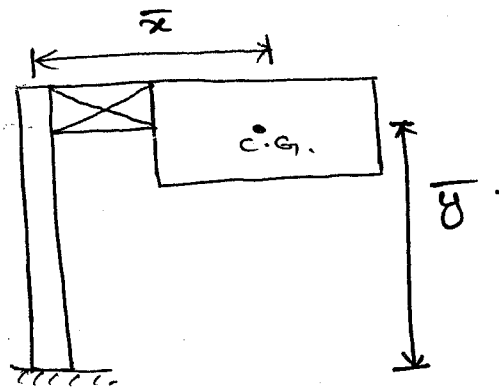
$$= \frac{T^2 \times \frac{D^2}{4}}{\left(\frac{\pi D^4}{32}\right)^2 \times 4G} \times \frac{\pi D^2}{4} \times l.$$

$$= \frac{T^2 \times \pi \left(\frac{D^4}{32}\right) \times l}{\left(\frac{\pi D^4}{32}\right)^2 \times 2G}.$$

$$U = \frac{T^2 \times l}{2GJ}.$$

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* Expression for Equivalent Torque and Equivalent B.M when it is subjected to combined Torque and B.M.



$$T = P \times \bar{x}$$

$$M = P \times \bar{y}$$

$$P = \frac{M}{\bar{y}}$$

$$\frac{T}{R} = \frac{T}{J}$$

$$T = \frac{T \times R}{J}$$

$$= \frac{T \times D/2}{\left(\frac{\pi D^4}{32}\right)}$$

$$T = \frac{16T}{\pi D^3}$$

$$\frac{M}{H} = \frac{f}{y}$$

$$\frac{M \times y}{H} = \sigma_x$$

$$\sigma_x = \frac{M}{\left(\frac{\pi D^4}{64}\right)} \times \frac{D}{2}$$

$$\sigma_x = \frac{32M}{\pi D^3}$$

$$\sigma_{1,2} = \frac{\sigma_x}{2} \pm \frac{1}{2} \sqrt{\sigma_x^2 + 4T^2}$$

$$= \frac{\left(\frac{32M}{\pi D^3}\right)}{2} \pm \frac{1}{2} \sqrt{\left(\frac{32M}{\pi D^3}\right)^2 + 4\left(\frac{16T}{\pi D^3}\right)^2}$$

$$= \frac{\left(\frac{32M}{\pi D^3}\right)}{2} \pm \frac{1}{2} \sqrt{\left(\frac{32M}{\pi D^3}\right)^2 + \left(\frac{32T}{\pi D^3}\right)^2}$$

$$= \frac{32M}{\pi D^3} \pm \frac{32}{\pi D^3} \sqrt{M^2 + T^2}$$

$$= \frac{32}{\pi D^3} \times \frac{1}{2} \left[M \pm \sqrt{M^2 + T^2} \right]$$

$$\sigma_{1,2} = \frac{16}{\pi D^3} \left[M \pm \sqrt{M^2 + T^2} \right]$$

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For Resultant Bending.

$$\frac{M_{eq}}{I} = \frac{f}{y}$$

$$\frac{M_{eq}}{\left(\frac{\pi \times D^4}{64}\right)} = \frac{\sigma_1}{\left(\frac{D}{2}\right)}$$

$$M_{eq} \times \frac{64}{\pi D^4} \times \frac{D}{2} = \sigma_1 = \frac{M_{eq}}{\frac{\pi D^3}{32}}$$

$$\boxed{\sigma_1 = \frac{32 M_{eq}}{\pi D^3}}$$

$$\frac{16}{\pi D^3} (M + \sqrt{M^2 + T^2}) = \frac{32 M_{eq}}{\pi D^3}$$

$$\boxed{M_{eq} = \frac{1}{2} (M + \sqrt{M^2 + T^2})}$$

$$\frac{T_{max}}{R} = \frac{\sigma_1 - \sigma_2}{2}$$

$$= \frac{1}{2} \times \frac{16}{\pi D^3} \times (M + \sqrt{M^2 + T^2} - M + \sqrt{M^2 + T^2})$$

$$= \frac{1}{2} \times \frac{16}{\pi D^3} \times 2 (\sqrt{M^2 + T^2})$$

$$\boxed{T_{max} = \frac{16}{\pi D^3} \sqrt{M^2 + T^2}}$$

Resultant Torsion,

$$\frac{T_{max}}{R} = \frac{T_{eq}}{J}$$

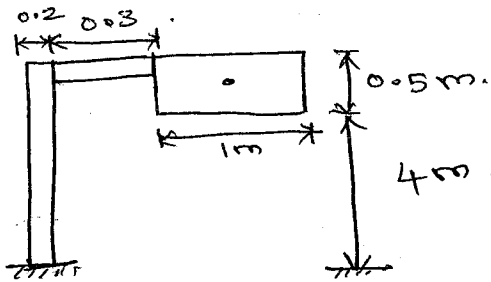
$$\frac{\frac{16}{\pi D^3} \sqrt{M^2 + T^2}}{\frac{D}{2}} = \frac{T_{eq}}{\frac{\pi D^4}{32}}$$

$$\frac{32}{\pi D^4} \sqrt{M^2 + T^2} = \frac{32}{\pi D^4} T_{eq}$$

$$\boxed{T_{eq} = \sqrt{M^2 + T^2}}$$

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1. A vertical post $D_o = 200\text{mm}$. $D_i = 100\text{mm}$.
 Supports a sign board of $1\text{m} \times 0.5\text{m}$ subjected wind
 pressure 1.5KPa . board is connected to the
 post 200 by 300mm long angles. the under
 side of board is 4m above G.L. Determine
 M_{eq} , T_{eq} , σ_1 , T_{max} , Bending stress due to,
 Bending B.M only, τ due to Torsion only.



$$\bar{x} = \frac{0.2}{2} + 0.3 + \frac{1}{2}$$

$$\boxed{\bar{x} = 0.9\text{m}}$$

$$\bar{y} = \frac{0.5}{2} + 4$$

$$\boxed{\bar{y} = 4.25\text{m}}$$

$$P = 1.5 \times 1 \times 0.5$$

$$\boxed{P = 0.75\text{KN}}$$

$$M = P \times \bar{y}$$

$$= 0.75 \times 4.25$$

$$\boxed{M = 3.1875\text{KN}\cdot\text{m}}$$

$$T = P \times \bar{x}$$

$$= 0.75 \times 0.9$$

$$\boxed{T = 0.675\text{KN}\cdot\text{m}}$$

$$M_{eq} = \frac{1}{2} \left[M + \sqrt{M^2 + T^2} \right]$$

$$= \frac{1}{2} \left[3.1875 + \sqrt{(3.1875)^2 + (0.675)^2} \right]$$

$$\boxed{M_{eq} = 3.22\text{KN}\cdot\text{m}}$$

$$T_{eq} = \sqrt{M^2 + T^2}$$

$$= \sqrt{(3.1875)^2 + (0.675)^2}$$

$$\boxed{T_{eq} = 3.26\text{KN}\cdot\text{m}}$$

$$\sigma_1 = \frac{3.2 M_{eq}}{\pi D_o^3}$$

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$$\sigma_1 = 16$$

$$\sigma_x = \frac{M}{I} \times y.$$

$$= \frac{3.1875 \times \frac{200}{2}}{\frac{\pi}{64} (200^4 - 100^4)}$$

$$= \frac{3.1875 \times \left(\frac{0.2}{2}\right)}{\frac{\pi}{64} (0.2^4 - 0.1^4)}$$

$$\sigma_x = 64.93 \text{ N/mm}^2 \quad \boxed{\sigma_x = 4.33 \text{ N/mm}^2}$$

$$\tau = \frac{T}{J} \times R.$$

$$= \frac{0.675 \times 10^6 \times \frac{200}{2}}{\frac{\pi}{32} (200^4 - 100^4)}$$

$$\tau = 6.688 \text{ N/mm}^2 \quad \boxed{\tau = 0.46 \text{ N/mm}^2}$$

$$\sigma_1 = \frac{\sigma_x}{2} + \frac{1}{2} \sqrt{\sigma_x^2 + 4\tau^2}$$

$$= \frac{4.33}{2} + \frac{1}{2} \sqrt{(4.33)^2 + 4(0.46)^2}$$

$$= 2.165 + 2.213$$

$$= 4.378$$

$$\boxed{\sigma_1 = 4.38 \text{ N/mm}^2}$$

$$\sigma_2 = 2.165 - 2.213$$

$$\sigma_2 = -0.02$$

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{4.38 + 0.02}{2}$$

$$\boxed{\tau_{max} = 2.202 \text{ N/mm}^2}$$

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- 2.) A solid shaft is subjected to torque 4.5 kNm $\theta = 1^\circ$ $l = 20 \text{ dia}$
 $\tau_{\text{max}} = 80 \text{ MPa}$. Take $G = 0.8 \times 10^5 \text{ N/mm}^2$
 Determine the dia.

Solution:

$$\frac{T}{J} = \frac{\tau}{R}$$

$$\frac{4.5 \times 10^6}{\frac{\pi}{64} D^4} = \frac{80}{D/2}$$

$$D^3 = \frac{4.5 \times 10^6}{\frac{\pi}{16} \times 80}$$

$$D = 65.92 \text{ mm}$$

$$\frac{\tau}{R} = \frac{G\theta}{l} \quad \frac{T}{J} = \frac{G\theta}{l}$$

$$\frac{4.5 \times 10^6}{\frac{\pi}{32} D^4} = \frac{0.8 \times 10^5 \times \frac{1 \times \pi}{180}}{20 \times D}$$

$$D^3 = \frac{4.5 \times 10^6 \times 20}{0.8 \times 10^5 \times \frac{\pi}{180} \times \frac{\pi}{32}}$$

$$D = 86.91 \text{ mm}$$

Adopt the greater value.

$$D = 87 \text{ mm}$$

* Power :

$$P = \frac{2\pi NT}{60}$$

P - Watts
T - KN.m

$$\begin{aligned} \% \text{ of saving material} &= \frac{W_s - W_H}{W_s} \times 100 \\ &= \frac{D_s^2 - (D_o^2 - D_i^2)}{D_s^2} \times 100 \end{aligned}$$

% of wastage of material

$$= \frac{W_H - W_s}{W_H} \times 100$$

* Hoop stress, longitudinal stress

$$\text{Hoop stress } (\sigma_x) = \frac{Pd}{2t}$$

$$\text{Longitudinal stress } (\sigma_y) = \frac{Pd}{4t}$$

$$\text{Absolute Max stress} = \frac{\sigma}{2}$$

* If the torque is acting @ the middle of the bar then, $\theta_{net} = 0$

$$\theta_1 = \theta_2$$

* If the torque is acting in opposite direction in two different bars, then, θ_{net}

$$\theta_{net} = \theta_1 - \theta_2$$

39) A solid shaft has to transfer a mean power of 1000 kW @ 300 rpm. $T_{max} = 60 \text{ MPa}$. determine dia of shaft. If $T_{max} = 25\%$ more than T_{mean} . Also determine % saving in material. If hollow shaft of inner dia $0.6 D_o$ is used

Solution:

$$P = \frac{2\pi NT}{60}$$

$$1000 \times 10^3 = \frac{2\pi NT}{60}$$

$$T_{\text{mean}} = 31.83 \text{ KN}\cdot\text{m}$$

$$T_{\text{max}} = 1.25 \times 31.83$$

$$T_{\text{max}} = 39.78 \text{ KN}\cdot\text{m}$$

$$\frac{T}{J} = \frac{T_{\text{max}}}{R}$$

$$\frac{39.78 \times 10^6}{\frac{\pi}{32} \times D^4} = \frac{60}{D/2}$$

$$D^3 = \frac{39.78 \times 10^6}{\left(\frac{\pi}{16} \times 60\right)}$$

$$D = 150 \text{ mm}$$

For hollow shaft.

$$\frac{39.78 \times 10^6}{\frac{\pi}{32} \times (D^4 - (0.6D)^4)} = \frac{60}{D/2}$$

$$\frac{39.78 \times 10^6}{\frac{\pi}{32} \times 0.8704 D^4} = \frac{60 \times 2}{D}$$

$$D^3 = 3.879 \times 10^6$$

$$D = 157 \text{ mm}$$

Mat

$$\% \text{ of saving material} = \frac{W_s - W_H}{W_s} \times 100.$$

$$= \frac{D_s^2 - (D_o^2 - D_i^2)}{D_s^2} \times 100$$

$$= \frac{150^2 - (157^2 - (0.6 \times 157)^2)}{150^2} \times 100.$$

$\% \text{ of saving of material} = 29.89\%$
--

$$\% \text{ of wastage of material} \} = \frac{W_H - W_s}{W_H} \times 100.$$

18/12/2015

SOM

1. A thin cylinder $d = 300\text{mm}$, $t = 10\text{mm}$, $T = 100\text{ N}\cdot\text{m}$.
inner fluid pressure = 1 MPa . Determine
shear stress due to torque. Max. principle
stress, absolute maximum stress.

$$\frac{T}{J} = \frac{\tau}{R}$$

$$R = \frac{300}{2} = 150\text{mm}$$

$$\tau = \frac{T \times R}{J}$$

$$J = \frac{\pi}{4} d^3 t$$

$$= \frac{100 \times 150 \times 10^3}{\frac{\pi}{4} \times 300^3 \times 10}$$

$$\tau = 0.07\text{ N/mm}^2$$

$$\text{Hoop stress} = \frac{Pd}{2t}$$

$$\sigma_x = \frac{1 \times 300}{2 \times 10}$$

$$\sigma_x = 15\text{ N/mm}^2$$

$$\text{Longitudinal stress} = \frac{Pd}{4t}$$

$$= \frac{1 \times 300}{4 \times 10}$$

$$= 7.5\text{ N/mm}^2$$

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$$

$$= \left(\frac{15 + 7.5}{2} \right) + \frac{1}{2} \sqrt{(15 - 7.5)^2 + 4(0.07)^2}$$

$$\sigma_1 = 15\text{ N/mm}^2$$

$$\sigma_2 = \left(\frac{\sigma_x + \sigma_y}{2} \right) - \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$$

$$\sigma_2 = 7.5\text{ N/mm}^2$$

$$\begin{aligned}\tau_{\max} &= \frac{\sigma_1 - \sigma_2}{2} \\ &= \frac{15 - 7.5}{2}\end{aligned}$$

$$\boxed{\tau_{\max} = 3.75 \text{ N/mm}^2}$$

$$\begin{aligned}\text{Absolute max. stress} &= \frac{\sigma_1}{2} \\ &= \frac{15}{2} \\ &= 7.5 \text{ N/mm}^2.\end{aligned}$$

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2. Simply supported beam span = 4m
 subjected to central point load 2kN.
 $T = 1.5 \text{ kN}\cdot\text{m}$ $\phi = 100 \text{ mm}$. Find. M_{eq} , σ_1 ,
 T_{eq} , T_{max} ,

Solution:

$$M = \frac{wL}{4} = \frac{2 \times 4}{4} = 2 \text{ kN}\cdot\text{m}.$$

$$M_{eq} = \frac{1}{2} \left[M + \sqrt{M^2 + T^2} \right]$$

$$= \frac{1}{2} \left[2 + \sqrt{2^2 + 1.5^2} \right].$$

$$M_{eq} = 2.25 \text{ kN}\cdot\text{m}.$$

$$T_{eq} = \sqrt{M^2 + T^2}$$

$$= \sqrt{2^2 + 1.5^2}.$$

$$T_{eq} = 2.5 \text{ kN}\cdot\text{m}.$$

$$\frac{M_{eq}}{I} = \frac{\sigma_{max}}{y}.$$

$$y = \frac{D}{2}$$

$$\sigma_{max} = \frac{M_{eq} \times y}{I}$$

$$= \frac{2.25 \times 10^6 \times 50}{\frac{\pi}{64} \times 100^4}.$$

$$\sigma_{max} = 22.92 \text{ N/mm}^2$$

$$\frac{T_{eq}}{J} = \frac{T_{max}}{R}.$$

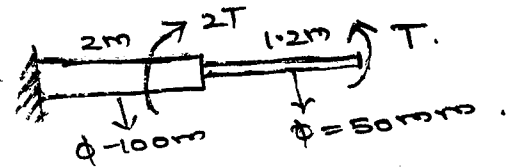
$$T_{max} = \frac{T_{eq} \times R}{J}$$

$$= \frac{2.5 \times 10^6 \times 50}{\frac{\pi}{32} \times 100^4}.$$

$$T_{max} = 12.73 \text{ N/mm}^2$$

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3. A shaft is shown in figure below determine value of T if $\tau_{max} \leq 80$ MPa stress must not exceed 80 MPa $G = 0.08 \times 10^5$ MPa. Determine angle of twist also.



Solution:

Hint : If Torque is safer for smaller dia then sure it will be safer for larger dia.

$$\frac{T}{J} = \frac{\tau_{max}}{R}$$

$$T = \frac{80 \times \frac{\pi}{32} \times 50^4}{\frac{50}{2}}$$

$$T = 1.96 \times 10^6 \text{ N}\cdot\text{mm}$$

$$\frac{T}{J_1} = \frac{\theta_1}{l_1}$$

$$\theta_1 = \frac{1.96 \times 10^6 \times 2000}{0.08 \times 10^5 \times \left(\frac{\pi \times 50^4}{32} \right)}$$

$$\theta_1 = 0.0479$$

$$\theta_2 = \frac{1.96 \times 10^6 \times 2000}{0.08 \times 10^5 \times \left(\frac{\pi \times 100^4}{32} \right)}$$

$$\theta_2 = 0.005$$

$$\theta_{net} = \theta_1 - \theta_2$$

$$= 0.0479 - 0.005$$

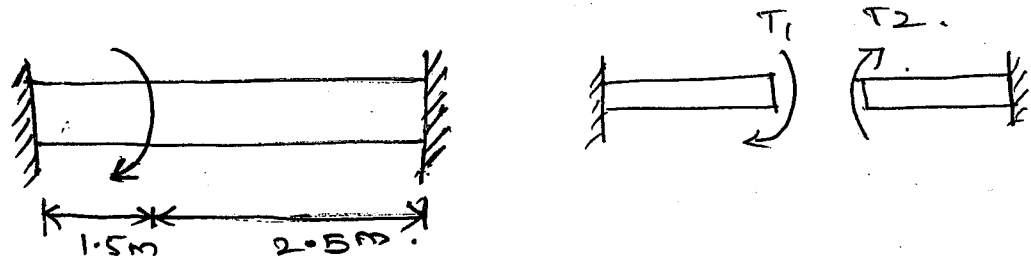
$$\theta_{net} = 0.043$$

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4. solid shaft $\phi = 50\text{mm}$. $T = 1\text{KN}\cdot\text{m}$

@ 1.5m from left support. Determine the Torque developed @ fixed end.

Max τ_{max} , θ ,



$$T = T_1 + T_2.$$

$$1 = T_1 + T_2.$$

$$1 = 1.67T_2 + T_2.$$

$$T_2 = \frac{1}{2.67}.$$

$$\boxed{T_2 = 0.375 \text{ KN}\cdot\text{m}}$$

$$T_1 = 1 - 0.375.$$

$$\boxed{T_1 = 0.625 \text{ KN}\cdot\text{m}}$$

$$\theta_1 = \frac{T_1 l_1}{GJ} = \left(\frac{0.625 \times 10^3 \times 1500}{0.8 \times 10^5 \times \frac{\pi}{32} \times 50^4} \right)$$

$$\boxed{\theta_1 = 0.019}$$

$$\frac{\tau_{\text{max}}}{R} = \frac{T_{\text{max}}}{J}$$

$$\tau_{\text{max}} = \frac{0.625 \times 10^3 \times \frac{50}{2}}{\frac{\pi}{32} \times 50^4}$$

$$\boxed{\tau_{\text{max}} = 25.46 \text{ N/mm}^2}$$

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27/12/2015

STRENGTH OF MATERIAL

1. A hollow shaft ^{Inner dia inner dia} with 60% of outer dia. is compared to the solid shaft of same material and equal weight. Determine the ratio of torque resistance. $T = \frac{\tau}{R} \times J$.

Solution:

$$\frac{W_H}{W_S} = \frac{\cancel{\rho} \times \text{Volume of hollow shaft}}{\cancel{\rho} \times \text{Volume of solid shaft}}$$

$$= \frac{\text{Area of hollow shaft} \times L}{\text{Area of solid shaft} \times L}$$

$$= \frac{\frac{\pi}{4} (D_H^2 - D_i^2)}{\frac{\pi}{4} D_o^2}$$

$$\frac{W_H}{W_S} = \frac{\frac{\pi}{4} \cdot 0.64 \cdot D_H^2}{\frac{\pi}{4} D_o^2} = \frac{0.64 D_H^2}{D_o^2}$$

$$W_H = 0.64 W_S$$

$$\boxed{D_o = 0.8 D_H}$$

$$\frac{T_H}{T_S} = \frac{\frac{\tau}{R_H} \times J}{\frac{\tau}{R_S} \times J}$$

$$= \frac{\cancel{\tau} \times (D_H^4 - D_i^4) \frac{\pi}{64}}{\cancel{\tau} \times D_S^4 \frac{\pi}{64}}$$

$$= \frac{D_H \times \frac{\pi}{64} (D_S^4)}{D_S \times \frac{\pi}{64} (0.8704 D_H^4)}$$

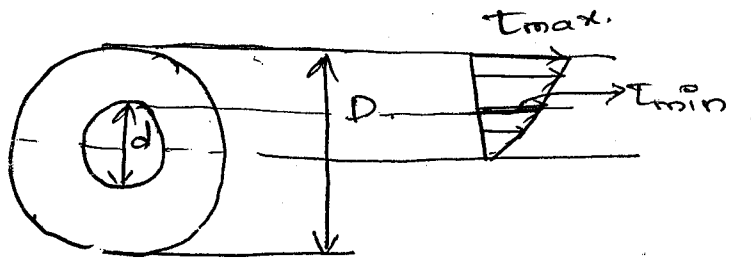
$$= \frac{D_H \times \frac{\pi}{64} (0.8704 D_H^4)}{0.8 D_H (0.8 D_H)^4}$$

$$= \frac{0.8 D_H (0.8704 D_H^4)}{D_H (0.8 D_H)^4}$$

$$= 1.7$$

$$\boxed{\frac{T_H}{T_S} = 1.7}$$

Isotropic State of stress is independent
of frame of reference (Reference Axis)



Note : If

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Note:

If 2 different shafts are connected in series then the torque will be same for both the shafts and the angle of twist will not be same.

If 2 shafts are in parallel then angle of twist will be same but torque will not be same. (i.e) the torque is shared by both the shafts.

2. A steel shaft of dia 40 mm is coaxially fitted with hollow bronze shaft. The inner dia of hollow shaft is equal to dia of steel shaft. The maximum permissible shear stress is 40 MPa for bronze and 60 MPa for steel. The total length of the shaft is 4 m. Determine outer dia of hollow shaft. Total torque applied on combined shaft. Angle of twist.
- $G_{\text{steel}} = 0.8 \times 10^5 \text{ N/mm}^2$ $G_{\text{bronze}} = 0.4 \times 10^5 \text{ N/mm}^2$

$$\theta_s = \theta_b \quad \frac{T}{R} = \frac{G\theta}{l}$$

$$\left(\frac{T \times l}{R \times G} \right)_s = \left(\frac{T \times l}{R \times G} \right)_b \quad \theta = \frac{T \times l}{R \times G}$$

$$\left(\frac{60 \times 4}{\frac{40}{2} \times 0.8 \times 10^5} \right) = \frac{40 \times 4}{\frac{D_b}{2} \times 0.4 \times 10^5}$$

$$D_b = 53.33 \text{ mm (outer dia)}$$

$$D_b = 40 \text{ mm (inner dia)}$$

$$\frac{T_b}{J} = \frac{T}{R}$$

$$T_b = \frac{40}{D_b/2} \times \frac{\pi}{32} (D_{b0}^4 - D_{b1}^4)$$

$$= \frac{40}{\left(\frac{53.33}{2}\right)} \times \frac{\pi}{32} (53.33^4 - 40^4)$$

$$T_b = 814.24 \times 10^3 \text{ N}\cdot\text{mm}$$

$$T_s = \frac{T_s}{R} \times J$$

$$= \frac{60}{(40/2)} \times \frac{\pi}{32} \times 40^4$$

$$T_s = 753.98 \times 10^3 \text{ N}\cdot\text{mm}$$

$$\frac{T_s}{J} = \frac{G\theta}{l}$$

$$\theta = \frac{753.98 \times 10^3}{\frac{\pi}{32} (40^4)} \times \frac{4000}{0.8 \times 10^5}$$

$$\theta = 0.15$$

$$\frac{T_b}{J} = \frac{G\theta}{l}$$

$$\theta = \frac{814.24 \times 10^3 \times 4000}{\frac{\pi}{32} (53.33^4 - 40^4) \times 0.4 \times 10^5}$$

$$\theta = 0.15$$

$$\text{Total torque} = T_1 + T_2$$

$$= 814.24 \times 10^3 + 753.98 \times 10^3$$

$$= 1568.22 \text{ kN}\cdot\text{mm}$$

28/12/2015

STRENGTH OF MATERIAL.

STRAIN ENERGY OF HOLLOW SHAFT.

$$U = \frac{T^2 l}{2GJ}$$

$$T = \frac{\tau}{R} \times J$$

$$= \frac{\tau^2 J^2}{R^2} \times l$$

$$= \frac{\tau^2 \cdot \frac{\pi}{32} (D^4 - d^4) \times l}{\left(\frac{D}{2}\right)^2 \times 2G}$$

$$= \frac{\tau^2 \cdot \frac{\pi}{4} (D^2 - d^2) \times \frac{1}{8} (D^2 + d^4) \times l}{\left(\frac{D}{2}\right)^2 \times 2G}$$

$$= \frac{\tau^2 \times A \times l \times \frac{1}{8} \frac{(D^2 + d^4)}{D^2}}{2G \times \frac{D}{4}}$$

$$U = \frac{\tau^2}{4G} \times \text{Volume} \left(\frac{D^2 + d^4}{D^2} \right)$$

$$\frac{U}{\text{Volume}} = \frac{\tau^2}{4G} \cdot \left(\frac{D^2 + d^4}{D^2} \right)$$

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1. Determine torsional energy per unit volume of a hollow shaft of $D_o = 100\text{mm}$ and $d_i = 60\text{mm}$. torsional stress 60MPa and $G = 84\text{GPa}$.

$$\begin{aligned} \frac{U}{V} &= \frac{\tau^2}{4G} \times \frac{D_o^2 + d_i^2}{D^2} \\ &= \frac{(60)^2}{4 \times 84 \times 10^9 \cdot \frac{10^6}{10^6}} \times \left(\frac{100^2 + 60^2}{100^2} \right) \\ &= 0.0146 \frac{\text{N}}{\text{mm}^2} \times \frac{\text{mm}}{\text{mm}} \\ &= 0.0146 \frac{\text{N} \cdot \text{mm}}{\text{mm}^3} \\ &= 0.0146 \cdot 10^{-3} \frac{\text{Joule}}{\text{mm}^3} \end{aligned}$$

$$\boxed{\frac{U}{V} = 0.0146 \cdot 10^6 \frac{\text{Joule}}{\text{m}^3}}$$

2. A shaft $d = 75\text{mm}$ $l = 1\text{m}$ is connected to another shaft of $d = 50\text{mm}$ of $l = 2\text{m}$. a power of 45KW is given to 75mm dia shaft and power of 15KW is taken away @ the junction of two shaft. Determine T_{max} anywhere in the shaft and total angle of twist. Take revolution of shaft as 200rpm .

$$\begin{aligned} P &= \frac{2\pi NT}{60} \\ 45 \times 10^3 &= \frac{2 \times 11 \times 200 \times T_1}{60} \end{aligned}$$

$$T_1 = 2148.59 \text{ N} \cdot \text{mm}$$

$$\boxed{T_1 = 2148.59 \times 10^3 \text{ N} \cdot \text{mm}}$$

$$\frac{T_1}{J} = \frac{T}{R}$$

$$T = \frac{2148.59 \times 10^3}{\frac{\pi}{32} \times 75^4} \times \frac{75}{2}$$

$$\boxed{T = 25.96 \text{ N/mm}^2}$$

$$\tau = \frac{T}{J} = \frac{G\theta}{l}$$

$$\theta_1 = \frac{2148.59 \times 10^3}{\frac{\pi}{32} \times 75^4} \times \frac{1000}{0.8 \times 10^5}$$

$$\theta_1 = 8.646 \times 10^{-3} \text{ radians}$$

$$P_2 = \tau_2 \frac{2\pi r^3 \pi}{60}$$

$$(45-15) \times 10^3 = \frac{2 \times 200 \times T_2 \pi}{60}$$

$$T_2 = 1432.39 \text{ N m}$$

$$T_2 = 1432.39 \times 10^3 \text{ N mm}$$

$$\frac{T_2}{J} = \frac{T}{R}$$

$$T = \frac{1432.39 \times 10^3}{\frac{\pi}{32} \times (50^4)} \times \frac{50}{2}$$

$$\boxed{T = 58.36 \text{ N/mm}^2} \quad \checkmark$$

$$\frac{T}{J} = \frac{G\theta}{l}$$

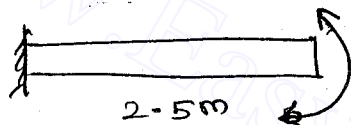
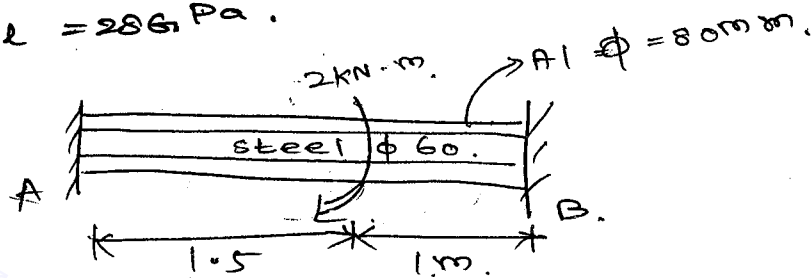
$$\theta_2 = \frac{1432.39 \times 10^3 \times 2000}{0.8 \times 10^5 \times \frac{\pi}{32} \times (50^4)}$$

$$\boxed{\theta_2 = 0.0583} \text{ radians}$$

$$\theta = \theta_1 + \theta_2 = 8.646 \times 10^{-3} + 0.0583 = 0.0670 \text{ radians}$$

3. A steel shaft of $\phi 60$ mm is coaxially fitted with aluminium shaft of 80mm. Both ends of shaft are fixed. Torque of $2 \text{ kN}\cdot\text{m}$ is applied at a distance 1 m from right support and 1.5 m from left support. Determine the torque developed @ support and maximum T develops in shaft. $G_s = 84 \text{ GPa}$

$$G_{Al} = 28 \text{ GPa}$$



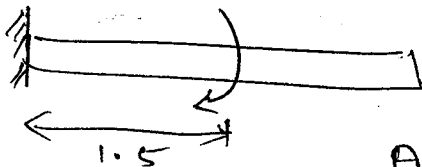
$$\theta_B = \frac{T_B \times l}{GJ}$$

$$= \frac{T_B \times 2500}{G_s J_s + G_{Al} J_{Al}}$$

$$= T_B \times 2500 \cdot \left(\frac{32}{84 \times 10^3 \times \pi \times 60^4} \right) + \left(\frac{32}{28 \times 10^3 \times \pi \times 80^4} \right)$$

$$\theta_B = 1.12 \times 10^{-14} T_B$$

$$\theta_B = 5.587 \times 10^{-8} T_B$$



$$\theta_A = \frac{2 \times 10^6 \times 1500}{G_s J_s + G_{Al} J_{Al}}$$

$$\theta_A = 0.0670$$

$$\theta_A = \theta_B$$

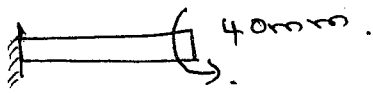
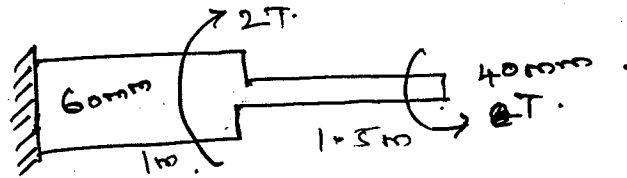
$$0.0670 = 5.587 \times 10^{-8} T_B$$

$$T_B = 1.2 \times 10^6 \text{ N}\cdot\text{m}$$

39) A steel shaft of 16mm ϕ is coaxially fitted with aluminium. $\phi = 20\text{mm}$ Both ends of shaft are fixed. A torque of 2 kN-m is applied at a distance x m from.

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- 4.) Figure shows a stepped steel shaft subjected to torque "T" @ free end, and 2T @ in the opp. direction @ the junction. Determine the total angle of twist if.
 $\tau_{max} = 80 \text{ MPa}$. $G = 80 \text{ GPa}$.



$$\frac{T_1}{J} = \frac{T_2}{R}$$

$$\frac{T}{J} = \frac{\tau_{max}}{R}$$

$$= \frac{80}{(40/2)} \times \frac{\pi}{32} 40^4$$

$$T = 1.005 \times 10^6 \text{ N.m}$$

$$T = 1.005 \text{ KN.m}$$

$$\frac{G\theta}{l} = \frac{T}{J}$$

$$\theta = \frac{T}{G}$$

$$= \frac{1.005}{\pi}$$

3.73

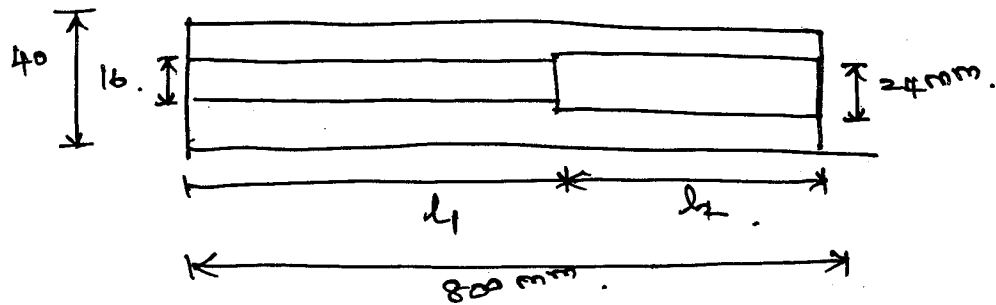
$$\theta = \frac{T}{G} \left(\frac{l_1}{J_1} - \frac{l_2}{J_2} \right)$$

$$= \frac{1.005 \times 10^6}{80 \times 10^3} \left[\frac{1000}{\frac{\pi}{32} 60^4} - \frac{1500}{\frac{\pi}{32} 40^4} \right]$$

$$\theta = 3.71^\circ$$

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5. Figure shows a hollow shaft determine the power transmitted @ 200 rpm $\tau_{max} = 70 \text{ MPa}$. Find the lengths of the shaft portions. If twist produced in two shafts are equal.



Solution:

$$\tau = \frac{16T}{\pi} \left(\frac{D}{D^4 - d^4} \right)$$

The maximum value of shear stress will reach in the portion in which "Dia" is higher.

$$P = \frac{2\pi NT}{60}$$

$$\tau = \frac{16T}{\pi} \left(\frac{D}{D^4 - d^4} \right)$$

$$70 = \frac{16 \times T}{\pi} \left(\frac{40}{40^4 - 24^4} \right)$$

$$T = 765.64 \times 10^3 \text{ N}\cdot\text{mm}$$

$$P = \frac{2\pi NT}{60} = \frac{2\pi \times 200 \times 765.64}{60}$$

$$P = 16.03 \text{ kW}$$

$$\frac{Tl}{JG} = \theta$$

$$\theta_1 = \theta_2$$

$$\frac{Tl_1}{JG} = \frac{Tl_2}{JG}$$

$$l_1 = l_2 \cdot \frac{\frac{\pi}{32} (40^4 - 16^4)}{\frac{\pi}{32} (40^4 - 24^4)}$$

$$l_1 = 1.12 l_2$$

$$l_1 + l_2 = 800$$

$$1.12 l_2 + l_2 = 800$$

$$l_2 = 377.45 \text{ mm}$$

$$l_1 = 422.55 \text{ mm}$$

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+
S.O.M.

* Σ Normal stresses acting @ any 2
 \perp^r plane remains constant.

$$\text{i.e. } \sigma_x + \sigma_y = \sigma_1 + \sigma_2.$$

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} = \frac{\sigma_1 + \sigma_2}{2}$$

$$T = \frac{P_1 - P_2}{2}$$

$$\sigma_r = \sqrt{\frac{P_1^2 + P_2^2}{2}}$$

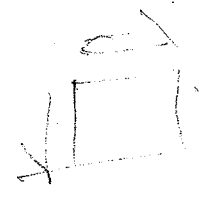
$$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2} \right) + \left(\frac{\sigma_x - \sigma_y}{2} \right) \cos 2\theta + T \sin \theta.$$

$$T_\theta = - \left(\frac{\sigma_x - \sigma_y}{2} \right) \sin 2\theta + 2T \cos 2\theta.$$

Complex stresses

$$\sigma_n = \sigma_x \sin^2 \theta + \sigma_y \cos^2 \theta + 2\tau \sin \theta \cos \theta$$

$$\tau_\theta = \left(\frac{\sigma_x - \sigma_y}{2} \right) \sin 2\theta + \tau \cos 2\theta$$



$$\sigma_x = \sqrt{\sigma_n^2 + \tau_\theta^2}$$

$$\theta_r = \tan^{-1} \left(\frac{\tau_\theta}{\sigma_n} \right) \text{ wr. to } \sigma_n$$

Prin

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$$

$$\tan(180 - 2\theta) = \frac{2\tau}{\sigma_x - \sigma_y} \quad \theta = 90 - \frac{1}{2}$$

$$\theta = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau \cos 2\theta$$

$$-\tau \frac{\cos \theta}{\sin \theta} = \frac{(\sigma_x - \sigma_y)}{2}$$

$\sigma_x > \sigma_y$ $\theta = 90 - 45$

$\sigma_x = \sigma_y$ $\theta = 45$

$\sigma_x < \sigma_y$ $\theta = 245$

$$+\tau \tan = \frac{\sigma_x - \sigma_y}{2}$$

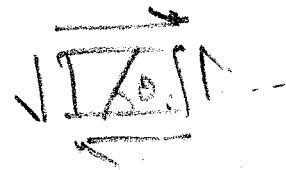
$$\tan 2\theta = \frac{2\tau}{\sigma_x - \sigma_y}$$



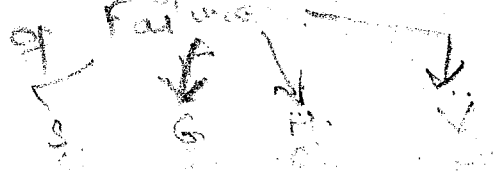
$$\tan(180 - 2\theta) = \frac{2\tau}{\sigma_x - \sigma_y}$$

$$\tau_{max} = \frac{\sigma_x - \sigma_y}{2} \quad \theta = 45^\circ$$

$$\theta = \tan 2\theta = \frac{2\tau}{\sigma_x - \sigma_y}$$



Theories of Failure



$$\sigma_1 < \sigma_y$$

Green

$$\epsilon_1 < \epsilon_y$$

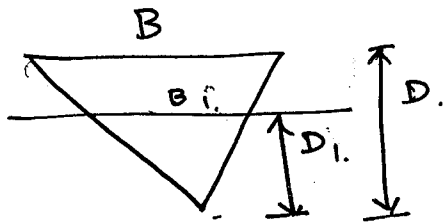
$$U < U_y$$

$$\sqrt{\frac{\sigma_1^2 + \sigma_2^2}{2} + \tau^2} < \sigma_y$$

08/1/2016

STEEL

1.)



$$\frac{B}{D} = \frac{B_1}{D_1}$$

$$B_1 = \frac{B D_1}{D}$$

$$z_p = ?$$

$$A_1 = A_2$$

$$\frac{(B+B_1) \times (D-D_1)}{2} = \frac{1}{2} \times B_1 \times D_1$$

$$\left(B + \frac{B D_1}{D}\right) \times (D-D_1) = \frac{B D_1}{D} \times D_1$$

$$\frac{B}{D} (D+D_1) \times (D-D_1) = \frac{B}{D} \times D_1^2$$

$$D^2 - D_1^2 = D_1^2$$

$$2D_1^2 = D^2$$

$$D_1 = \frac{D}{\sqrt{2}}$$

$$z_p = A_1 y_1 + A_2 y_2$$

$$= \frac{(B+B_1) \times (D-D_1)}{2} \times \dots$$

$$A_1 = \frac{(B + \frac{B D_1}{D}) \times (\frac{B + B D_1}{D}) \times (\frac{D - D_1}{2})}{(B + 2B)}$$

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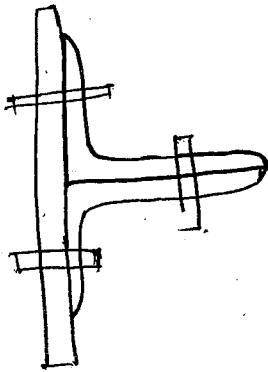
LUG ANGLES.

- * In order to increase the efficiency of outstanding leg in single angle. and to decrease the ~~strengths~~ a length of the end connection. sometimes a short length angle at the ends are connected to the gusset and the outstanding leg of the main angle directly.
- * By using lug angle there will be saving in gusset plate but additional plates and angle member are required. Hence nowadays it is not preferred.
- * As per IS 800 2007. specification of lug angle are
 - a) minimum of two bolt (or) equivalent weld should be used for. attached lug angle to the gusset, if the main member is an angle.
 - (b) The whole area of member shall be taken as effective area rather than net section.
 - (c) The load on the lug angle with gusset plate shall be 20% excess of load acting on outstanding leg of the main angle
 - (d) The connection lug angle ~~and~~ with main angle shall be based on 40% excess of load acting on outstanding leg of main angle.
 - (e) In case of channel as main tension member. \rightarrow Lug angle - Gusset = 10% excess of load acting on outstanding leg of channel section.

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(f) Lug angle - Main channel. } = 20% of load of
 member } outstanding leg of
 main channel.

1. Design the lug angle and its connection for a. angle $125 \times 75 \times 10$ carrying a. $P_u = 300 \text{ kN}$. Use $20 \text{ mm } \phi$ bolt.



Step 1:

$$P = 300 \text{ kN}$$

$$A_g = (125 + 75 - 10) \times 10 = 1900 \text{ mm}^2$$

$$A_1 = (125 - 10) \times 10$$

$$A_2 = (75 - 5) \times 10$$

$$A_1 = 1200 \text{ mm}^2$$

$$A_2 = 700 \text{ mm}^2$$

$$P_1 = \frac{P}{A_g} \times A_1$$

$$= \frac{300 \times 1200}{1900}$$

$$P_1 = 189.47 \text{ kN}$$

$$P_2 = 300 - 189.47$$

$$P_2 = 110.53 \text{ kN}$$

Step 2: ~~Find~~ Find the area of lug angle.

$$1.2P = f \times A$$

$$1.2 \times 110.53 = \frac{250}{1.1} \times A$$

$$A = 583.5 \text{ mm}^2$$

Assume.

ISA 75 x 75 x 8 mm.

Step 3: Design of lug angle - angle section.

$$n = \frac{1.4 \times 110.53}{45.26}$$

$$= 3.418$$

$$n \approx 4 \text{ bolt.}$$

Step 4: Design of lug angle - gusset plate

$$n = \frac{1.2 \times 110.53}{45.26}$$

$$= 2.93$$

$$n \approx 3 \text{ bolt.}$$

Step 5: Design of connected leg.

$$n = \frac{189.47}{45.26}$$

$$= 4.19$$

$$n = 5 \text{ bolt.}$$

2.

An angle section $150 \times 75 \times 10 \text{ mm}$ is connected to gusset plate with 5 bolt of $20 \text{ mm } \phi$ bolt take $g = 100 \text{ mm}$ from the corner. Determine ultimate Load.

Solution:

Yielding criteria.

$$P_{at} = \frac{f_y}{1.1} \times A_g.$$

$$= \frac{250}{1.1} \times (150 + 75 - 10) \times 10.$$

$$P_{at} = 488.64 \text{ kN}$$

Rupture

criteria.

$$P_{at} = \phi \frac{0.9 f_u}{1.25} A_{nc} + \beta \left(\frac{f_y}{1.1} \right) \times A_g.$$

$$= \frac{0.9 \times 410}{1.25} \times \left(150 - \frac{22}{5} \right) \times 10.$$

$$P_{at} = 317.86 \text{ kN} + 1.11 \left(\frac{250}{1.1} \right) \times (75 - 5) \times 10$$

Block Shear.

$$\beta = 1.4 - 0.076 \left(\frac{w}{t} \right) \left(\frac{f_y}{f_u} \right) \left(\frac{b_s}{L} \right).$$

$$= 1.4 - 0.076 \left(\frac{75}{10} \right) \left(\frac{250}{410} \right) \left(\frac{165}{4 \times 50} \right).$$

$$\beta = 1.11.$$

$$P_{at} = 538.096 \text{ kN}$$

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$$T_{db} = \left. \begin{aligned} & \frac{A_{vg}}{\sqrt{3}} \frac{f_y}{\gamma_{m0}} + A_{tn} \frac{0.9f_u}{\gamma_{m1}} \\ & \left(\frac{A_{vn}}{\sqrt{3}} \cdot \frac{0.9f_u}{\gamma_{m1}} \right) + A_{tg} \frac{f_y}{\gamma_{m0}} \end{aligned} \right\} \text{least.}$$

$$A_{vg} = L_{vg} \times t = [30 + 4(50)] \times 10$$

$$A_{vg} = 2300 \text{ mm}^2$$

$$A_{vn} = [230 - 4 \cdot 5(22)] \times t$$

$$A_{vn} = 1310$$

$$A_{tg} = L_{tg} \times t = 50 \times 10 = 500 \text{ mm}^2$$

$$A_{tn} = (50 - \frac{22}{2}) \times 10$$

$$A_{tn} = 390 \text{ mm}^2$$

$$A_{tn} = 390 \text{ mm}^2$$

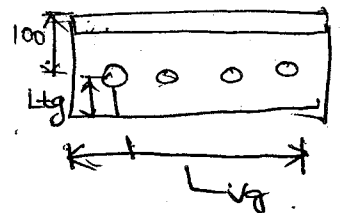
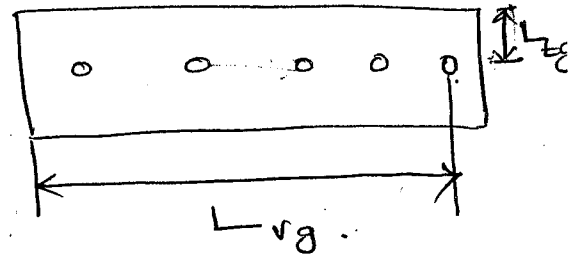
$$T_{db} = \left(\frac{2300}{\sqrt{3}} \times \frac{250}{1.1} \right) + \left(\frac{390}{1.25} \times 0.9 \times 410 \right)$$

$$= 564.52 \text{ kN} = 416.92 \text{ kN}$$

$$T_{db} = \left(\frac{1310 \times 0.9 \times 410}{\sqrt{3} \times 1.25} \right) + \left(\frac{500}{1.1} \times \frac{250}{1.1} \right)$$

$$T_{db} = 336.9 \text{ kN}$$

$$\text{Ultimate Load} = 336 \text{ kN}$$



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8/12/2015

STEEL

TENSION MEMBER.

- * For preliminary design of tension member IS 800 recommends following formula for design shearing strength of net section.

$$T_{dn} = \alpha \frac{A_n f_u}{\gamma_m}$$

where

$\alpha = 0.6$ for one or 2 bolt.

$\alpha = 0.7$ for 3 bolt

$\alpha = 0.8$ for 4 and more no. of bolt

$\alpha = 0.8$ for weld length.

- * Maximum Slenderness Ratio: (stiffness requirement)

→ A tension member in which reversal of direct stress due to other than wind or seismic load. (due to live load and dead load)

$$\lambda \nless 180$$

→ Members subjected to reversal of stress due to wind load or seismic load $\lambda \nless 350$.

→ For any other than except Pre stress (pre-tension) $\lambda \nless 400$.

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1. A flat ties 150ISF 16 is carrying load subjected to reversal of stress other than wind load. Determine limiting lengths of flange.

Solution:

$$\frac{l}{r_{min}} \leq 180$$

$$r_{min} = \sqrt{\frac{I}{A}}$$

$$= \sqrt{\frac{(b^3/12)}{b \times t}}$$

$$= \sqrt{\frac{t^2}{12}}$$

$$r_{min} = \sqrt{\frac{16^2}{12}}$$

$$\frac{l}{\sqrt{\frac{16^2}{12}}} \leq 180$$

$$l \leq 180 \times \frac{16}{\sqrt{12}}$$

$$l \leq 831.38$$

$$\boxed{l = 831 \text{ mm}}$$

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Tension splice:

* It is joint for tension member.

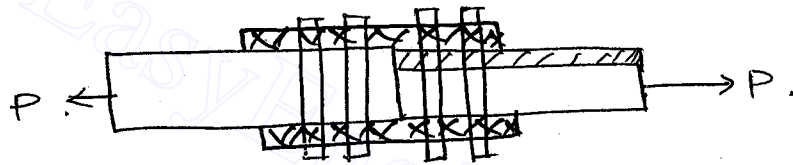
* ~~It~~ Tension splice is provided.

when length of the member required is lesser than the available length.

from Indian rolling mill, or factory.
(or).

* when two lengths of a tension member has different thickness are to be ~~filled~~ connected with fillet plate.

* Tension splices are provided on the both sides of a member jointed in the form of cover plates.



* The strength of splice, plate, and bolt/weld connecting them should have strength atleast equal to design load.

* The design shear capacity of bolt carrying shear through packing plate in excess of 6mm shall be decreased by a factor.

$$\beta_{PKG} = 1.0 - [0.0125t_{PKG}]$$

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1. 2 plates 8mm and 16mm are to be jointed by a double cover butt joint with a packing plate and cover plate of $t = 8\text{mm}$. what is the factor. effect of packing on design shear strength of bolt. The factor is.

$$\beta_{\text{pkg}} = 1 - 0.0125 (8)$$

$$\beta_{\text{pkg}} = 0.9$$

Inference: The design shear strength is reduced by 10%.

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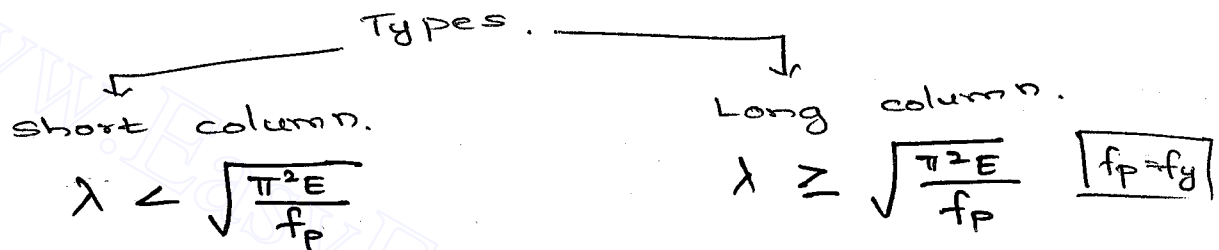
COMPRESSION MEMBERS.

* A compression member is a structural member which is subjected to two equal and opposite compressive forces applied at its ends.

* Example:

Top chords of trusses, Bracing members, boom in a crane, compression flanges of built-up beams, and rolled beams.

* Types.



* short columns subjected to axial compression fails by yielding or crushing.

* Very long column fails by elastic buckling by Euler's Load.

* Intermediate column generally fails by inelastic buckling.

Design compressive strength of member (P_d)

* slenderness ratio and yield stress are the factors affecting ultimate strength of axially loaded column.

$$P_d = A_e f_{cd}$$

A_e - effective cross section of column.

f_{cd} - design stress in compression.

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* IS 800 propose 4 multiple curves for column, namely a, b, c, d. based on Perry Robertson approach.

* code recommends

$$f_{cd} = \frac{f_y / \gamma_{m0}}{\phi + (\phi^2 - \lambda^2)^{0.5}}$$

7.1.2.1

$$f_{cd} = \chi (f_y / \gamma_{m0}) \leq \frac{f_y}{\gamma_{m0}}$$

$$\chi = \frac{1}{\phi + (\phi^2 - \lambda^2)^{0.5}}$$

where

$$\phi = 0.5 [1 + \alpha (\lambda - 0.2) + \lambda^2]$$

$$\lambda = \sqrt{\frac{f_y}{f_{cc}}}$$

$$f_{cc} = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}$$

α - imperfection factor

χ - stress reduction factor for residual stresses.

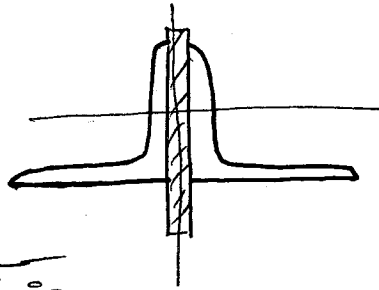
Table - 7 of IS: 800:

Buckling class.	a	b	c	d.
α .	0.21	0.34	0.49	0.76.

f_{cc} - Euler buckling stress.

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1. A principle strut of a roof truss composed of 2 equal angle ISA 75x75x6 or are connected back to back, on either side of 10mm thick gusset plate the c/s area of each angle is 866 mm² M. I $I_{zz} = I_{yy} = 457000 \text{ mm}^4$. the distance of centroid $C_{zz} = C_{yy} = 20.6 \text{ mm}$. what is the r_{\min} .



$$r_{\min} = \sqrt{\frac{I_{\min}}{A}}$$

$$= \sqrt{\frac{914 \times 10^3}{866 \times 2}}$$

$$r_{\min} = 22.99 \text{ mm.}$$

$$I_{z-z} = 2 [I_{zz} + A y^2]$$

$$= 2 [457000]$$

$$I_{z-z} = 914 \times 10^3 \text{ mm}^4.$$

$$I_y = 2 [I_{yy} + A (20.6 + \frac{10}{2})^2]$$

$$= 2 [457000 + 866 (25.6)^2]$$

$$I_y = 958.33 \times 10^3 \text{ mm}^4$$

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A tubular column section $D_o = \sqrt{3}D$, $D_i = D$. The column is effectively held in position and unrestrained against rotation. $\lambda_{eff} = 200$ what is $\frac{l}{d}$ ratio.

Solution:

$$r_{min} = \sqrt{\frac{I}{A}}$$

$$= \sqrt{\frac{\frac{\pi}{64} ((\sqrt{3}d)^4 - d^4)}{\frac{\pi}{4} ((\sqrt{3}d)^2 - d^2)}}$$

$$a^4 - b^4$$

$$(a^2)^2 - (b^2)^2$$

$$(a^2 + b^2)(a^2 - b^2)$$

$$= \sqrt{\frac{\frac{1}{16} ((\sqrt{3}d)^2 - d^2) ((\sqrt{3}d)^2 + d^2)}{(\sqrt{3}d)^2 - d^2}}$$

$$= \sqrt{\frac{1}{16} (3d^2 + d^2)}$$

$$= \sqrt{\frac{4d^2}{16}}$$

$$= \sqrt{\frac{d^2}{4}}$$

$$r_{min} = \frac{d}{2}$$

$$\lambda_{eff} = \frac{l}{r_{min}}$$

$$200 = \frac{l}{(d/2)}$$

$$\frac{l}{d} = \left(\frac{200}{2}\right)$$

$$\boxed{\frac{l}{d} = 100}$$

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2. Determine the design axial load on the column ISMB ³⁵⁰/₆₀₀ height = 3m. both ends are pinned. $f_y = 250 \text{ N/mm}^2$. $f_u = 410 \text{ N/mm}^2$. $E = 2 \times 10^5 \text{ N/mm}^2$.

Properties of ISMB 350 are:

$$A_f = 6670 \text{ mm}^2, \quad t_f = 14.2 \text{ mm}$$

$$t_w = 8.1 \text{ mm}$$

$$b_f = 140 \text{ mm}$$

$$r_{zz} = 143$$

$$r_{yy} = 28.4 \text{ mm}$$

Solution:

Buckling about z-z axis is a.

Buckling about y-y axis is b.

$$P_d = A_e \times f_{cd}$$

$$f_{cd} = \gamma (f_y / \gamma_{m0})$$

$$\chi = \frac{1}{\phi + (\phi^2 - \lambda^2)^{0.5}}$$

$$\phi = 0.5 [1 + \alpha (\lambda - 0.2) + \lambda^2] \quad \lambda = \sqrt{\frac{f_y}{f_{cc}}}$$

$$f_{cc} = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 \times 2 \times 10^5}{\left(\frac{1 \times 3000}{28.4}\right)^2}$$

$$f_{cc} = 176.9 \text{ N/mm}^2$$

$$\lambda = \sqrt{\frac{250}{176.9}} = 1.19$$

$$\lambda = 1.19$$

$$\phi = 0.5 [1 + 0.34 (1.19 - 0.2) + 1.19^2]$$

$$\phi = 1.37$$

$$\gamma = \frac{1}{1.37 + (1.37^2 - 1.19^2)^{0.5}}$$

$$\gamma = 0.48$$

$$f_{cd} = 0.48 \times \frac{250}{1.1}$$

$$f_{cd} = 109.09 \text{ N/mm}^2$$

$$P = A_e \times f_{cd}$$
$$= 6670 \times 109.09$$

$$P = 727.63 \text{ kN}$$

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* f_{cd} shall be assumed as following.

for angle strut, $f_{cd} = 90 \text{ MPa}$.

for rolled steel
beam section, $f_{cd} = 135 \text{ MPa}$.

column with heavy factored
load, $f_{cd} = 200 \text{ MPa}$.

* Maximum slenderness ratio for
compression member:

→ A member carrying D.L & L.L $\nless 180$.

→ Member subjected to W.L $\nless 250$
or S.L

→ Compression flange of a
beam restrained against
torsional buckling. $\nless 800$.

3.) A strut of roof truss $60 \times 60 \times 6$ are
connected to 10 mm gusset plate. Is.
subjected compressive load resulting from
wind load c/s Area of each angle.

$$684 \text{ mm}^2 \quad I_{zz} = I_{yy} = 226000 \text{ mm}^4$$

$$I_{uu} = 36000 \text{ mm}^4 \quad I_{vv} = 91000 \text{ mm}^4$$

Determine the maximum length of a
strut of a truss.

$$\frac{l}{r_{\min}} \leq 250$$

$$\frac{l}{11.53} \leq 250$$

$$l \leq 250 \times 11.53$$

$$l \leq 2883.58$$

$$l = 2880 \text{ mm}$$

$$r_{\min} = \sqrt{\frac{I_{\min}}{A}}$$

$$= \sqrt{\frac{91000}{684}}$$

$$= 11.53$$

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8/12/2015

STEEL

COLUMN.

BUILT UP COLUMNS:

* It is used when rolled steel sections do not provide required sectional area (or) large radius of gyration of column is required to different direction.

* whenever two members are used as column, then they must be connected by lacings or batten systems.

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Lacing and Built-up section:

The different components of built up section are placed in such a way that the built-up section has same radius of gyration about both the axis.

Different component of built up section they are laced up or connected together so that they act as a single column.

Lacing is preferred in eccentric load and battens are preferred in concentric load.

LACING SPECIFICATIONS:

* Flat bars, angle, channel and tubular section are used for lacing.

* It should be continued upto the top of the column.

* It should be shadow of each other.

* It should not be varied about the lengths.

* Tie plates should be provided at the ends and where lacing systems are interrupted.

Lacing (Design specification):

* Effective slenderness ratio can be increased by 5% to account for shear deformation due to unbalanced horizontal forces.

* Angle of inclination $40^\circ - 70^\circ$.

* $\lambda_{eff} > 145$.

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$$* t_{\min} = \frac{l}{40} \quad - \text{single lacing.}$$

$$t_{\min} = \frac{l}{60} \quad - \text{double lacing.}$$

* Effective length:

$$l_e = l \quad - \text{single lacing.}$$

$$l_e = 0.7l \quad - \text{Double lacing}$$

$$l_e = 0.7l \quad - \text{welded lacing.}$$

* Spacing:

$$\frac{S_{\max}}{r_{\min}} \neq 0.7 \times \lambda_{\text{column.}}$$

$$\left(\frac{S_{\max}}{r_{\min}} \right) \neq 50.$$

* Minimum width:

$$b_{\min} = 3 \times d$$

* Load on lacing:

The lacing should be designed to resist transverse shear of 2.5% of column load.

$$V (\text{transverse shear load}) = 2.5\% \text{ of design axial load.}$$

* Lacing should be designed to resist in addition shear due to bending if the column carries bending.

* For single lacing the force in the

$$\text{lacing bar } F = \frac{V}{N \sin \theta}.$$

$$\text{Double lacing } F = \frac{V}{2N \sin \theta} \quad (\text{If } N=2)$$

$$\text{Double lacing, } F = \frac{V}{4 \sin \theta}$$

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1. A built column made up of ISMC 450. Channels placed back to back. carries a factored load of 2250 kN single. Lacing with $\theta = 45^\circ$. with longitudinal axis. what is design load transverse shear on lacing.

$$V = \frac{2.5}{100} \times 2250$$

$$\boxed{V = 56.25 \text{ kN.}}$$

2. A built column consists of ISMC 300 placed back to back @ a spacing = 200 mm and carries an working axial load of 1500 kN. the Double lacing provided with angle 45° with longitudinal axis. what is the design axial load of lacing.

$$\text{Ultimate load} = 1500 \times 1.5$$

$$\text{ultimate load} = 2250 \text{ kN.}$$

$$V = 2.5\% \times 2250$$

$$= \frac{2.5}{100} \times 2250$$

$$\boxed{V = 56.25 \text{ kN}}$$

$$F = \frac{V}{4 \sin \theta}$$

$$= \frac{56.25 \times 10^3}{4 \sin 45}$$

$$\boxed{F = 19.89 \text{ kN.}}$$

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19/12/2015

STEEL

BATTEN

* Minimum 3 bays there should be 3 bays of battening. (i.e) the no. of battens should be such that the member should be divided not less than 3 parts longitudinally.

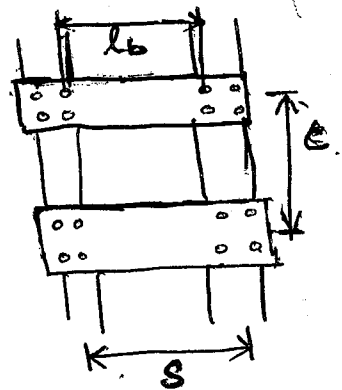
* Flat plates are used by as battens.

* λ_{eff} should be increased by 10%.

* Spacing

$$\frac{C}{r_{min}} \geq 50.$$

$$\frac{C}{r_{min}} \geq 0.7 \times \lambda_{column}.$$



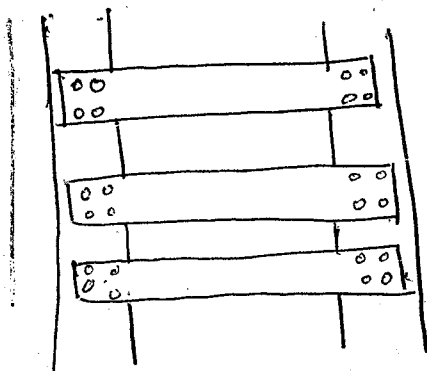
* s is transverse distance between centroid of the bolt group.

* Battens will be designed to carry B.M and S.F arising from a transverse shear, 2.5% of Design ^{axial} load where.

$$V_1 = \frac{VC}{Ns}$$

$$M = \frac{VC}{2N}$$

N = No. of parallel plates of battens.



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* thickness :

$$t > \frac{l_b}{50}$$

* Effective depth (d)

$d > a$ for end batter.

$d > 2b$ for any batter.

$d > \frac{3a}{4}$ for intermediate batter.

a - distance b/w. ~~Centres~~ ^{centre to centre} of flange ~~of~~ the column section.

b - width of the flange of column section.

6. 2 channels IS 44 350 are placed. are placed back to back total length = 6m one end of the restrained against rotation. only. but the other end is restrained against rotation and translation. Determine spacing of channel, slenderness ratio if batt ten system is provided, safe compressive load as per WSM and LSM

Pro $A = 4949 \text{ mm}^2$ $I_{xx} = 9312.6 \times 10^4 \text{ mm}^4$

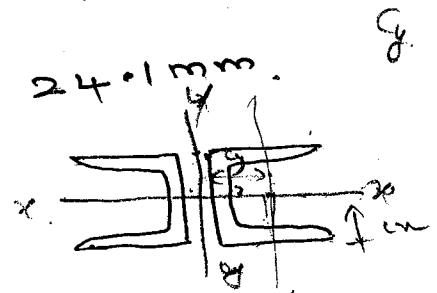
$I_{yy} = 394.6 \times 10^4 \text{ mm}^4$ $C_y = 24.1 \text{ mm}$

Solution:

$$r_y \geq r_x$$

$$\sqrt{\frac{I_{yy}}{A}} \geq \sqrt{\frac{I_{xx}}{A}}$$

$$I_y \geq I_x$$



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$$I_{yy} + Ah^2 \geq I_{xx}$$

$$(39.4 \cdot 6 \times 10^4 + 4947 \left[\frac{24.1}{2} \right]^2) \geq 19312.6 \times 10^4$$

$$S = 220 \text{ mm}$$

$$\lambda = \frac{kL}{r_{\min}} = \frac{1.2 \times 6000}{\sqrt{\frac{I_{\min}}{A}}}$$

Guided roller $k=1.2$

$$= \frac{1.2 \times 6000}{\sqrt{\frac{2(9312.6 \times 10^4)}{2 \times 4947}}}$$

$$\lambda = 52.48$$

$$\lambda_{\text{batten}} = 1.1 \times \lambda$$

$$= 52.48 \times 1.1$$

$$\lambda = 57.7 \text{ mm}$$

L.S.M.

$$P_a = A_e \cdot f_{cd}$$

$$171.42$$

$$= 2 \times 4947 \times 171.42$$

$$1696$$

$$P_a = 1.628 \times 10^6 \text{ N}$$

W.S.M

$$P_c = \sigma_{ac} \times A_{\text{total}}$$

$$\sigma_{ac} = \frac{0.6 f_{cc} f_y}{(f_{cc}^n + f_y^n)^{1/n}}$$

$$n = 1.4$$

Constant

$$f_{cc} = \frac{\pi^2 E}{\lambda^2} = \frac{\pi^2 \times 2 \times 10^5}{(57.7)^2} = 592.89 \text{ N/mm}^2$$

$$\sigma_{ac} = \frac{592.89 \times 250 \times 0.6}{((592.89)^{1.4} + (250)^{1.4})^{1/1.4}} = 207.45 \text{ N/mm}^2$$

$$\sigma_{ac} = 124.47 \text{ N/mm}^2$$

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1. Design lacing and batten for axial load of 1000kN. longitudinal. ~~to~~ Load
 A. Transverse load, Moments
 no. of batten plate. 4, $S = 400\text{mm}$

Solution:

Design of lacing:

(i) Longitudinal load = 2.5% load on column

$$= \frac{2.5}{100} \times 1000.$$

$$= 25\text{ kN}.$$

(ii) Transverse load.

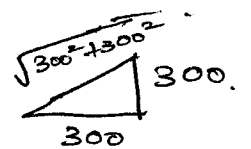
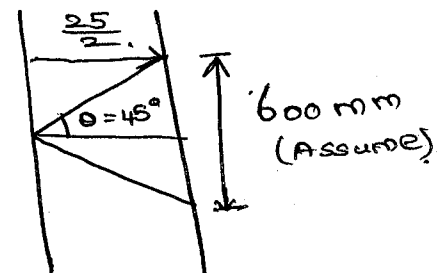
Assume $\theta = 45^\circ$.



$$\sin \theta = \frac{(25/2)}{F_{\text{transverse}}}$$

$$F_{\text{transverse}} = \frac{12.5}{\sin 45^\circ}$$

$$F_{\text{transverse}} = 17.68\text{ kN}$$



(iii) width:

$$b = 3 \times d \text{ of bolt.}$$

$$= 3 \times 20.$$

$$b = 60\text{ mm}$$

(iv) thickness:

$$t = \frac{l}{40} l_{\text{eff}}$$

$$= \frac{l}{40} \times 424 = 26.$$

Assume $l_{\text{eff}} = \sqrt{300^2 + 300^2}$
 $l_{\text{eff}} = 424 = 26\text{ mm}$

$$t = 10.61\text{ mm}$$

$$t = 12\text{ mm}$$

(v)

$$I_{xx} = \frac{bt^3}{12}$$

$$= \frac{60 \times 12^3}{12}$$

$$I_{xx} = 8640 \text{ mm}^4$$

$$I_{yy} = \frac{b^3t}{12}$$

$$= \frac{12^3 \times 60}{12}$$

$$I_{yy} = 216 \times 10^3 \text{ mm}^4$$

$$r_{\min} = \sqrt{\frac{8640}{60 \times 12}}$$

$$r_{\min} = 3.464$$

$$\lambda_{\text{lacing}} \leq 145$$

$$\frac{l_{\text{eff}}}{r_{\min}} = \left(\frac{4240.26}{3.464} \right)$$

$$= 1220.48 \leq 145 \text{ mm}$$

Hence safe.

Batten:

10/12/2015

STEEL

COLUMN BASES

* The main function of base plate is to spread the column load ^{over a} sufficiently wide area and keep the footing from over stressed.

* Types of column bases:

→ slab base

→ Gusseted base.

* Slab base:

For purely axially load, a plane square steel plate or a slab base attached to the column is adequate.

* Gusseted Base

when there is a large moment in relation to the vertically applied a gusseted base may be required.

* The ~~minimum~~ minimum thickness (t_s) of rectangular slab bases under axial compression shall be for I, H, channel

Box,

$$\underline{\text{LSM.}} \quad t_s = \sqrt{\frac{2.5W(a^2 - 0.3b^2)}{f_y / \gamma_{mo}}}$$

$> t_f$
(thickness of column flange)

$$\underline{\text{WSM.}} \quad t_s = \sqrt{\frac{3W(a^2 - 0.3b^2)}{0.75 f_y}}$$

$> t_f$

(As per IS 800:2008 Annexure G)

Column Base:

Main Functions:

- To spread the column load over a wide area.
- To avoid footing from over stressed.

*

Types → Slab base.
→ Gusseted Base.

*

slab base
suitable.

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$$w = \frac{\text{load in column}}{\text{size of base plate}}$$

w - pressure on underside of the base plate.

a, b - larger and smaller projection of the slab base beyond the column.

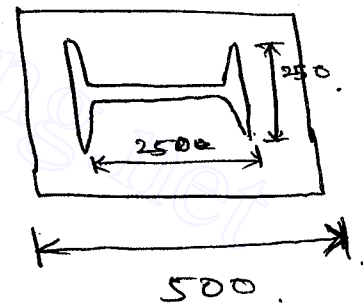
1. A slab base of size 500×500 mm is to be provided below a column section IS HB 250 $D_f = 97$ mm. $b_f = 250$ mm. Supports a load of 2000 kN $f_y = 250$. $f_u = 410$ $\gamma_{m0} = 1.1$ $\gamma_{m1} = 1.25$. Find the t_s

Solution:

$$w = \frac{2000 \times 10^3}{500 \times 500} = 8 \text{ N/mm}^2.$$

$$a = \frac{500 - 250}{2} = 125 \text{ mm}$$

$$b = \frac{500 - 250}{2} = 125 \text{ mm}$$



$$t_s = \sqrt{\frac{2.5 \times 8}{(250/1.1)} \times (125^2 - 0.3(125)^2)}$$

$$t_s = 31.7 \text{ mm} > 9.7 \text{ mm}$$

Provide 35 mm. thick base plate.

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Gusseted Base:

* The design bending stress @ critical section.

$$M_d = \frac{1.2 f_y \times Z}{\gamma_{m0}}$$

* Thickness of gusseted base.

$$t = C \sqrt{\frac{2.75 w}{f_y}}$$

C - cantilever projection.

COLUMN SPLICES

* The design load on the splice section is dependent on end condition of column to column connect

→ If ends are machine ended } = 50% of the column load will be transferred to lower portion of column & 50% will be carried by splice plate.

→ If ends are hand cut. } = 100% of the column is shared by splice plate.
(or) not machine ended.
(no complete bearing).

* If the moment is applied on the column then it is converted into load that load is totally resisted by one splice plate only (i.e) Load on one splice

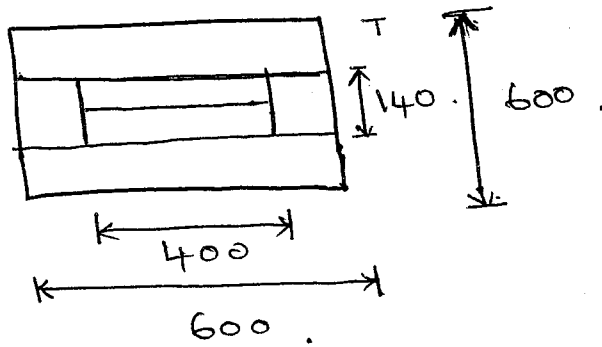
$$P_1 = \frac{P_{\text{column}}}{4} + \frac{M}{D} \quad (\text{Machine ended})$$

$$P_2 = \frac{P_{\text{column}}}{2} + \frac{M}{D} \quad (\text{Not machine ended})$$

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* splice plate is designed as compression member.

1. what is the "C" value for the gusseted base plate shown in the figure.



Cantilever projection $C = \frac{600 - 140}{2}$
 $= 230 \text{ mm}$

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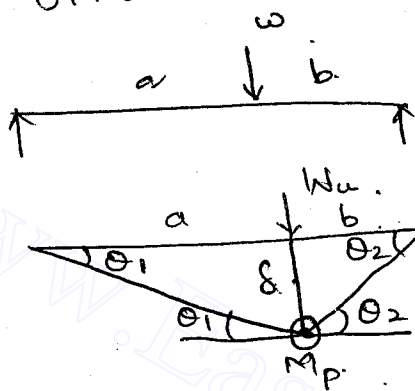
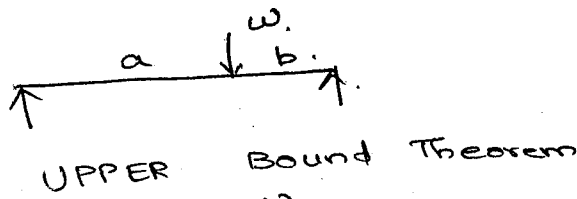
24/12/2015

STEEL

PLASTIC MOMENT CAPACITY

PLASTIC MOMENT THEORY.

1. Find out the collapse load and M_p for. S.S beam subjected to eccentric load.



$$\delta = a\theta_1 = b\theta_2$$

$$\theta_1 = \frac{b\theta_2}{a}$$

$$EWD = IES.$$

$$W_u \times \delta = M_p (\theta_1 + \theta_2)$$

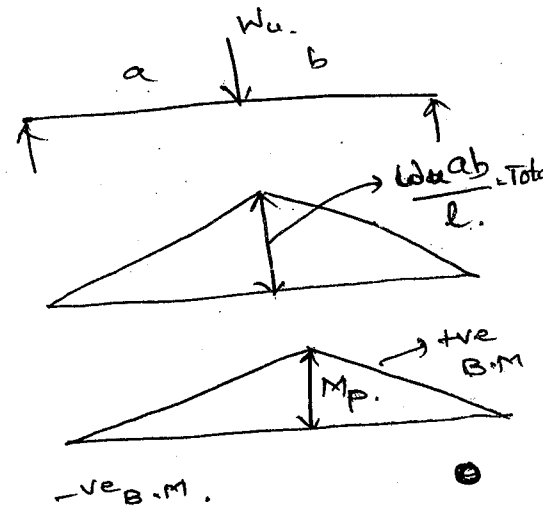
$$W_u \times a\theta_1 = M_p \left(\theta_1 + \frac{\theta_1 a}{b} \right)$$

$$W_u \times a\theta_1 = M_p \theta_1 \left(\frac{b+a}{b} \right)$$

$$W_u = \frac{M_p l}{ab}$$

$$M_p = \frac{W_u ab}{l}$$

Lower Bound Theorem



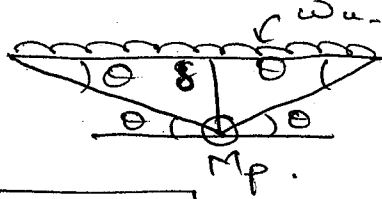
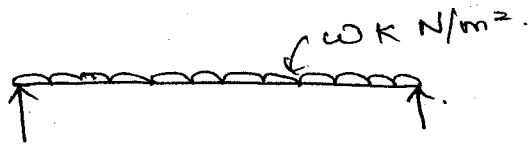
$$\text{Total B.M.} = (\text{+ve B.M.}) + (-\text{ve B.M.})$$

$$\frac{W_u ab}{l} = M_p$$

$$W_u = \frac{M_p l}{ab}$$

2.)

Upper bound Theorem



$$\delta = \frac{\theta l}{2}$$

E.W.D = I.E.S.

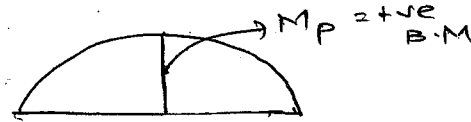
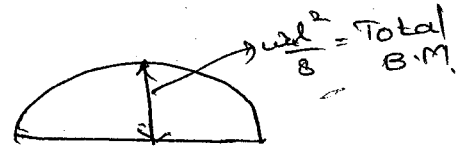
$$w_u \times \frac{\delta}{2} = M_p(\theta + \theta)$$

$$w_u \times l \times \frac{\theta l}{4} = 2M_p \theta$$

$$M_p = \frac{w_u l^2}{8}$$

$$w_u = \frac{8M_p}{l^2}$$

Lower bound Theorem



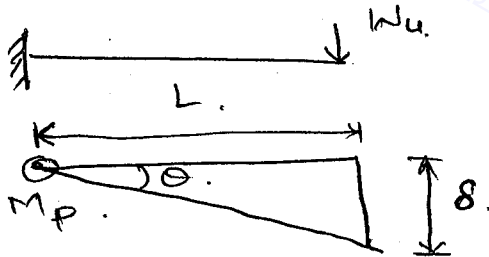
-ve B.M.

Total B.M
= (+ve B.M)
+ (-ve B.M)

$$\frac{w_u l^2}{8} = M_p$$

$$w_u = \frac{8M_p}{l^2}$$

3.)



$$\delta = \theta l$$

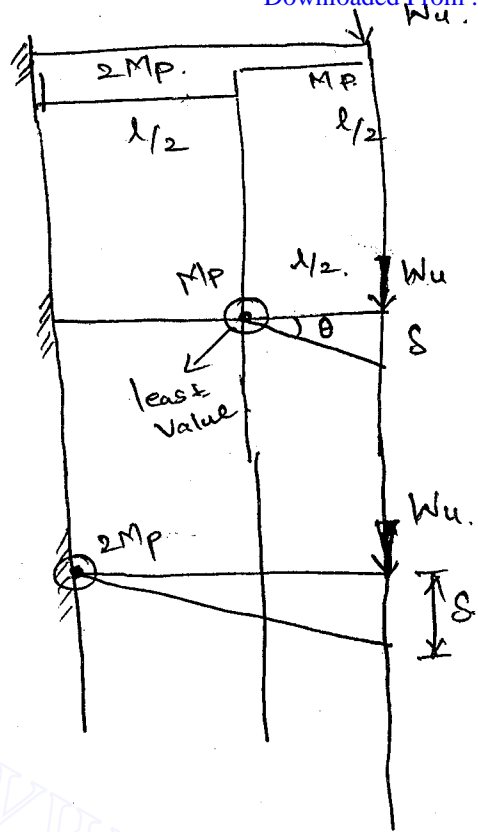
E.W.D = I.E.S.

$$w_u \times \delta = M_p \times \theta$$

$$w_u \times \theta l = M_p \theta$$

$$M_p = w_u l$$

4.)



case ①

$$\delta = \theta l/2$$

$$EWD = IES.$$

$$W_u \times \delta = M_p \times \theta$$

$$W_u \times \frac{\theta l}{2} = M_p \theta$$

$$\boxed{W_u = \frac{2M_p}{l}}$$

case ②

$$\delta = \theta l$$

$$EWD = IES.$$

$$W_u \times \delta = 2M_p \theta$$

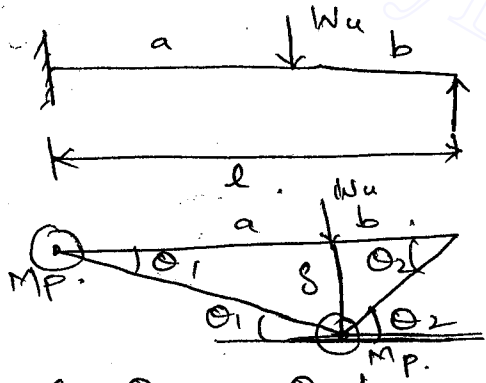
$$W_u \times \theta l = 2M_p \theta$$

$$\boxed{W_u = \frac{2M_p}{l}}$$

Least of case ① and case ②

$$W_u = \frac{2M_p}{l}$$

5.)



$$\delta = \theta_1 a = \theta_2 b$$

$$\boxed{\theta_1 = \frac{b}{a} \theta_2}$$

$$EWD = IES.$$

$$W_u \times \delta = M_p \theta_1 + M_p (\theta_1 + \theta_2)$$

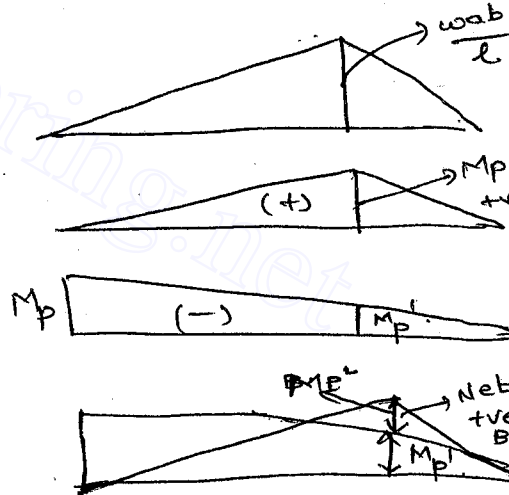
$$W_u \times \theta_2 b = 2M_p \cdot \frac{b}{a} \theta_2 + M_p \theta_2$$

$$W_u \times \frac{\theta_2 b}{2} = M_p \cdot \frac{\theta_2}{2} \left[\frac{2b}{a} + 1 \right]$$

$$= M_p \frac{b + b + a}{b \times a}$$

$$\boxed{W_u = M_p \left(\frac{l+b}{ab} \right)}$$

Lower Bound Theorem



$$\boxed{M_p' = \frac{M_p \times b}{l}}$$

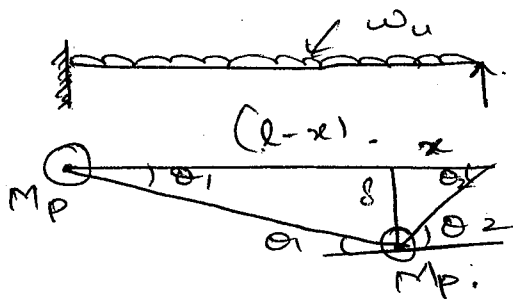
$$\frac{w_{ab} l}{2} = M_p + M_p'$$

$$\frac{w_{ab} l}{2} = M_p \left[1 + \frac{b}{l} \right]$$

$$w_{ab} = M_p \left(\frac{l+b}{l} \right) \times \frac{l}{ab}$$

$$\boxed{w_{ab} = M_p \left(\frac{l+b}{ab} \right)}$$

6.)



$$\delta = \theta_1(l-x) = \theta_2 x.$$

$$\theta_1 = \theta_2 \left(\frac{x}{l-x} \right)$$

8 EWD = IES.

$$w_u \times \frac{\delta}{2} = M_p \theta_1 + M_p(\theta_1 + \theta_2)$$

$$w_u \times \frac{\theta_1(l-x)}{2} = 2M_p \theta_1 + M_p \left(\frac{l-x}{x} \right) \theta_1$$

$$w_u \times \frac{\theta_1(l-x)}{2} = M_p \theta_1 \left[2 + \frac{l-x}{x} \right]$$

$$w_u = \frac{2M_p}{l-x} \left(\frac{2x+l-x}{x} \right)$$

$$w_u = \frac{2M_p(l+x)}{(lx-x^2)}$$

Diff. w.r.t to x and equate to zero

$$\frac{\partial w_u}{\partial x} = 0$$

$$2M_p \frac{d}{dx} \left(\frac{l+x}{lx-x^2} \right) = 0$$

$$\frac{(lx-x^2)(1) - (l+x)(l-2x)}{(lx-x^2)^2} = 0$$

$$lx - x^2 - [l^2 - 2lx + xl - 2x^2] = 0$$

$$lx - x^2 - l^2 + 2lx + 2x^2 = 0$$

$$x^2 + 2lx - l^2 = 0$$

~~$$x(x+2l) = 0$$~~

~~$$x^2 + 2lx - l^2 = 0$$~~

~~$$x(x+l) = l(x+l) = 0$$~~

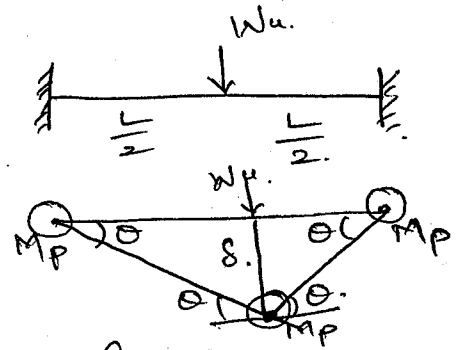
~~$$(x+l)(x-l) = 0$$~~

$$x = 0.414l$$

$$w_u = \frac{2M_p(-l + 0.414l)}{(l \times 0.414l) - (0.414l)^2} = \frac{2M_p \times 1.414l}{0.2426l^2}$$

$$w_u = 11.66 \frac{M_p}{l}$$

7.)



$$\delta = \frac{\theta L}{2}$$

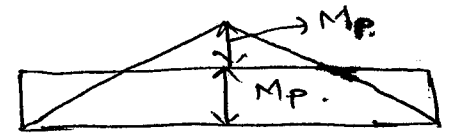
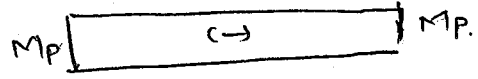
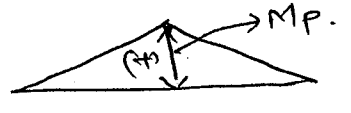
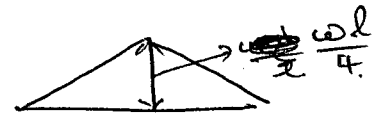
EWD = IES.

$$W_u \times \delta = M_p \theta + M_p (\theta + \theta) + M_p \theta$$

$$W_u \times \frac{\theta L}{2} = 4 M_p \theta$$

$$W_u = \frac{8 M_p}{L}$$

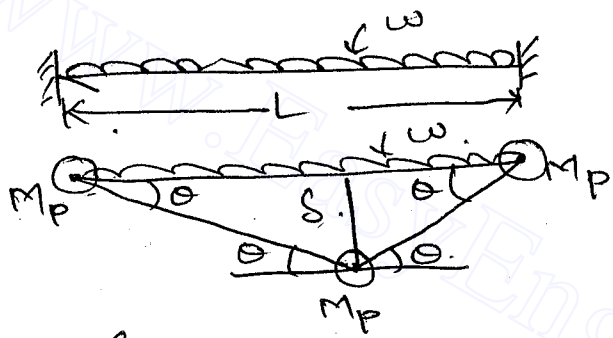
Lower Bound Theorem.



$$\frac{W_u L}{4} = 2 M_p$$

$$W_u = \frac{8 M_p}{L}$$

8.)



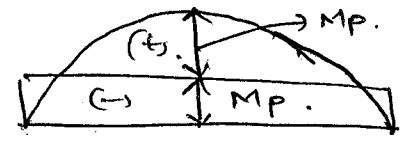
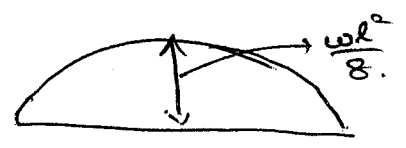
$$\delta = \frac{\theta \times L}{2}$$

EWD = IES.

$$w_u \times L \times \frac{\delta}{2} = M_p \theta + M_p (\theta + \theta) + M_p \theta$$

$$w_u \times \frac{L}{2} \times \frac{\theta L}{2} = 4 M_p \theta$$

$$w_u = \frac{16 M_p}{L^2}$$



$$\frac{w L^2}{8} = 2 M_p$$

$$w_u = \frac{16 M_p}{L^2}$$

KN/m

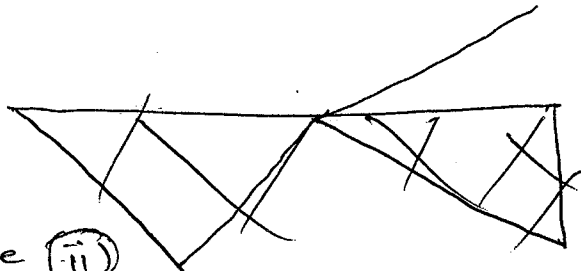
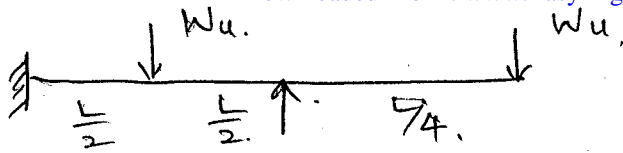
$$W = w_u \times L$$

$$W = \frac{16 M_p}{L}$$

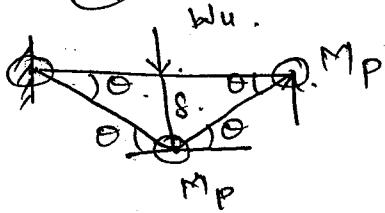
KN

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9.)



case (ii)



$$\delta = \theta \times \frac{L}{2}$$

$$W_u \times \theta \times \frac{L}{2} = 4M_p \theta$$

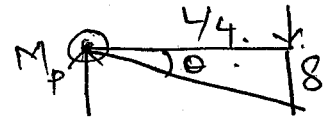
$$W_u = \frac{8M_p}{L}$$

Total Load.

$$W_u = \frac{8M_p}{L} + \frac{4M_p}{L}$$

$$W_u = \frac{12M_p}{L}$$

Case (i).



$$\delta = \theta \frac{L}{4}$$

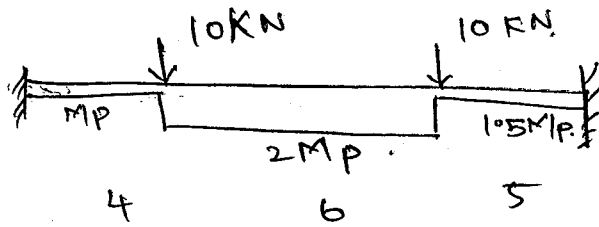
$$W_u \times \delta = M_p \theta$$

$$W_u \times \frac{L}{4} = M_p \theta$$

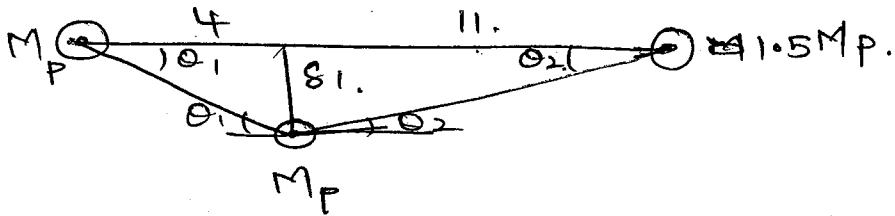
$$W_u = \frac{4M_p}{L}$$

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10.)



Case 1.



$$\delta_1 = 4\theta_1 = 11\theta_2.$$

$$\theta_1 = \frac{11}{4}\theta_2$$

$$\text{EWD} = \text{IES.}$$

$$10 \times \frac{11\theta_2}{4} = M_p\theta_1 + M_p\theta_1 + M_p\theta_2 + 1.5M_p\theta_2.$$

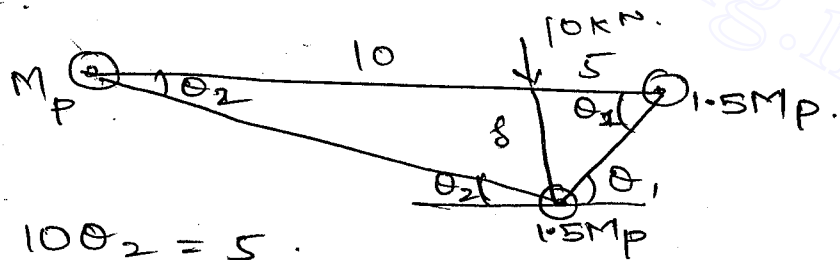
$$\frac{110}{4}\theta_2 = 2M_p\theta_1 + 2.5M_p\theta_2$$

$$\frac{110}{4}\theta_2 = 2 \times \frac{11}{4}\theta_2 M_p + 2.5M_p\theta_2$$

$$\frac{110}{4}\theta_2 = M_p\theta_2 \left[\frac{22}{4} + 2.5 \right]$$

$$M_p = 13.75 \text{ kN}\cdot\text{m}$$

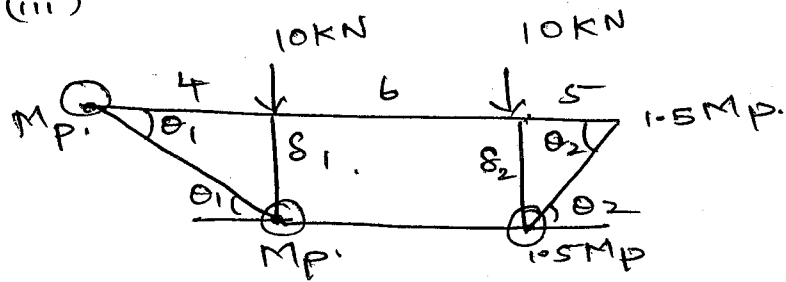
Case 2:



$$\delta = 10\theta_2 = 5.$$

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Case (iii)



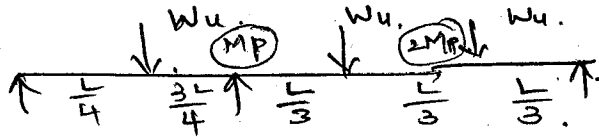
$$\delta_1 = 4\theta_1$$

$$\delta_2 = 5\theta_2$$

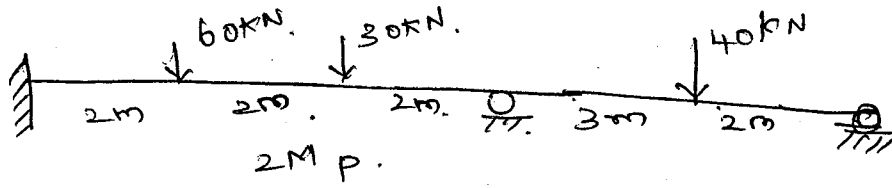
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11.)



12.)

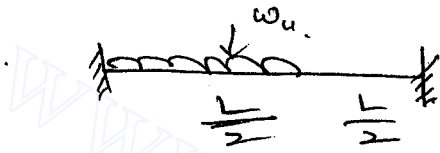


Determine Elastic Section modulus,

Take S.F = 1.2 F.O.S = 1.5, $f_y = 250\text{MPa}$

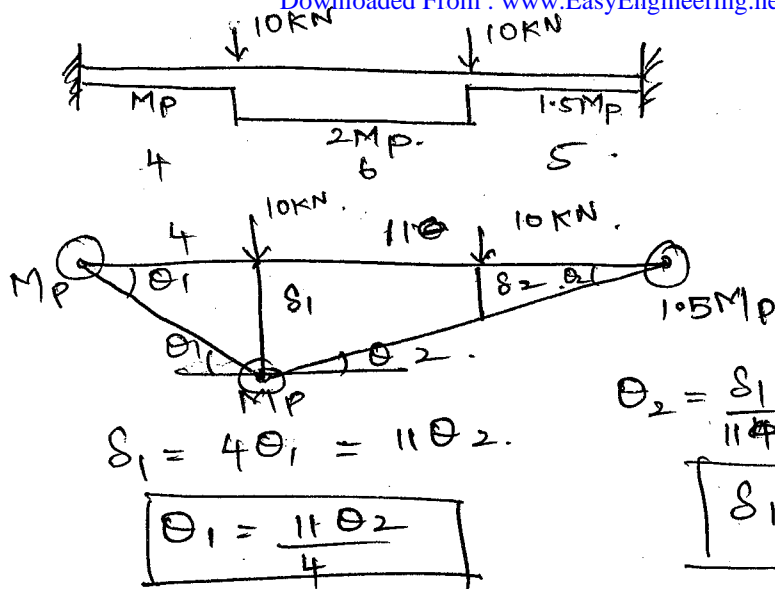
All given loads are working loads.

13.)



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10)

EWD = Σ ES.

$$(W_{u_1} \times \delta_1) + (W_{u_2} \times \delta_2) = 2M_p\theta_1 + 2.5M_p\theta_2 + \cancel{1.5M_p\theta_2}$$

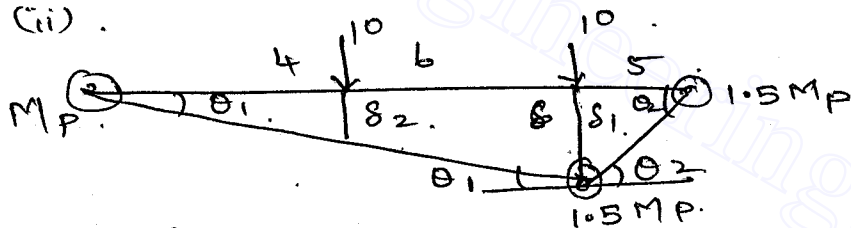
$$\cancel{10 \times 11\theta_2} \frac{10 \times 11\theta_2}{4.5} + 10 \times \frac{5\delta_1}{11} = 2M_p \frac{11\theta_2}{4} + 2.5M_p\theta_2$$

$$110\theta_2 + \frac{50}{4} \frac{11\theta_2}{4.5} = M_p\theta_2 \left[\frac{11}{2} + 2.5 \right]$$

$$122.5\theta_2 = M_p\theta_2 \cdot 8.$$

$$M_p = 15.3125 \text{ kN}\cdot\text{m}$$

case (ii).



$$\delta_1 = 5\theta_2 = 10\theta_1 \quad \delta_2 = \frac{\delta_1}{10} \times 4$$

$$\theta_1 = \frac{\theta_2}{2}$$

$$\delta_2 = \frac{2\delta_1}{5}$$

$$(10 \times \delta_1) + (10 \times \delta_2) = 2.5M_p\theta_1 + 3M_p\theta_2$$

$$(10 \times 5\theta_2) + (10 \times \frac{2}{5} \times 5\theta_2) = 2.5M_p \cdot \frac{\theta_2}{2} + 3M_p\theta_2$$

$$70\theta_2 = 4.25M_p\theta_2$$

$$M_p = 16.47 \text{ kN}\cdot\text{m}$$

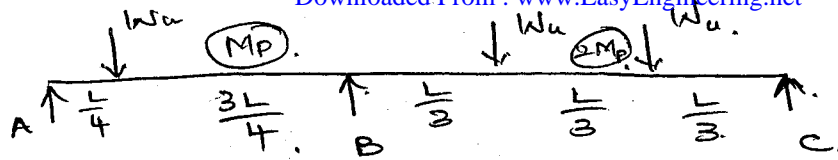
$$M_p = 16.47 \text{ kN}\cdot\text{m}$$

$$2M_p = 32.94 \text{ kN}\cdot\text{m}$$

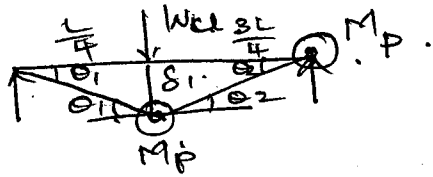
$$1.5M_p = 24.705 \text{ kN}\cdot\text{m}$$

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11)



Beam AB.



$$\delta_1 = \frac{L}{4} \theta_1 = \frac{3L}{4} \theta_2$$

$$\theta_1 = 3\theta_2$$

$$W_u \times \delta_1 = M_p \theta_1 + 2M_p \theta_2$$

$$W_u \times \frac{3L\theta_2}{4} = M_p \cdot 3\theta_2 + 2M_p \theta_2$$

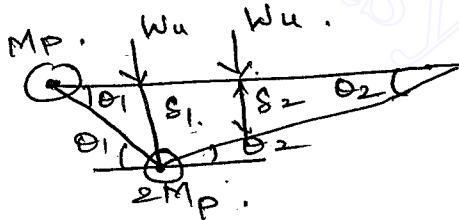
$$\frac{3 \cdot W_u L}{4} \theta_2 = 5M_p \theta_2$$

$$M_p = \frac{3W_u L}{20}$$

$$M_p = \frac{3W_u L}{20}$$

$$2M_p = \frac{3W_u L}{10}$$

Beam BC.



$$\delta_1 = \frac{L}{3} \theta_1 = \frac{2L}{3} \theta_2$$

$$\theta_1 = 2\theta_2$$

$$\delta_2 = \frac{\delta_1 \sqrt{3}}{2\sqrt{3}}$$

$$\delta_2 = \delta_1 / 2$$

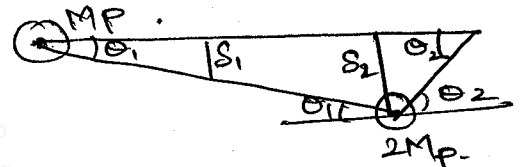
$$(W_u \times \delta_1) + (W_u \times \delta_2) = 3M_p \theta_1 + 2M_p \theta_2$$

$$W_u \frac{2L}{3} \theta_2 + W_u \frac{\delta_1}{2} = 6M_p \theta_2 + 2M_p \theta_2$$

$$\frac{2L}{3} W_u \theta_2 + \frac{2L}{3} \cdot \frac{\theta_2}{2} = 8M_p \theta_2$$

$$\frac{2L}{3} W_u \theta_2 \left[\frac{3}{2} \right] = 8M_p \theta_2$$

$$M_p = \frac{W_u L}{8}$$



$$\delta_2 = \frac{2L}{3} \theta_1 = \frac{L}{3} \theta_2$$

$$2\theta_1 = \theta_2$$

$$\delta_1 = \frac{\delta_2 \sqrt{3}}{2\sqrt{3}} \quad \delta_1 = \frac{\delta_2}{2}$$

$$(W_u \times \delta_1) + (W_u \times \delta_2) = 3M_p \theta_1 + 2M_p \theta_2$$

$$\frac{W_u \cdot 2L \theta_1}{2 \cdot 3} + W_u \cdot \frac{2L \theta_1}{3}$$

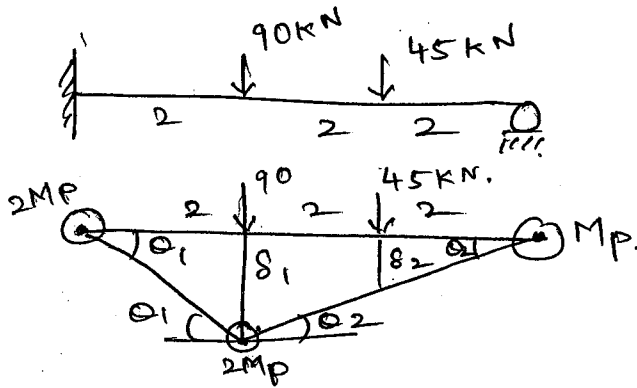
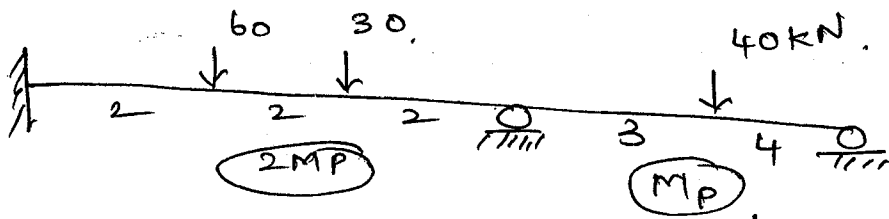
$$= 3M_p \theta_1 + 4M_p \theta_1$$

$$\frac{2L}{3} W_u \theta_1 \left(\frac{3}{2} \right) = 7M_p \theta_1$$

$$M_p = \frac{W_u L}{7}$$

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(2)



$$\delta_1 = 2\theta_1 = 4\theta_2$$

$$\theta_1 = 2\theta_2$$

$$\delta_2 = \frac{2\delta_1}{4}$$

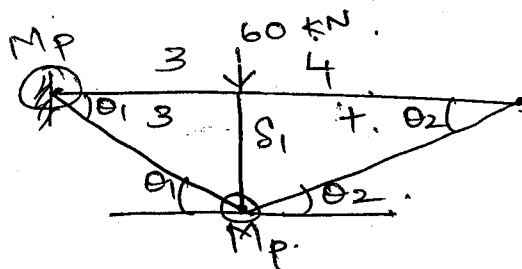
$$\delta_2 = \frac{\delta_1}{2}$$

$$(90 \times \delta_1) + (45 \times \delta_2) = 4M_p\theta_1 + 3M_p\theta_2$$

$$(90 \times 4\theta_2) + \left(\frac{45 \times 4\theta_2}{2}\right) = 11M_p\theta_2$$

$$360\theta_2 + 90\theta_2 = 11M_p\theta_2$$

$$M_p = 40.91 \text{ kN}\cdot\text{m}$$



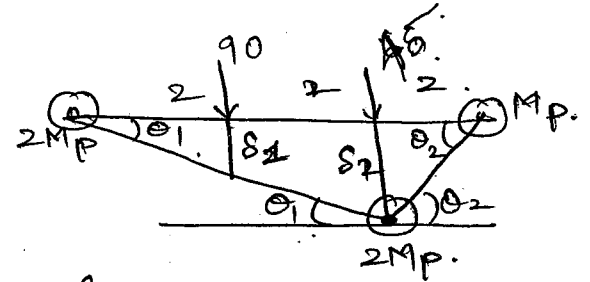
$$\delta_1 = 3\theta_1 = 4\theta_2$$

$$\theta_1 = \frac{4}{3}\theta_2$$

$$60 \times \delta_1 = 2M_p\theta_1 + M_p\theta_2$$

$$60 \times 4\theta_2 = \frac{2 \times 4}{3} M_p\theta_2 + M_p\theta_2$$

$$M_p = 65.45 \text{ kN}\cdot\text{m}$$



$$\delta_2 = 2\theta_2 = 4\theta_1$$

$$\theta_2 = 2\theta_1$$

$$\delta_2 = \delta_1$$

$$(90 \times \delta_1) + (45 \times \delta_2) = 4M_p\theta_1 + 3M_p\theta_2$$

$$(45 \times 4\theta_1) + (45 \times 4\theta_1) = 10M_p\theta_1$$

$$M_p = 36 \text{ kN}\cdot\text{m}$$

$$M_p = 65.45 \text{ kN}\cdot\text{m}$$

$$Z_p = \frac{M}{f_y} = \frac{I}{y}$$

$$\frac{M}{I} = \frac{f}{y} = z$$

$$Z_p = \frac{M_p}{f_y} = \frac{65.45 \times 10^6}{250}$$

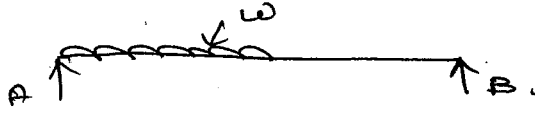
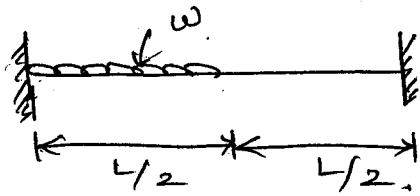
$$Z_p = 261.8 \times 10^3 \text{ mm}^3$$

$$Z_e = \frac{Z_p}{S.F} = \frac{261.8 \times 10^3}{1.2}$$

$$Z_e = 218.17 \times 10^3 \text{ mm}^3$$

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13)



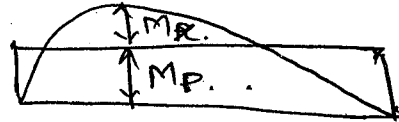
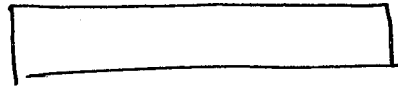
$$\sum M_A = 0$$

$$R_B \times l = w_c \frac{l^2}{8}$$

$$R_B = \frac{w_c l}{8}$$

$$R_A = \frac{w_c l^2}{2} - \frac{w_c l}{8}$$

$$R_A = \frac{3w_c l}{8}$$



$$R_{xx} \cdot V_{xx} = 0$$

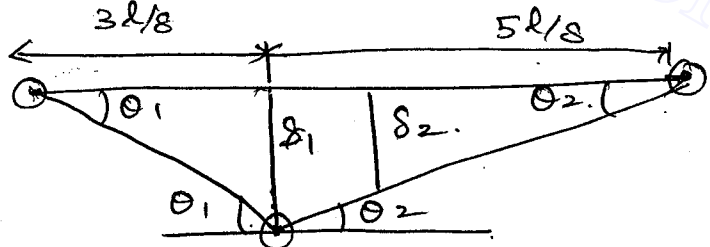
$$\frac{3w_c l}{8} - w_c x = 0$$

$$w_c x = \frac{3w_c l}{8}$$

$$x = \frac{3l}{8}$$

where B.M is max.

$$B.M_{max} = \frac{3w_c l}{8} \times \frac{3l}{8} - w_c \frac{3l}{8} \times \frac{3l}{16}$$



$$\delta_1 = \frac{3l\theta_1}{8} = \frac{5l\theta_2}{8}$$

$$\theta_1 = \frac{5\theta_2}{3}$$

EWD = I.E.S.

$$\left(w_c \times L \times \frac{\delta_1}{2} \right) - \left(w_c \times \frac{L}{2} \times \frac{\delta_2}{2} \right) = 2M_p \theta_1 + 2M_p \theta_2$$

$$\delta_2 = \frac{\delta_1 \times k}{\frac{5k}{8}}$$

$$w_c \times \frac{L}{2} \times \frac{5l\theta_2}{8} - \frac{w_c L}{2} \left(\frac{4\delta_1}{5} \right) = 2M_p \frac{5\theta_2}{3} + 2M_p \theta_2$$

$$\delta_2 = \frac{4\delta_1}{5}$$

$$\frac{5}{16} w_c l^2 \theta_2 - \frac{w_c L}{8} \frac{5l}{8} \theta_2 = \left(\frac{10}{3} + 2 \right) M_p \theta_2$$

$$\left(\frac{5w_c l^2}{16} - \frac{2w_c l^2}{8} \right) \theta_2 = \frac{16}{3} M_p \theta_2$$

$$\frac{3w_c l^2}{16} = \frac{16}{3} M_p$$

$$M_p = \frac{3w_c l^2 \times 3}{16}$$

$$w = \frac{28.4 M_p}{l}$$

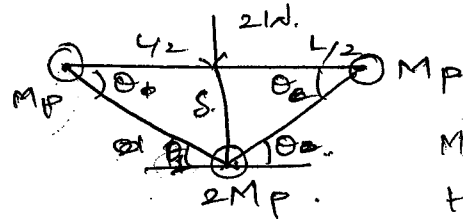
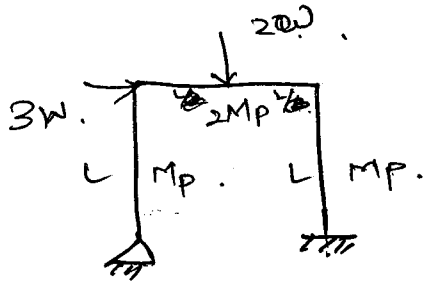
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25/12/2015

STEEL.

PLASTIC MOMENT CAPACITY.

1.)



$$M_p \theta + 2M_p(\theta + \theta) + M_p \theta = 6M_p \theta$$

$$\delta = \frac{L}{3} \theta$$

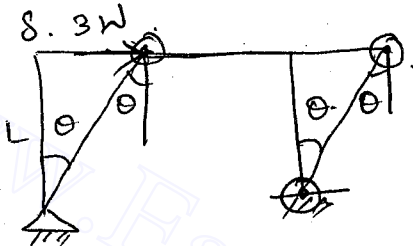
$$2W \times \delta = 6M_p \theta$$

$$2W \times \frac{L}{3} \theta = 6M_p \theta$$

$$M_p = \frac{WL}{3}$$

$$W_u = \frac{3M_p}{L}$$

Sway Mechanism



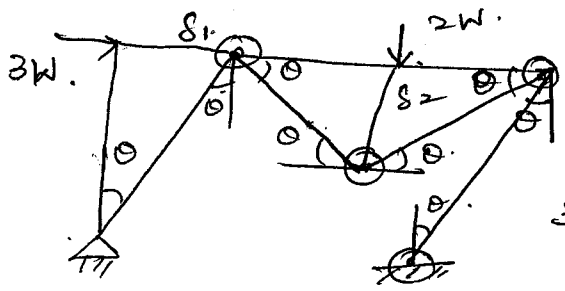
$$\delta = \theta L$$

$$3W \times \delta = 3M_p \theta$$

$$W \theta L = M_p \theta$$

$$\frac{M_p}{L} = W$$

combined Mechanic



$$\delta_1 = 3L\theta$$

$$\delta_2 = \frac{L}{3}\theta$$

$$3W_u \delta_1 + 2W_u \delta_2 = M_p \theta + 2M_p(\theta + \theta) + M_p \theta + M_p \theta$$

$$(3W_u L \theta) + (2W_u \frac{L}{3} \theta) = 7M_p \theta$$

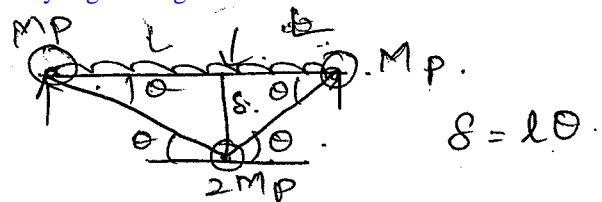
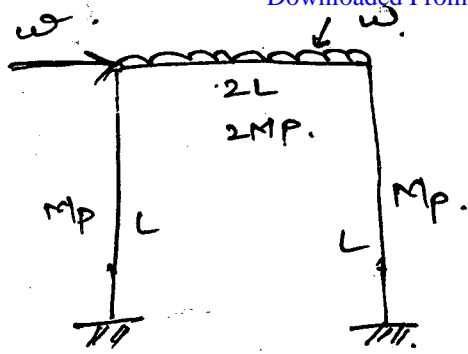
$$5W_u L \theta = 7M_p \theta$$

$$W_u = \frac{7M_p}{5L}$$

$$W_u = \frac{M_p}{L}$$

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2.)

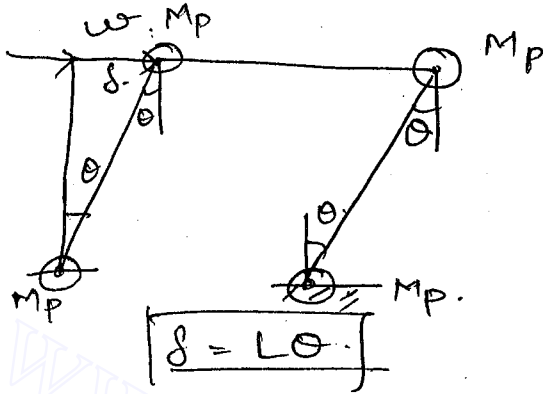


(2)

$$\int w \times l \times \frac{\delta}{2} = 6M_p \theta$$

$$w \times l \times l \times \theta = 6M_p \theta$$

$$w = \frac{6M_p}{l^2}$$

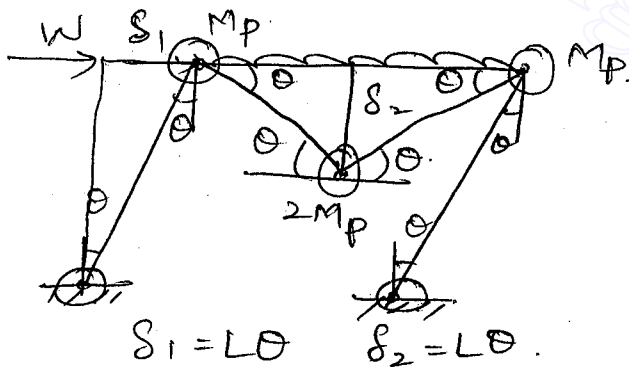


$$\delta = L\theta$$

$$W \times \delta = 4M_p \theta$$

$$W \times L\theta = 4M_p \theta$$

$$W = \frac{4M_p}{L}$$



$$\delta_1 = L\theta \quad \delta_2 = L\theta$$

$$W \times \delta_1 + w \times L \times \frac{\delta_2}{2} = M_p \theta + M_p \theta + 4M_p \theta + M_p \theta + M_p \theta$$

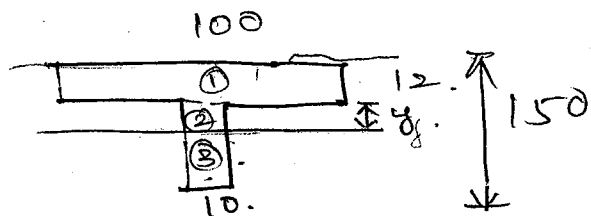
$$(W \times L\theta) + w \times L \times \frac{L\theta}{2} = 8M_p \theta$$

$$\frac{2WL}{2} = 8M_p \theta$$

$$MP = W_u = \frac{8MP}{L}$$

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3)



Equal Area AXIS.

$$A_1 + A_2 = A_3$$

$$(100 \times 12) + (10 \times y) = 10(138 - y)$$

$$1200 + 10y = 1380 - 10y$$

$$20y = 1380 - 1200$$

$$y = 9 \text{ m.}$$

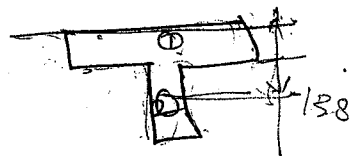
EAA.

$$z_p = A_1 y_1 + A_2 y_2 + A_3 y_3$$

$$= [1200 \times (6 + 9)] + (10 \times 9 \times 4.5)$$

$$+ (10 \times 138 \times 64.5)$$

$$z_p = 101.61 \times 10^3$$

z_e:

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$= \frac{(100 \times 12 \times 6) + (10 \times 138 \times 81)}{(1200 + 1380)}$$

$$\bar{y} = 46.12 \text{ mm. from top.}$$

$$I_{xx} = I_{x1} + A(\bar{y} - y_1)^2 + I_{x2} + A(\bar{y} - y_2)^2$$

$$= \frac{100 \times 12^3}{12} + 1200(46.12 - 6)^2$$

$$+ \frac{10 \times 138^3}{12} + 1380(46.12 - 81)^2$$

$$= 4.858 \times 10^6$$

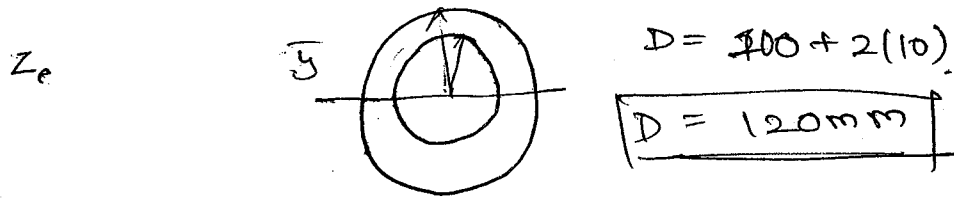
$$= 5.8143 \times 10^6 \text{ mm}^4$$

$$z_e = \frac{5.8143 \times 10^6}{103.88} = 55.97 \times 10^3 \text{ mm}^3$$

$$S.F = \frac{z_p}{z_e} = \frac{101.61 \times 10^3}{55.97 \times 10^3} = 1.82$$

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4) Find s.f. of ring having inner dia. 100mm⁽¹⁴⁾
thickness 10mm.

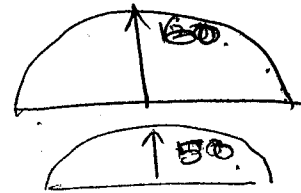


$$Z_e = \frac{I}{y}$$

$$= \frac{\frac{\pi}{64} (120^4 - 100^4)}{120/2}$$

$$Z_e = 87.83 \times 10^3 \text{ mm}^3$$

$$Z_p = 2[A_1 x y_1 - A_2 x y_2]$$



$$= 2 \left[\left[\frac{\pi r_1^2}{4} \times \frac{4r_1}{3\pi} \right] - \left[\frac{\pi r_2^2}{2} \times \frac{4r_2}{3\pi} \right] \right]$$

$$= 2 \left[\left(\frac{\pi \times 60^2}{2} \times \frac{4 \times 60}{3\pi} \right) - \left(\frac{\pi \times 50^2}{2} \times \frac{4 \times 50}{3\pi} \right) \right]$$

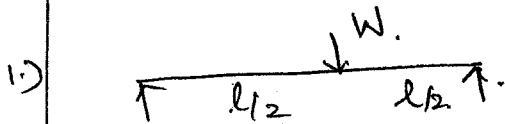
$$= 2 \left[60.67 \times 10^3 \right]$$

$$Z_p = 121.33 \times 10^3 \text{ mm}^3$$

$$S.F = \frac{Z_p}{Z_e} = \frac{121.33}{87.83} = 1.381$$

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STEEL STRUCTURE.



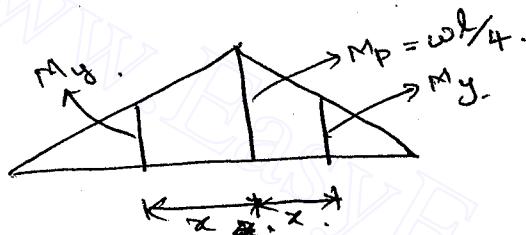
M_p will remain. @ mid span.

$$M_p = f_y Z_p.$$

$$M_p = f_y \frac{bd^2}{4}$$

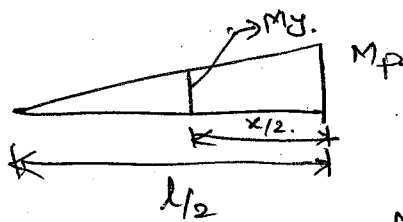
$$M_y = f_y \times z_e.$$

$$M_y = f_y \frac{bd^2}{6}$$



$$M_p > M_y.$$

length of plastic hinge.



$$\frac{M_p}{M_y} = \frac{l/2}{\frac{l}{2} - x}$$

$$\frac{f_y \frac{bd^2}{4}}{f_y \frac{bd^2}{6}} = \frac{l/2}{\frac{l}{2} - x}$$

$$x = \frac{l}{6}$$

$$\begin{aligned} \text{Plastic hinge length} &= 2x \\ &= 2 \times \frac{l}{6} \\ &= \frac{l}{3}. \end{aligned}$$

Plastic hinge

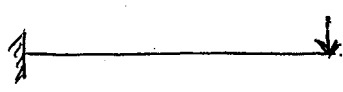
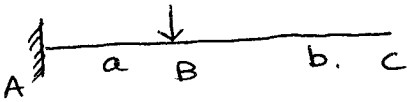
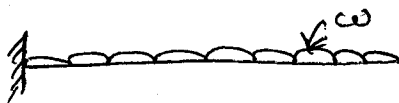

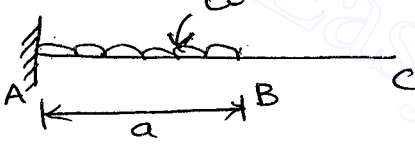
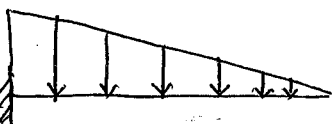
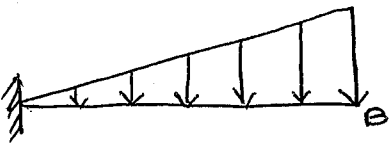
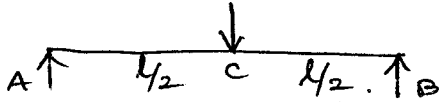
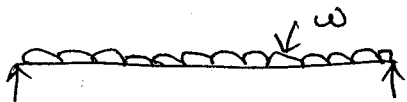
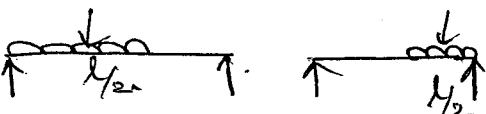
when 3 (or) 4 members meet
@ point plastic hinge is formed
in all the member.

In elastic method of analysis.
Equilibrium eqn is used

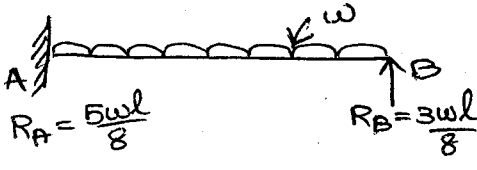
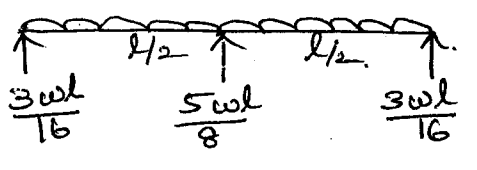
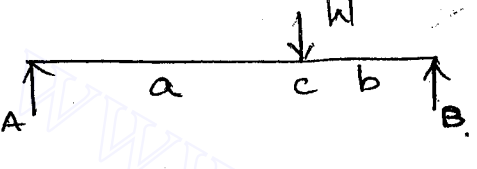
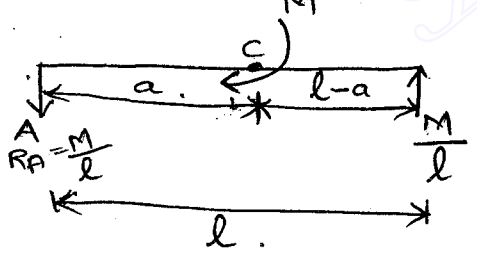
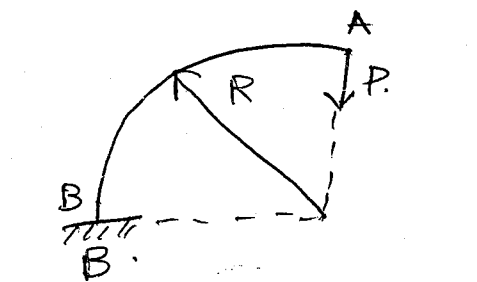
In plastic method of analysis
equilibrium yield and mechanism is
used.

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29/12/2015

Beam	θ	δ
1. 	$\frac{Wl^2}{2EI}$	$\frac{Wl^3}{3EI}$
2.) 	$\theta_B = \theta_C = \frac{Wa^2}{2EI}$	$\delta_B = \frac{Wa^3}{3EI}$ $\delta_C = \frac{Wa^3}{3EI} + \frac{Wa^2}{2EI}(l-a)$
3.) 	$\frac{Wl^3}{6EI}$	$\frac{Wl^4}{8EI}$
4.) 	$\frac{ML}{EI}$	$\frac{ML^2}{2EI}$
5.) 	$\theta_B = \theta_C = \frac{Wa^3}{6EI}$	$\delta_B = \frac{Wa^4}{8EI}$ $\delta_C = \frac{Wa^4}{8EI} + \frac{Wa^3}{6EI}(l-a)$
6.) 	$\theta_B = \frac{wl^3}{24EI}$	$\delta_B = \frac{wl^4}{30EI}$
7.) 	$\theta_B = \frac{wl^3}{8EI}$	$\delta_B = \frac{11}{120} \frac{wl^4}{EI}$
8.) 	$\theta_A = \theta_B = \frac{Wl^2}{16EI}$	$\delta_C = \frac{Wl^3}{48EI}$
9.) 	$\theta_A = \theta_B = \frac{Wl^3}{24EI}$	$\delta_C = \frac{5}{384} \frac{Wl^4}{EI}$
10.) 	-	$\delta_C = \frac{1}{2} \left(\frac{5}{384} \frac{Wl^4}{EI} \right)$

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Beam	θ	δ
 <p> $R_A = \frac{5wl}{8}$ $R_B = \frac{3wl}{8}$ </p>		$\delta_{max} @ 0.42l$ from B $\delta_{max} = 0.005415 \frac{wl^4}{EI}$
 <p> $\frac{3wl}{16}$ $\frac{5wl}{8}$ $\frac{3wl}{16}$ </p>		$\delta_{max} @ 0.21l$ from end support. $\delta_{max} = 3.85 \times 10^{-4} \frac{wl^4}{EI}$
 <p> a c b </p>		$\delta_c = \frac{wa^2b^2}{3EI}$ $\delta_{max} @ x = \sqrt{\frac{l^2 - b^2}{3}}$ from A. $\delta_{max} = \frac{wb(a^2 + 2ab)^{3/2}}{9\sqrt{3}EI}$
 <p> a $l-a$ </p>	$\theta_A = \frac{M}{6EI} (2l^2 - 6al + 3a^2)$ $\theta_B = \frac{M}{6EI} (3a^2 - l^2)$	$\delta_c = \frac{Ma(l-a)(l-2a)}{3EI}$
<p>when</p> $a = b = l/2$	$\theta_B = \theta_A = \frac{Ml}{24EI}$	$\delta_c = 0$
		$\delta_V = \frac{PR^3}{4EI}$ $\delta_H = \frac{PR^3}{2EI}$

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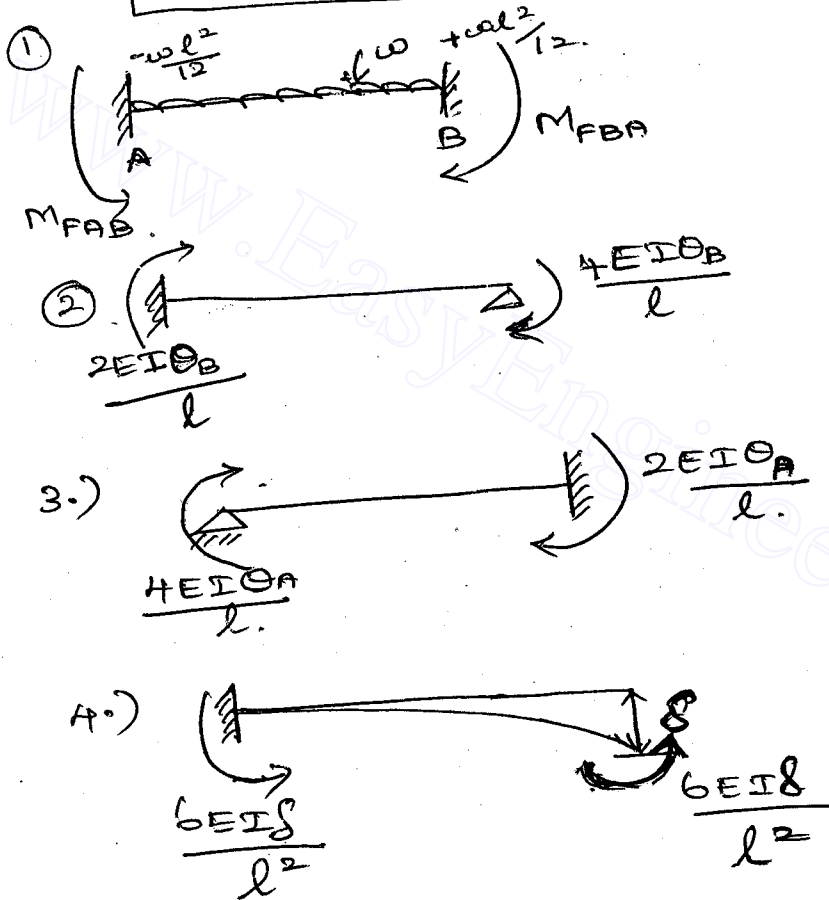
6/1/2016

STRUCTURAL ANALYSIS
SLOPE AND DEFLECTION
METHOD

Generalized equation.

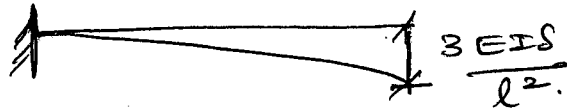
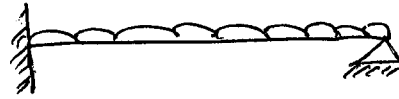
$$M_{AB} = M_{FAB} + \frac{2EI}{l} \left(2\theta_A + \theta_B - \frac{3\delta}{l} \right)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l} \left[\theta_A + 2\theta_B - \frac{3\delta}{l} \right]$$



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one End Fixed and other end hinged. ②



$$M_{AB} = M_{FAB} - \frac{3 E I \delta}{l^2} + \frac{3 E I \theta_A}{l}$$

$$M_{AB} = M_{FAB} + \frac{3 E I}{l} \left[\theta_A - \frac{\delta}{l} \right]$$

$$M_{BA} = 0$$

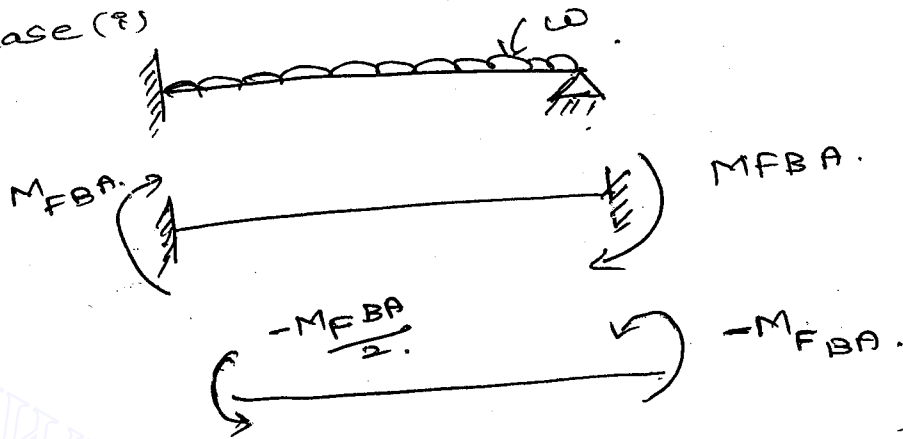
Explanation:

||

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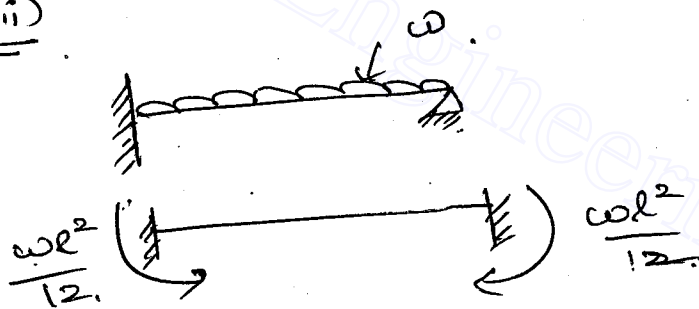
Determine the fix end moment @ A in case of end being hinged from the fixed end moments. corresponding to the case of both end fixed.

Case (i)



$$\text{Fixed End Moment } \left. \begin{array}{l} \text{at A} \\ \text{at B} \end{array} \right\} = M_{FAB} - \frac{M_{FBA}}{2}$$

Case (ii)



~~M_{FAB}~~

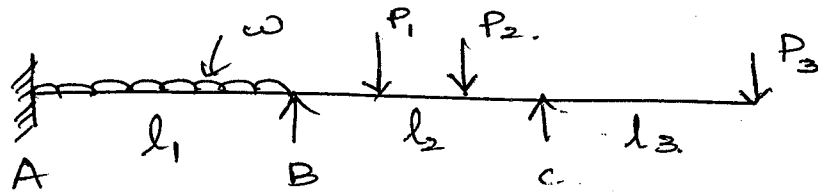
$$= M_{FAB} - \frac{M_{FBA}}{2}$$

$$= -\frac{wL^2}{12} - \frac{wL^2}{24}$$

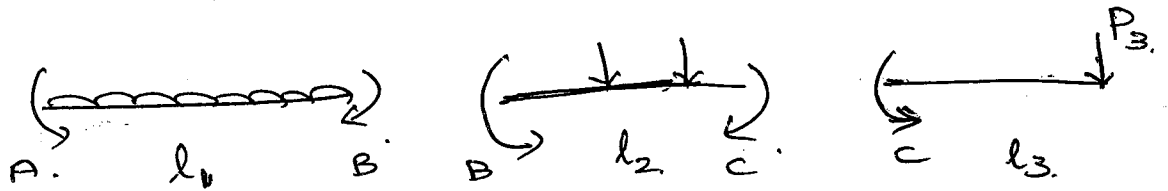
$$\text{Fixed end moment } \left. \begin{array}{l} \text{at A} \\ \text{at B} \end{array} \right\} = \frac{wL^2}{8}$$

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Method to find out. Find End Moment ⁽⁴⁾



Joint B Equilibrium Equation.



$$\begin{aligned} M_{BA} + M_{BC} &= 0 \\ M_{CB} - P_3 l_3 &= 0 \end{aligned}$$

Shear Equation:

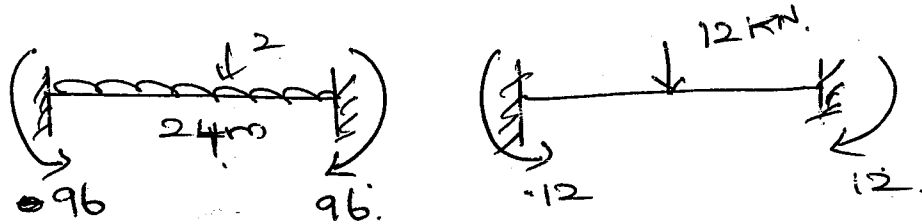
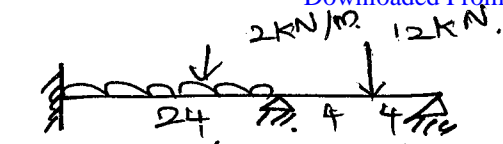
$$\sum V = 0$$

$$R_A + R_B + R_C - [w l_1 + P_1 + P_2 + P_3] = 0$$

$$R_A + R_B + R_C = [w l_1 + P_1 + P_2 + P_3]$$

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1.)



$$M_{FAB} = -96 \text{ kN}\cdot\text{m} \quad M_{FBC} = -12 \text{ kN}\cdot\text{m}$$

$$M_{FBA} = 96 \text{ kN}\cdot\text{m} \quad M_{FCB} = 12 \text{ kN}\cdot\text{m}$$

$$M_{AB} = M_{FAB} + \frac{2EI}{l} \left[2\theta_A + \theta_B - \frac{3\delta}{l} \right]$$

$$M_{AB} = -96 + \frac{2EI}{24} \theta_B$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l} \left[\theta_A + 2\theta_B - \frac{3\delta}{l} \right]$$

$$M_{BA} = 96 + \frac{4EI\theta_B}{l}$$

$$M_{BC} = M_{FCB} + \frac{2EI}{l} \left[2\theta_B + \theta_C - \frac{\delta}{l} \right]$$

$$M_{BC} = -12 + \frac{4EI\theta_B}{l} + \frac{2EI\theta_C}{l}$$

$$\begin{aligned} M_{CB} &= 0 + \frac{2EI}{l} \left[\theta_B + 2\theta_C - \frac{3\delta}{l} \right] \\ &= \frac{2EI\theta_B}{l} + \frac{4EI\theta_C}{l} + 12 \end{aligned}$$

Joint Equilibrium Equation:

$$M_{BA} + M_{BC} = 0$$

$$84 + \frac{4EI\theta_B}{24} + \frac{4EI\theta_B}{8} + \frac{2EI\theta_C}{8} = 0$$

$$84 + \frac{16EI\theta_B + 6EI\theta_C}{24} = 0$$

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$$16 EI \theta_B + 6 EI \theta_C = -84 \times 24$$

$$8 EI \theta_B + 3 EI \theta_C = \frac{-84 \times 24}{2}$$

$$8 \theta_B + 3 \theta_C = \frac{-1008}{EI} \quad \text{--- (1)}$$

$$M_{CB} = 0$$

$$\frac{2 EI \theta_B}{8} + \frac{4 EI \theta_C}{8} = -12$$

$$2 \theta_B + 4 \theta_C = \frac{-12 \times 8}{EI}$$

$$\theta_B + 2 \theta_C = \frac{-12 \times 8}{2 \times EI}$$

$$\theta_B + 2 \theta_C = \frac{-48}{EI} \quad \text{--- (2)}$$

$$\theta_B = \frac{-144}{EI}$$

$$\theta_C = \frac{48}{EI}$$

$$M_{AB} = -96 + \frac{2EI}{24} \times \frac{-144}{EI}$$

$$= -108 \text{ KN}\cdot\text{m}$$

$$M_{BA} = 96 + \frac{4EI}{24} \times \left(\frac{-144}{EI} \right)$$

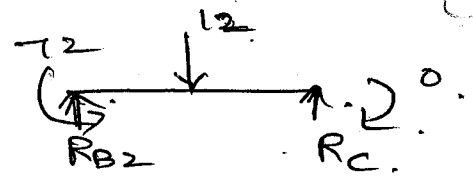
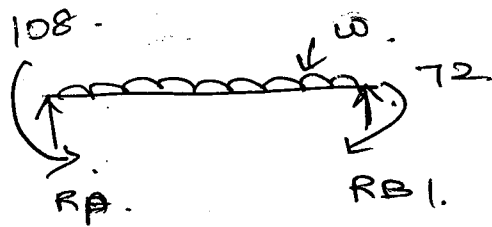
$$= 72 \text{ KN}\cdot\text{m}$$

$$M_{BC} = -12 + \frac{4 \times EI}{8} \left(\frac{-144}{EI} \right) + \frac{2 \times EI}{8} \left(\frac{48}{EI} \right)$$

$$= -72 \text{ KN}\cdot\text{m}$$

$$M_{CB} = 0$$

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$$\sum M_A = 0.$$

$$R_B \times 24 - \left(\frac{2 \times 24^2}{2} \right) - 72 + 108 = 0.$$

$$R_B = 22.5$$

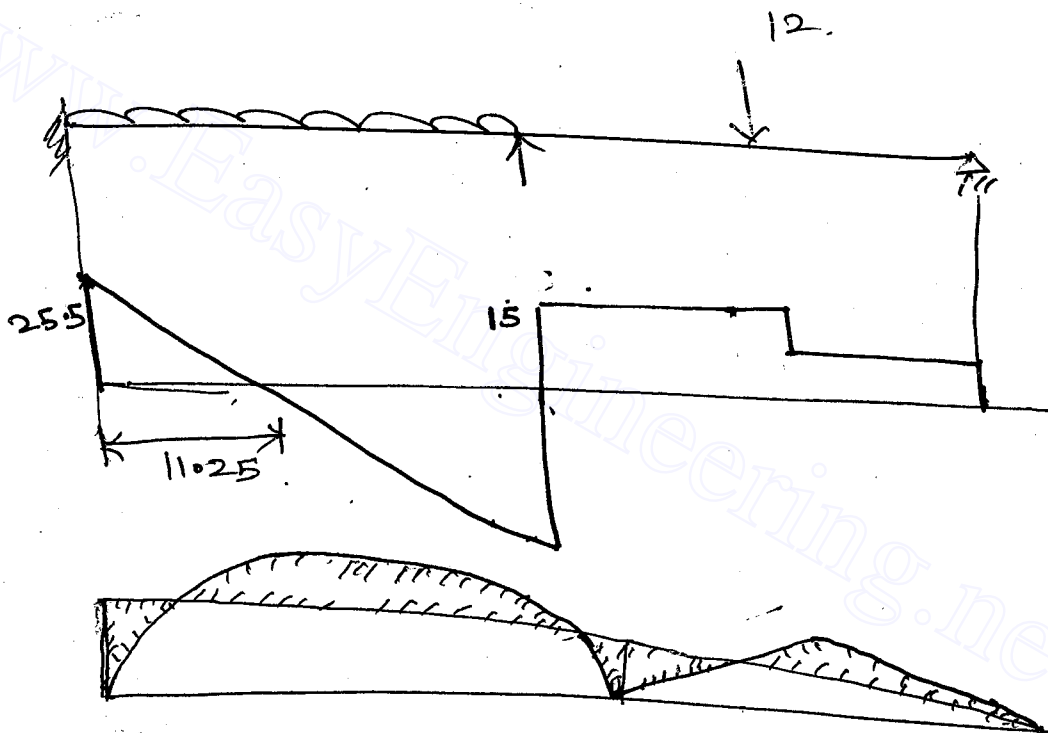
$$R_A = 25.5 \text{ kN.}$$

$$\sum M_C = 0.$$

$$R_{B2} \times 8 - 72 - (12 \times 4) = 0$$

$$R_{B2} = 15 \text{ kN}$$

$$R_C = -3 \text{ kN}$$



$$V.F = 0.$$

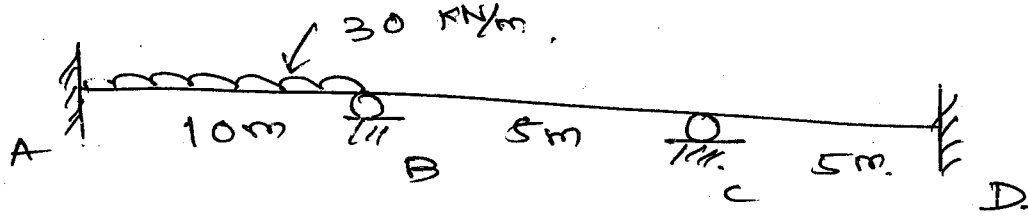
$$R_A = 2 \times x.$$

$$25.5 - 22.5 = x$$

$$x = 11.25 \text{ m.}$$

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2)



$$E = 2 \times 10^5 \text{ N/mm}^2$$

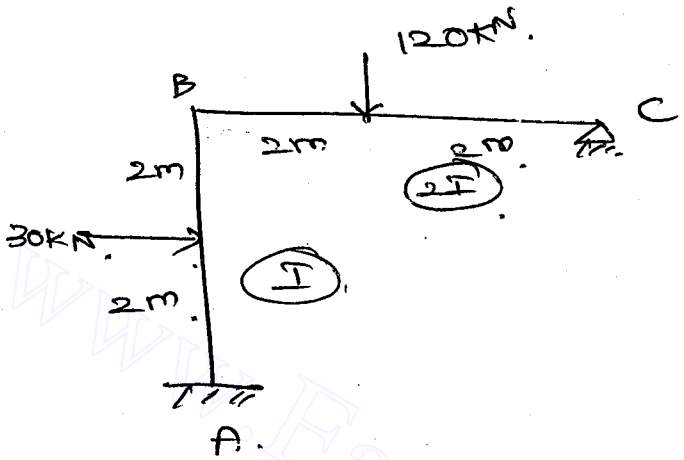
$$I = 60 \times 10^6 \text{ mm}^4$$

$$S_x = 30 \text{ cm}$$

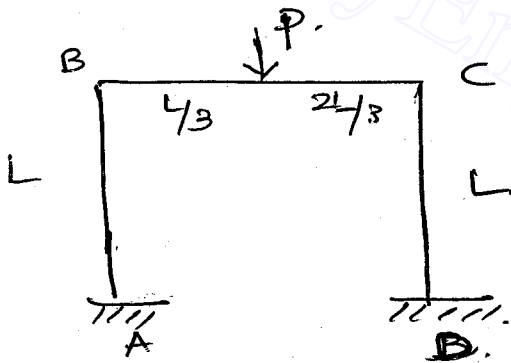
$$\delta l_B = +30$$

$$\delta l_{CD} = -30$$

3)



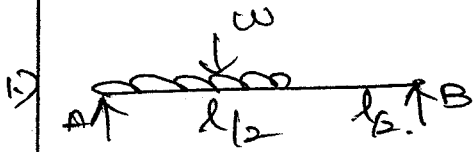
4)



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9/1/2016

Structural Analysis



$$\theta_A = \theta_B = \frac{\omega l^3}{128 EI}$$

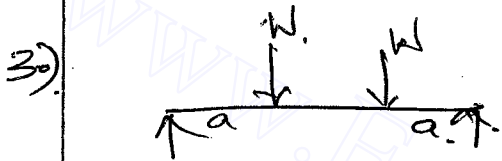
$$\theta_A = \frac{3}{128} \frac{\omega l^3}{EI}$$

$$\theta_B = \frac{1}{384} \frac{\omega l^3}{EI}$$



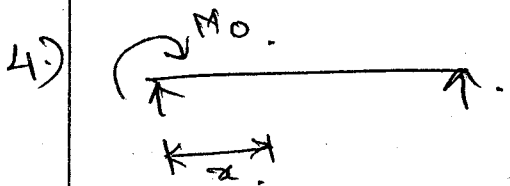
$$\theta_A = \frac{\omega a^2}{24 EI} (2l - a)^2$$

$$\theta_B = \frac{\omega a^2}{24 EI} (2l^2 - a^2)$$



$$\theta_A = \theta_B = \frac{\omega a (l - a)}{2 EI}$$

$$\delta_C = \delta_{\max} = \frac{\omega a}{24 EI} (3l^2 - 4a^2)$$

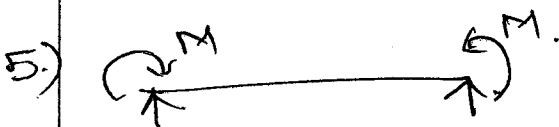


$$\theta_A = \frac{ML}{3EI}$$

$$\theta_B = \frac{ML}{6EI}$$

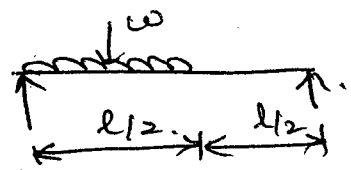
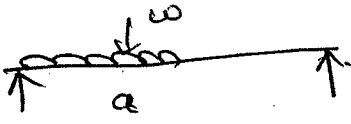
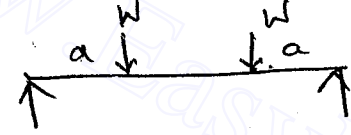
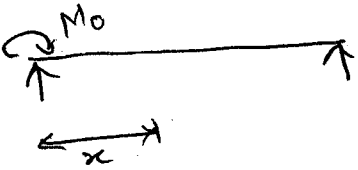

$$@ x = 0.42$$

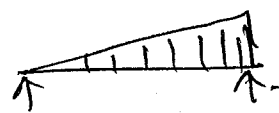

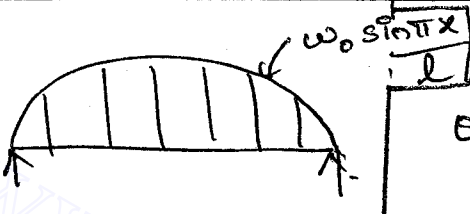
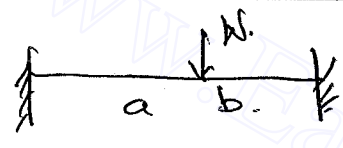
$$\delta_{\max} = \frac{ML^2}{9\sqrt{3}EI}$$



$$\theta_A = \theta_B = \frac{ML}{2EI}$$

$$\delta_C = \frac{ML^2}{8EI}$$

S.NO	Beam	θ	δ
1.		$\theta_A = \frac{3}{128} \cdot \frac{wl^3}{EI}$ $\theta_B = \frac{7}{384} \frac{wl^3}{EI}$	
2.)		$\theta_A = \frac{wa^2}{24EI} (2l-a)^2$ $\theta_B = \frac{wa^2}{24EI} (2l^2-a^2)$	
3.)		$\theta_A = \theta_B = \frac{wa(l-a)}{2EI}$	$\delta_C = \delta_{max} = \frac{wa}{24EI} (3l^2 - 4a^2)$
4.)		$\theta_A = \frac{ML}{3EI}$ $\theta_B = \frac{ML}{6EI}$	$\text{@ } x = 0.42$ $\delta_{max} = \frac{ML^2}{9\sqrt{3}EI}$
5.)		$\theta_A = \theta_B = \frac{ML}{2EI}$	$\delta_C = \frac{ML^2}{8EI}$

6)		$\theta_A = \frac{7}{360} \frac{wl^3}{EI}$	$\delta_c = \frac{5}{768} \frac{wl^4}{EI}$
		$\theta_B = \frac{wl^3}{45EI}$	$\delta_{max} @ 0.519L$ $\delta_{max} = 0.00652 \frac{wl^4}{EI}$
7)		$\theta_A = \theta_B = \frac{5}{192} \frac{wl^3}{EI}$	$\delta_c = \delta_{max} = \frac{wl^4}{120EI}$
8)		$\theta_A = \theta_B = \frac{w_0 l^3}{\pi^3 EI}$	$\delta_c = \delta_{max} = \frac{w_0 l^4}{\pi^4 EI}$
9.			$\delta = \frac{wab^3}{3EIL^3}$

A rectangular beam of section $b \times d$ is simply supported on a span l and carries a concentrated load W at its centre.

Strain energy due to shear $U = \frac{3}{20} \frac{w^2 l}{Gbd}$.

The deflection due to shear @ midspan is given by.

$$U = \frac{3}{20} \left[W \times \delta_c \right]$$

$$\delta_c = \frac{3}{10} \frac{WL}{Gbd}$$

A horizontal beam rests on two supports on the same level and carries a udl w . The supports are symmetrically placed. In order to greatest downward deflection may have a least value, the deflection @ its centre and @ the ends must be equal.

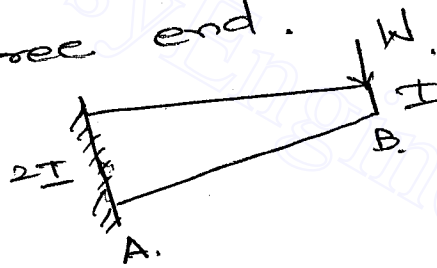
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The position of support, should be such that the distance b/w support = $0.554 \times \text{total beam length}$

In order to have both the supports @ same level @ horizontal position, the slope on each end must be zero.

The distance b/w the supports for above condition must be such that $0.5774 \times \text{total beam length}$

2) A cantilever beam fixed @ A and free @ B. changing moment of Inertia from $I(B)$ to $2I(A)$. carrying a concentrated load W @ free end.



$$\delta_B = \frac{WL^3}{8EI} (\log 2 - 0.5)$$

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26/12/2015

SURVEYING

LATITUDE AND DEPARTURE

1. * In a traverse, latitude and departure of the sides are calculated. It was calculated that $\Sigma L = 1.39$, $\Sigma D = -2.17$, calculate the length of bearing and closing error.

$$e = \sqrt{\Sigma L^2 + \Sigma D^2}$$

Solution:

$$\begin{aligned} \text{Closing error} &= \sqrt{(1.39)^2 + (-2.17)^2} \\ &= 2.57 \text{ m.} \end{aligned}$$

$$\tan \theta = \frac{\Sigma D}{\Sigma L}$$

$$\theta = 57^\circ 21' 29.83''$$

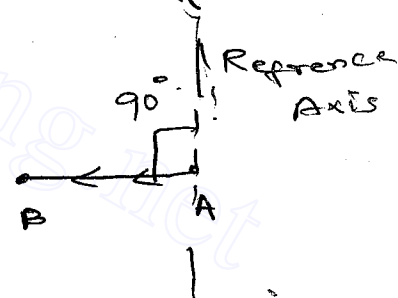
2. A man travels from A due west and reaches a point B. Distance b/w A and B ~~139.6m~~ ^{139.6m} calculated latitude and departure of line AB.

$$\theta = 90^\circ$$

$$\text{Latitude} = l \cos 90^\circ = 0$$

$$\text{Departure} = l \sin 90^\circ = l$$

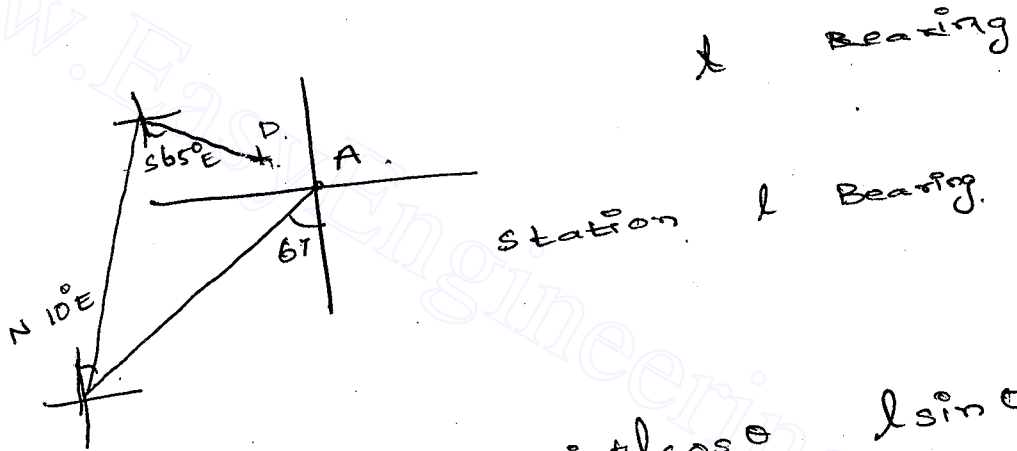
$$= 139.6 \text{ m}$$



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3

through the woods a survey with hand compass started from point A and walked 1000 steps S 67° W and reached point "B" and then he changed the direction and walked 512 steps in the direction N 10° E and then reached point "C" and then again changed the direction and walked 1504 steps S 65° E. The surveyor wants to reach A. In which direction we must go and how many steps to reach point A.



Station	l	Bearing	Direct	$l \cos \theta$	$l \sin \theta$
AB	1000	67° SW	-390.73	-920.50	
BC	512	10° NE	+504.22	+88.91	
CD	1504	65° SE	-635.62	+1363.08	
DA	x		$l \cos \theta$	$l \sin \theta$	

$$\sum l \cos \theta = -390.73 + 504.22 + 635.62 + l \cos \theta$$

$$l \cos \theta = +522.13$$

$$\sum l \sin \theta = 531.49$$

$$l \sin \theta = -531.49$$

$$(l \cos \theta)^2 + (l \sin \theta)^2 =$$

$$\frac{l \cos \theta}{l \sin \theta} = \frac{522.13}{531.49}$$

$$\theta = 44.491^\circ$$

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4. Determine the line EA.

Line	length	Bearing
AB	144.1	$85^{\circ}30'$
BC	201.2	15°
CD	168.4	$285^{\circ}30'$
DE	168.4 112.6	$195^{\circ}30'$
EA	?	P

5. Closed traverse having following length and bearing.

Line	Length	Bearing
AB	200m.	γ (Rotating east)
BC	98	178°
CD	Not obtained	270°
DA	86.4	1°

$l \cos \theta$	$l \sin \theta$
200 $\cos \theta$	200 $\sin \theta$
-99.94	3.420
$x \cos 270^{\circ} = 0$	-x
86.38	1.5078
<u>0</u>	

$$\sum l \cos \theta = 0$$

$$200 \cos \theta - 99.94 + x \cos 270^{\circ} + 86.38 = 0$$

$$\theta = 86^{\circ} 6' 44.51''$$

$$\sum l \sin \theta$$

$$200 \sin (86^{\circ} 112) + 3.420 - x + 1.5078 = 0$$

$$x = 204.467 \text{ m}$$

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6.) Co-ordinate A & B are given below,

A third point "C" has been chosen in such a way that AC and CB are $29^{\circ}30'$ and $45^{\circ}45'$. Calculate the length of AC and CB.

Point.	Northing.	Easting.
A	150.	200.
B	1500.	1300.

$$\begin{aligned} \text{Total latitude} &= 1500 - 150. \\ &= 1350. \end{aligned}$$

$$\begin{aligned} \text{Total Easting} &= 1300 - 200. \\ &= 1100. \end{aligned}$$

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7. A traverse is run to set out line "MQ" \angle 1900 m.
 @ right angle to a given line ~~PO~~.
 "MN" The length & bearings are observed
 calculate the length and bearing
 PQ.

Line	length	Bearing
MN.	—	360° .
MO.	830	120° .
OP.	1000	$86^\circ 30'$.
PQ.	?	?

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29/12/2015

SURVEY.

LOCAL ATTRACTION.

- * It denotes the influence which prevents the needle from pointing the magnetic north in a given locality. It may be due to electrical wires, steel structures, rail roads, underground rail pipes etc.,
- * It can be detected by absorbing the F.B and B.B of the line and finding the difference.

If $F.B \sim B.B = 180^\circ$ then both the stations are free from local attraction.

- * The amount of errors due to local attraction @ a station is same in all the bearings @ that place and in the same direction.

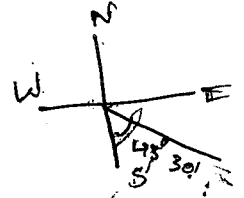
Elimination of Local Attraction:

1. By calculating the local attraction @ each station.
2. By included angles.

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1. The bearing absorbed in traversing with a compass @ a place where local attraction are suspected are given below. At what station, the suspected local attraction. Find the corrected bearing of the line.

AB.	S 45° 30' E.	N 45° 30' W.
BC	S 60° E.	N 60° 40' W.
CD	N 3° 20' E.	S 5° 30' W.
DA.	S 85° W.	N 83° 30' E.



AB.	134° 30'	314° 30'
BC	120°	299° 20'
CD	3° 20' + 4°	185° 30'
DA.	265°	83° 30'

To $134 \quad 314^{\circ} 30' - 134^{\circ} 30' = 180^{\circ}$

∴ There is no local attraction in AB.

∴ B.B in BC = $120^{\circ} + 180^{\circ}$
 = 300°

Correction in C = $300^{\circ} - 299^{\circ} 20'$
 = $40'$

Corrected F.B in C = $3^{\circ} 20' + 40'$
 = 4°

Corrected B.B in D = $4^{\circ} + 180^{\circ}$
 = 184°

Correction in D = $185^{\circ} 30' - 184^{\circ}$
 = $1^{\circ} 30'$

Corrected F.B Value = $265^{\circ} - 1^{\circ} 30'$
 = $263^{\circ} 30'$

	F.B	B.B.
CD AB	S 45° 30' E	N 45° 30' W
BC	S 60° E	N 60° W
CD	N 4° E	S 4° W
DA	S 83° 30' W	N 83° 30' E

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2. The following bearing was taken in a closed traverse.

Line	AB	BC	CD	DE	EA
FB	$48^{\circ}25'$	$117^{\circ}45'$	$104^{\circ}15'$	$165^{\circ}15'$	$259^{\circ}30'$
BB	230°	336°	$284^{\circ}55'$	$345^{\circ}15'$	79°

State the station, which are affected by local attraction by how much. Determine the correct bearing. Calculate the true bearing if the declination $1^{\circ}30' W$.

Solution:

Station affected by local attraction: A, B, C

B.B of A = $259^{\circ}30' - 180^{\circ}$
 $= 79^{\circ}30'$

Correction @ A = $79^{\circ}30' - 79^{\circ}$
 $= 30'$

Correction @ F.B A = $30' + 48^{\circ}25'$
 $= 57^{\circ}55'$

Corrected angle @ B = $48^{\circ}55' + 180^{\circ}$
 $= 228^{\circ}55'$

Correction @ B = $228^{\circ}55' - 230^{\circ}$
 $= -1^{\circ}5'$

Corrected F.B @ B = $117^{\circ}45' - 1^{\circ}5'$
 $= 116^{\circ}40'$

Corrected angle @ C = $116^{\circ}40' + 180^{\circ}$
 $= 296^{\circ}40'$

Correction @ C = $296^{\circ}40' - 356^{\circ}$
 $= 40'$

Corrected F.B @ C = $104^{\circ}15' + 40'$
 $= 104^{\circ}55'$

$$104^{\circ}55' + 180^{\circ} = \overset{284^{\circ}55'}{\underline{\underline{320^{\circ}55'}}$$

$$\text{Correction} = 284^{\circ}55' - 284^{\circ} \\ = 55'$$

Corrected angle is

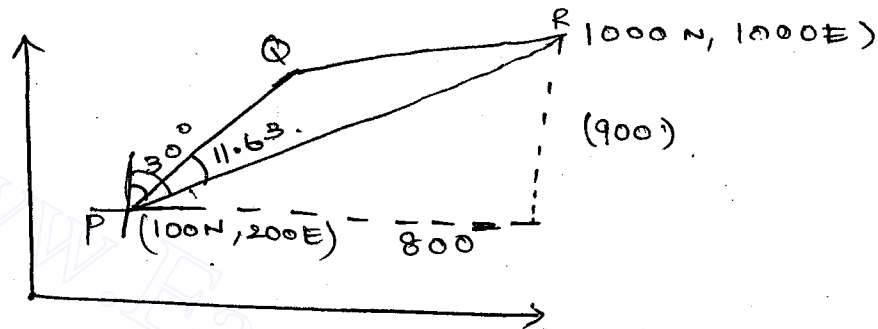
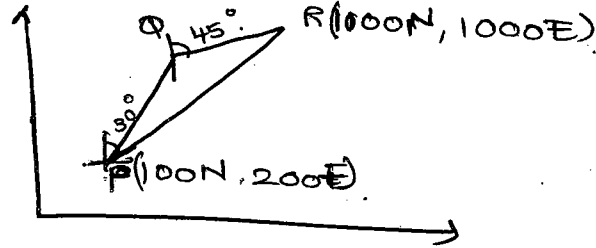
Corrected True Bearing Angle:

	F.B	B.B.
AB	$47^{\circ}25'$	$229^{\circ}25'$
BC	$176^{\circ}15'$	$336^{\circ}15'$
CD	$103^{\circ}25'$	$283^{\circ}25'$
DE	$163^{\circ}45'$	$343^{\circ}45'$
EA	$258^{\circ}27'$	$78^{\circ}27'$

2/2/16.

Survey.

- 1) In the figure given below. the length PQ (WCB 30°) and QR (WCB 45°) respectively. upto 3 places of decimal are.



$$PR = \sqrt{800^2 + 900^2} = 1204.159$$

$$\theta = \tan^{-1}\left(\frac{900}{800}\right) = 48.37$$

$$\sum L = 0$$

$$PQ \cos 30^\circ + QR \cos 45^\circ + 1204.159 \cos 48.37^\circ$$

$$\sum D = 0$$

$$PQ \sin 30^\circ + QR \sin 45^\circ + 1204.159 \sin 48.37^\circ$$

$$PQ = 273$$

$$QR = 9387$$

22) Method - II. (Projection Method.)

$$x \cos 30^\circ + y \cos 45^\circ = 900$$

$$x \sin 30^\circ + y \sin 45^\circ = 800$$

$$\begin{array}{l} x = 293 \\ y = 938 \end{array}$$

Method - III (sine rule)

$$\begin{array}{l} 1^\circ = 4' \\ 1' = 4'' \end{array}$$

2. The latitude and departure of a line AB are +78m and -45.1m respectively. The WCB, bearing of the line AB

$$l \cos \theta = 78$$

$$l \sin \theta = -45.1$$

$$\theta = \tan^{-1} \left(\frac{-45.1}{78} \right)$$

$$\boxed{\theta = 330^\circ}$$

3.)

Line	F.B	B.B.
AB	126° 45'	308° 00'
BC	45° 15'	227° 30'
CD	340° 30'	161° 45'
DE	288° 30'	78° 30'
ED.	216° 30'	31° 45'

$$31^\circ 45' - 36^\circ 30' = -4^\circ 45'$$

$$302^\circ - 308^\circ =$$

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6/1/2016.

SURVEYING.

1. A 30m chain was tested before a survey and found to have a length of 29.93. If the length of the line measured with this chain is 273.3m. Find the true length.

$$\begin{aligned} \text{True length} &= \text{Measured length} \times \left(\frac{L'}{L}\right) \\ &= 273.3 \times \left(\frac{29.93}{30}\right) \\ &= \underline{\underline{272.66 \text{ m}}} \end{aligned}$$

2. A true length of a line measured from a plan as per scale was 1276.54m. when the line was measured by 30m long chain, the length was measured as 1274.84m. Find the change in ~~true~~ ⁱⁿ length of the chain and also true length.

$$\text{True length} = \text{Measured length} \times \left(\frac{L'}{L}\right)$$

$$\frac{1276.54}{1274.84} = \left(\frac{L'}{30}\right)$$

$$L' = 30.04 \text{ m}$$

$$\begin{aligned} \delta L &= L' - L \\ &= 0.04 \text{ m. (too long)} \end{aligned}$$

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3. A 30m chain used to measure the length of a line was test before the line was measured and found to be 29.95m. The line was measured and recorded as 590.48m. The chain was tested again and found to be 30.08 m long. Find T.L

there is change of length of chain gradually and hence average length of chain must be used to find true length.

$$L'_{\text{avg}} = \frac{30.08 + 29.95}{2} = 30.015$$

$$T.L = 590.48 \times \frac{30.015}{30} = 590.7754 \text{ m}$$

- 4) A rectangular plot was measured with 20m chain 120cm too long the length of side of the rectangle 280x480. Find the true area of the plot.

$$\begin{aligned} \text{True area} &= \text{Measured area} \times \left(\frac{L'}{L}\right)^2 \\ &= 280 \times 480 \left(\frac{20.12}{20}\right)^2 \\ &= \underline{136017.63 \text{ m}^2} \end{aligned}$$

4.) A 30m chain was tested before starting the days work and found to be 20 cm too short.

After measuring a length of

1200m. The chain was tested again and was found to be

10cm too long @ the end of the days works the chain was

tested again and was found

to be 30cm too long. Find

the T.L. of line if the total length measured 2648 m.

$$L_{avg} = \frac{29.84}{1}$$