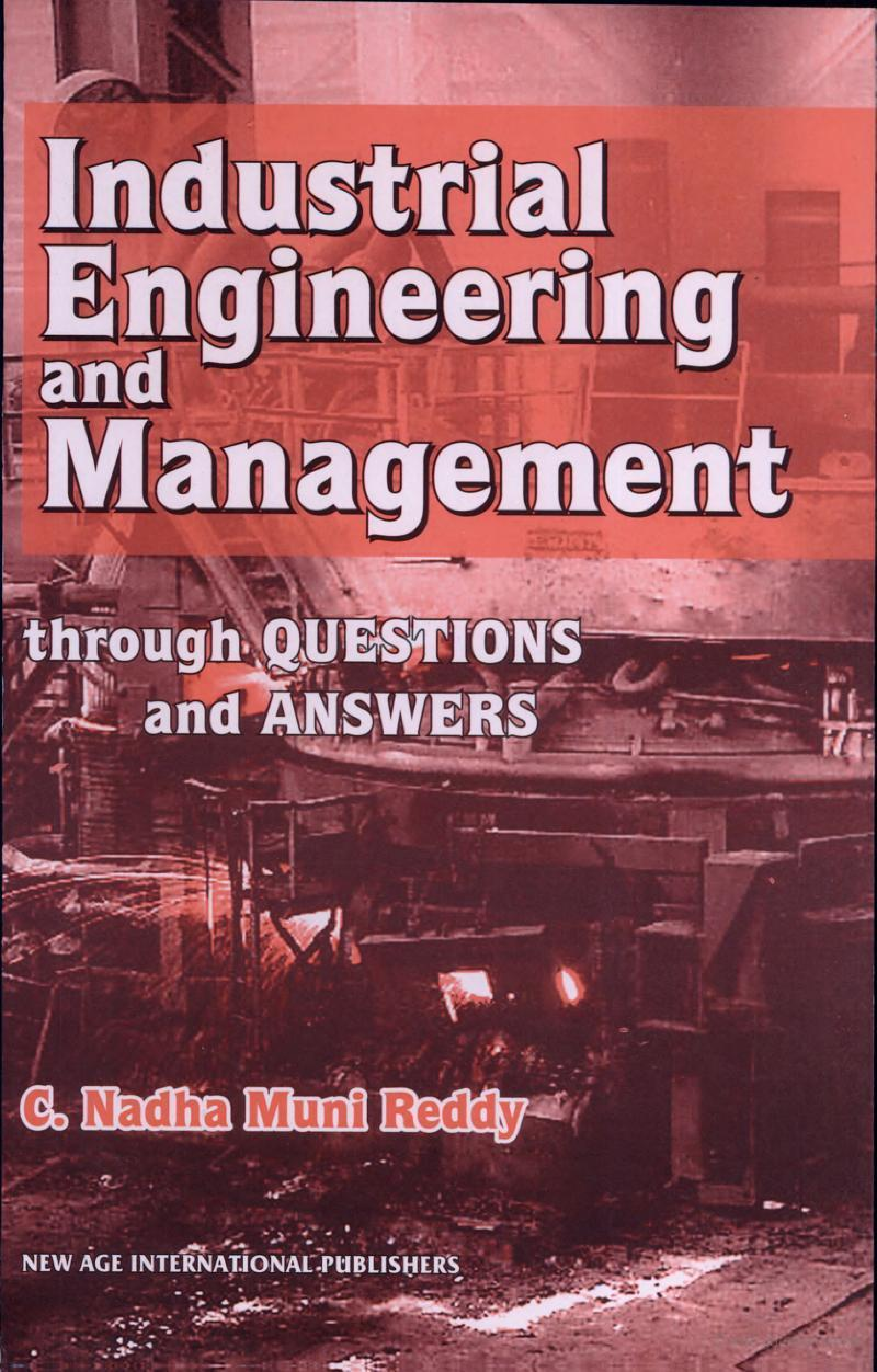

Industrial Engineering and Management

through QUESTIONS and ANSWERS

C. Nadha Muni Reddy

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encompasses an introductory treatment; learning enhancement within chapters with specially prepared questions and answers, cases and practice problems featuring an independent study approach.

Designed for undergraduate and postgraduate students of both engineering and management streams, it is hoped that this book would not only help them in preparing for examinations but would also enable them to emerge as successful managers.

Chinnagangu Nadha Muni Reddy

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PART I

Method Design & Work Measurement to Improve Productivity

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1.10 Differentiate among industrial engineering, productivity engineering and productivity management?

Ans. Industrial engineering is concerned with the design, improvement and installation of integrated systems of men, materials and equipment. It draws upon specialized knowledge and skill in the mathematical, physical and social sciences together with principles and methods of engineering analysis and design to specify, predict and evaluate the results to be obtained from systems. On one hand, they had to work closely with management and at the otherside with rank and file.

Productivity engineering is a part of industrial engineering function that is concerned with the design, installation and maintenance of productivity management, evaluation, planning and improvement of systems.

Productivity Management: It is a team or participative approach. It is a formal management process involving all levels of managers, workers and works for the ultimate objective of reducing cost through all the phases of cycle.

1.11 What different productivity measures are used for international comparisons?

Ans. Organization for European Economic cooperation (OEEC) used two measures (*a*) GDP per capita (*b*) GDP per employed civilian.

Rostas (1955) used (*a*) Gross output per one hour of labour (*b*) physical output per one unit of labour.

1.12 What different productivity measures are used at National Level?

Ans. *National Product:* Market value of output of final goods and services produced by Nation's economy. Final product refers to a product that is not resold. Intermediate goods means the products those are resold.

Gross national Product: Includes capital consumption allowances (reserves for depreciation, accidental damage to fixed capital).

Net National product: It excludes above such allowances. Bureau of labour statistics (BLS) use (*a*) Labour productivity Index (*b*) Capital Productivity Index (*c*) Labour and capital utility productivity index for this purpose.

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1.13 What are the advantages of productivity measurement at the industry level?

Ans. (a) It can be useful to know the economic performance of the country.

(b) It can be useful for the comparison of individual performance of different companies.

(c) It can also be used to forecast industry growth patterns, future conditions.

1.14 What different measures are used to measure the productivity at industry level?

Ans. 1. Siegel suggested index of unit labour cost = index of wages/index of productivity.

2. Mill's index = output/no. of wage earners.

3. Average production coefficient $= \Sigma q_i p_w / \Sigma q_f p_f =$ weighted average ratio of input to output.

Where $q_i =$ Quantity of materials and other commodities consumed.

$P_w =$ process of materials and commodities.

$q_f =$ quantities of final products.

$P_f =$ prices of final products.

1.15 What are the Lytton's measures used for productivity measurement in the service organizations?

Ans. 1. *Post-office Department:* Productivity = out put per person = Number of items handled/Net paid man years

2. *Social Security Administration:* Productivity = output per person = Work processed/Paid man years

3. *Revenue Service:* Productivity = Number of items processed/Average Employment.

1.16 What are disadvantages associated with partial productivities?

Ans. ⇒ One of the serious dangers of relying exclusively on partial productivity measures lies in over-emphasizing one input factor and the effect of other inputs is underestimated.

1.17 What is called 'Base Period' in connection with total productivity index measure?

Ans. Total productivity Index $= TP_{it}/TP_{io}$

Where $TP_{it} =$ Total productivity of t^{th} period, of product i

$TP_{io} =$ Total productivity in the base period, of product i

Base period is any normal period in which the production was not much different from the average.

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1.18 Give the relation between total productivity and partial productivity of a product?

Ans. $TPi = w_{ij} \cdot PP_{ij}$ where $w_{ij} = I_{ij}/\Sigma I_{ij}$ for all J

w_{ij} = Weightage of particular input J over all inputs for a product 'i'

PP_{ij} = partial productivity of J^{th} input for a product 'i'

TP_i = Total productivity of the product 'i'.

1.19 Give the relation between total productivity of firm and the total productivities of Individual products?

Ans. Total productivity of a firm, $TPF = \sum_{i=1}^{n} W_i \cdot TP_i$ where

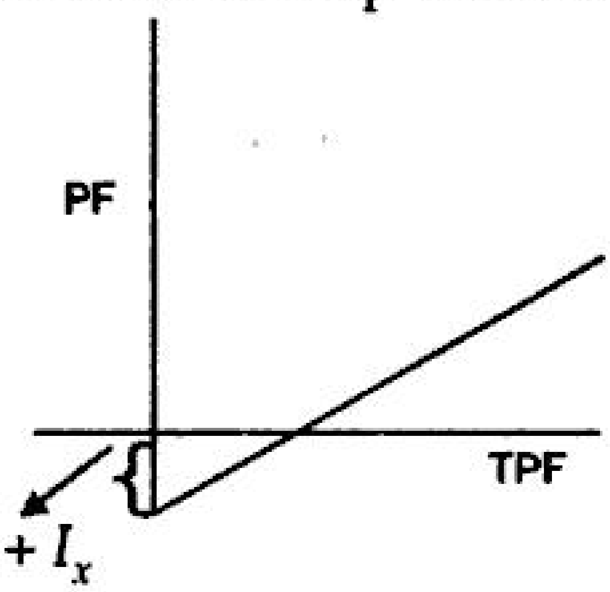
$W_i = I_i/\Sigma I_i$ = input i/sum of inputs of all products

TP_i = Total productivity of i th product

n = no. of varieties of products.

1.20 Give the relation between Profit of the firm, PF and total productivity of the firm, TPF?

Ans. $PF = a\,(TPF) - b$ where a = slope of the line; b = y-intercept.



1.21 What are the different technology based productivity improvement techniques?

Ans. (*a*) CAD (*b*) CAM (*c*) CIM (*d*) Robotics (*e*) Laser technology (*f*) Energy Technology (*g*) Group Technology (*h*) Computer graphics (*i*) Maintenance management (*j*) Re-engineering.

1.22. What are the different materials based productivity improvement techniques?

Ans. Inventory Control, Materials requirement planning, Quality engineering and control, Materials handling, Reuse and recycling.

1.23 List out the employee based productivity improvement techniques?

Ans. Financial incentives, non-financial and semi-financial and semi-financial incentives like fringe benefits, Employee

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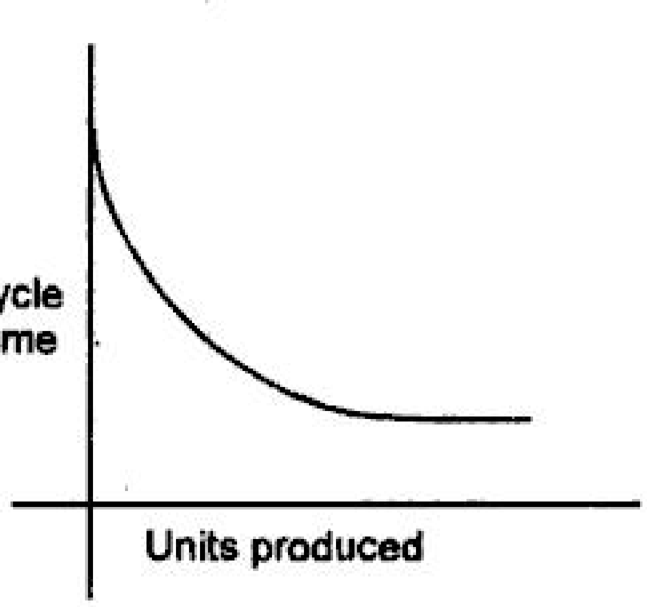
Promotion, Job enrichment, Job enlargement, Job rotation and Worker participation.

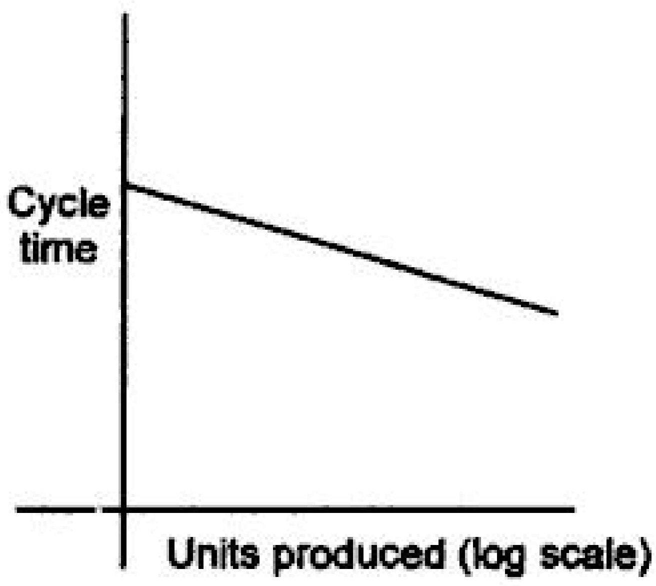
1.24 What is Job Enrichment?

Ans. It includes a variety assignments, feed back, Hygiene factors, Inter-Personal relations, Recognition, Status, Salary, Security etc.

1.25 What is learning curve?

Ans. Repetition of the same job results in decrease in cycle time and the curve showing the relation between the cycle time and the natural number of units produced is called learning curve.





$$T_i = T_1 * n^S$$

where T_i = cycle time for ith unit production

S = slope of the line in log scale

Learning rate, $$r = \frac{T_n}{Tn/2} = \frac{T_1 \cdot n^S}{T_1 \cdot (n/2)^S}$$

$$S = \log r / \log 2.$$

1.26 What is productivity planning?

Ans. It is concerned with setting up of productivity targets in future.

EXERCISE

1. Give the meaning of productivity?
2. Differentiate between production and productivity?
3. List out various types of partial productivities?
4. Explain the relation between partial productivities and total productivity?

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2

Work Method Design

There are numerous number of jobs to be designed in production organizations i.e, right procedures/economical procedures are to be evolved and the efforts made in this direction constitute method study. Work study consists of two parts: (*a*) method study and (*b*) work measurement. In work measurement, time standards will be established in the best method emerged through method study. Here in this chapter, concepts, tools and techniques of work method design are presented in logical sequence through questions and answers.

2.1 Define work-study?

Ans. Work-study is a generic term to indicate the techniques, particularly of method study and work measurement. It is used in the examination of human work in all its contexts. It involves systematic investigation of all the factors which affect the efficiency and economy of the situation being reviewed, in order to effect improvement.

2.2 What are the components of work-study and mention the significance and role of each?

Ans. Work study comprises of:

(*a*) *Method study*—To improve methods of production, resulting in more effective use of material, machines and manpower.

(*b*) *Work measurement*—To access human effectiveness, results in improved planning and control, marking time as a basis for sound incentive schemes.

2.3 What is work simplification?

Ans. It is the organized use of common sense to find easier and better ways of doing work.

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2.4 What are the primary objectives of work-study?

Ans. (*a*) To carry out the job in the most economical way.

(*b*) Standardization of the method used in the process.

(*c*) Determination of the time required by a skilled worker to perform the work at a normal speed.

(*d*) Planning of the training program for the workers in the new methods.

2.5 Define "Method Study"?

Ans. It refers to a systematic and scientific evaluation of existing and proposed methods of doing work as a means of developing the proposed method through analysis and critical examination.

2.6 Who is the originator for time study or work-measurement?

Ans. F.W. Taylor (1881) begun the time study in the machine shop of Midvale steel company.

2.7 What are the other achievements of Taylor apart from stop-watch time study?

Ans.
- Inventing high-speed steel.
- Discovering the factors affecting the cutting of metals.
- Functional type of organization.
- Scientific management.

2.8 Who originated 'Motion Study'?

Ans.
- Mr and Mrs Gilbreths
- Mr Gilbreth started his study on brick construction job in 1885

2.9 Definition of micro motion study?

Ans. It is the study of fundamental element or subdivisions of operation by means of a motion picture camera and a timing device, which accurately indicates the time intervals on the motion picture film.

2.10 What are the important phases of micro motion study?

Ans.
- Filming the operation
- Film analysis
- Graphic representation (simo chart)
- Examine, develop, install and maintain

2.11 What is a SIMO chart?

Ans. The charting for the micro motion study is done in the form of SIMO (simultaneous motion) chart. The elements and

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symbols used are called as 'Therbligs'. It consists of 17 basic elements. The motion elements are recorded against the time measured in terms of winks (1 wink = 1/2000 min).

2.12 Write the Therbligs for 'unavoidable delay' and "avoidable delay"?

Ans. Unavoidable delay UD 

Avoidable delay AD 

2.13 What are the Therbligs used for locating an object from a group?

Ans. Select → ST

2.14 What is cycle graph? Who is the first man to use this technique?

Ans. (*a*) It is a photographic record, on still film, of the path of motion of a body member taken with a light attached to it. The still photo has to be taken in a dark room.

(*b*) Gilberth.

(*c*) The cycle graph can provide the path of the movement as to note whether it is being jumbled up with quick changes of directions or is uniform.

2.15 What is chrono-cycle graph?

Ans. The light source used in cycle graph is made to 'on' and 'off' at a known frequency which can be varied by means of an electronic circuit. Thus the current flow to the bulbs is continuously interrupted and light is being caused to flash on at full intensity and to dim out slowly every time. The path of movement, when photographed by a still camera will be seen as a series of 'pear shaped dots. Depending on the distance between pear shaped dots, to know speed and acceleration also along with the direction of movement.

2.16 What is a process chart and what are the symbols used in it?

Ans. Process chart is a graphic representation of the sequence of events or steps that occur in the work method or procedure. The symbols used are:

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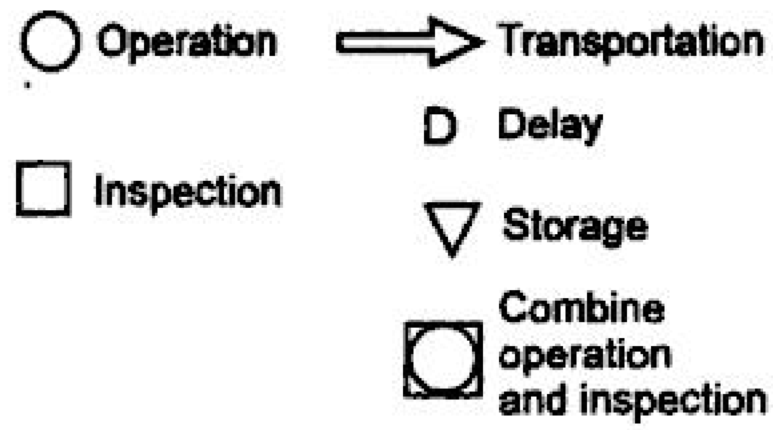

2.17 When do we use operation process chart?

Ans. It is a graphic representation of the 'operations' and 'Inspections', involved in a process in a chronological sequence. It uses only two symbols:

Operation — ○

Inspection — □

2.18 What is a flow process chart?

Ans. Flow process chart is graphic representation of all operations, transportations, inspections, delays and storage occurring during a process and also includes the information such as time required and distance moved etc., It is more informative than operation process chart. There are three types of flow process charts:

(*a*) material type

(*b*) man-type and

(*c*) Equipment process charts.

2.19 What are characteristics of a 'two handed process chart'?

Ans. This is a form of man type process chart on which the motions made on hand in relation with those made by the other hand are recorded. All the process chart symbols can be used. It is best suited for repetitive tasks, performed by an operator. It enables to develop proper synchronization and fairly equal distribution of movements and operations to be performed by both hands.

2.20 What are the characteristics of a 'Multiple activity chart'?

Ans. It is the graphic representation of coordinated activities of more than one subject (man or machine) many times, a process may require the coordination of various activities to be performed by men and machines. This chart indicates on a common time scale the working and idle time of persons, machines or any combination of men and machines.

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Position	P	P	Position
Release	RL	RL	Release
Reaches for lock washer	TE	TE	Reaches for lock washer
Selects	ST	ST	Selects
Grasps	G	G	Grasps
Moves to the fixture	TL	TL	Moves to the fixture
Position	P	P	Position
And release	RL	RL	And release
Reaches for bolt	TE	TE	Reaches for bolt
Select	ST	ST	Selects
Grasps	G	G	Grasps
Moves to the fixture	TL	TL	Moves to the fixture
Position bolt	P	P	Position bolt
Inserts bolt through washer	A	A	Inserts bolt through washer
With draws the assembly	DA	DA	With draws the assembly
Carry it to top of the chare	TL	TL	Carry it to top of the chare
Release it in the chare	RL	RL	Release it in the chare

2.28 What is a 'Flow diagram'? What is the importance of this tool?

Ans. It is a graphical/pictorial representation drawn to the scale, indicating the relative positions of the machines or work stations incorporating the paths of movements of materials, men or equipment. The symbols used in flow process chart may also be used in the flow diagram.

Flow diagrams act as powerful tools to develop the layouts, improving materials handling.

2.29 What is a string diagram? What are its uses?

Ans. String diagram consists of a scale plan of the shop and the equipments, in which a perk or pin is struck in the area representing the facility. A continuous coloured thread is used along the path of travel of material, men while performing the operations. The length of thread is equal to the total travel.

Repetitive movements and back tracking can be identified. When a group of persons are working, movements can be studied. By careful analysis, it is possible to determine better layouts and compare different feasible layouts.

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2.30. Who is responsible for the development of 'principles of motion economy'? What are the uses of this study?

Ans. Gilbreth is responsible for the development of the principles of motion economy. Barnes categorized these principles into three heads:

(*a*) Eight principles related to the use of human body.
(*b*) Eight principles related to the arrangement of work place and
(*c*) Five principles related to the design of tools and equipment. In industrial environment, the workers repeat a large number of varied physical motions. Hence, it is essential to study the basic patterns of these motions to improve and economize the motions. Studies also were made on the design of the display and control devices with a view to have better compatibility on the Man-Machine interactions.

2.31 Define Ergonomics?

Ans. Ergo means 'work'. Nomos means 'Natural Laws'. Ergonomics is defined as the scientific study of the human beings under work environment. It refers to the study of the natural laws of the physical and psychological aspects of human beings under work situation with a view to have better compatibility and effectiveness. It utilizes biological, physiological, behavioural, medical, bio-medical and technological sciences.

2.32 Describe in brief the scope of 'Ergonomics'?

Ans. The principles developed by 'Ergonomics study' have wide applications in:

1. Measurement of manual work.
2. Physical and psychological aspects of fatigue and stresses.
3. Design of Man-Machine system, work place layout, tools and equipment, controls, furniture etc. for better compatibility and effectiveness.

2.33 What is fatigue? What are its types?

Ans. It is the fatigue manifested by decrease in output, after continued performance of task. The work output is decreased due to:

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1. Improper job design.
2. Non-compatibility with work environment.
3. Morale problem indicating a desire for a change in routine.

2.34 What is Bio-energetics? What is the energy level required while performing work?

Ans.
- Bio energetics is concerned with the various levels of energy generation and consumption and utilization of human being at work.
- Human beings get the energy due to the combination of oxygen with nutrients (in the form of Glycogen) contained in the body. Human body gets energy from food equal to 4500-4800 k.cal per day.
- Energy requirements for Basal Metabolism – 500 kcal/day.
- Energy requirements for keeping the body active – 1500 kcal/day.
- Energy available for work = 2500 kcal/day
- Normally, male worker expends 5 kcal/min, female worker 4 kcal/min while doing work.
- If a person is working at a rate of energy expenditure of 5 kcal/min, one need not touch the reserve energy level (25 kcal). If the reserve energy is also tapped, then a person feels fatigue and requires rest.

2.35 Give the relation between rest period required, working rate and period of continuous work? If a person is expending energy at a rate of 7.5 kcal/min. what would be the continuous period of time of work for him? What rest period is required after this work?

Ans. Rest period, a = w. (b-s)/(b-1.5) where

b = average energy expenditure in kcal/min.
s = standard level of energy expenditure = 5 kcal/min.
w = continuous working time in minutes.

- In the example, current energy expenditure rate = 7.5
  Normal rate of energy expenditure = 5 kcal/min.
  Excess rate of energy expenditure = 7.5 – 5 = 2.5 kcal/min. is drawn from the reserve energy of 25 kcal.
  The time for which a person can work continuously = 25 kcal/2.5kcal/min = 10 min.

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i.e., $w = 10$ min.

The rest period required after working, $a = w.(b\text{-}s)/(b\text{-}1.5) = 10(7.5\text{-}5)/(7.5\text{-}1.5) = 25/6$ min.

2.36 What is human factors 'engineering'?

Ans. A production unit is usually considered as a Man-Machine integrated system performing under an environmental envelope. The environment efforts not only to the ambient conditions of temperature, humidity, noise etc., but also the arrangement of layout, facilities display and controls. The physical sensory and psychological parameters of human beings called as human factors being studied to synchronize effectively with the production system with the objective of achieving harmonious and efficient output from the system.

2.37 What is written standard practice? What are the improvements to record the method or operation available now?

Ans.
- Once the better method is determined with the help of method study, it is essential to record the method. This record is called "written standard practice". This helps the operator while performing job and also helps to train the new operator.
- Motion picture or videotape can record some complicated manual operations best. In fact, in may be more economical in certain cases to make the record in this manner than to rely entirely on written description of the job. Some times motion pictures are just sufficient to train the operators and in few cases, they serve as a supplement to the written standard practice.

2.38 What is 'work measurement'? What different techniques are available for this?

Ans.
- Work measurement is defined as a technique to establish the work content of a specific job, in terms of time required to carryout at a defined level of performance by a qualified worker.
- Different techniques of work measurement are:
  (*a*) stop-watch time study,
  (*b*) application of synthetic data,
  (*c*) application of predetermined motion time systems (PMTS)

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(*d*) *Micro-chronometers:* Gilbreth developed this spring wound watch. The big hand called sweep hand makes 20 revolutions per minute and the dial is graduated into 100 equal divisions. Thus, the least count = 1/20* 1/100 which is equal to 1/2000th of a minute = wink. There is also a small pointer moving at a speed of 2 revolutions per minute.

(*e*) *Wink counter:* Porter developed this motor driven equipment. Least count = 0.0005 minutes. The readings are in terms of winks.

(*f*) *Electronic digital stop watches:* Varieties of digital stop watches are available with different accuracies.

2.42 What is the importance of observation board with observation sheet in time study?

Ans.
- A lightweight board, slightly larger than the observation sheet is used to hold the paper and the stopwatch is mounted at the top right corner with the help of clamps.
- The observation sheet is a printed form with spaces provided for recording information about the operation being studied. This includes the detailed description of the operation, name of the operator, name of the observer, date and place of study etc. The form provides spaces for recording stop watch readings for each element of the operation, performance ratings of the operator and computations. Standard size is 8.5″ × 11″ as it can easily be bounded and can be easily filed

Observation sheet	*Study No.*
Operation	Op. No.
Part name	Part No.
Machine name	Dept.
Operator name and No.	Date
Experience on job	
Begin	Finish

			Trials								
Elements	Speed	Feed	1	2	3	4	5	6	7	8	Select time
1											
2											
3											
4											
⋮											

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Select time
Rating
Normal time
Personal allowance
Fatigue allowance
Total allowances
Standard time Timed by

2.43 What is the need of breaking job into elements and record the times for elements instead of timing the job entirely as one entity?

Ans. (*a*) One of the best ways to describe an operation is to break it down into definite and measurable elements and describe each separately. The begin and end of each element may be specifically indicated. Very often the elements taken from the time study can serve as the 'standard practice for the operation. Such lists of elements are helpful for training the operators.

(*b*) Standard values may be determined for the elements of the job, which can be used in the preparation of standard data.

(*c*) Analysis of an operation by elements may show slight variations in method that could not be detected so easily from an over all study.

(*d*) An operator may not work with same tempo and speed. Hence 'elements study' helps to create provision for giving separate ratings for different elements.

2.44. What are the guidelines/rules to be followed in dividing the job into elements?

Ans. 1. The elements should be as short in duration as can be accurately timed.

2. Handling time should be separated from the machine time.

3. Constant elements should be separated from variable elements. A constant element refers to those elements that are independent of the size, weight, length and shape of the piece.

2.45 What are the various methods available for timing?

Ans. The three most common methods are:

1. Continuous timing,

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2. Repetitive timing,
3. Accumulative timing.
   - *Continuous timing:* The observer starts the watch at the beginning of the first element and permits it to run continuously during the period of the study. The observer notes the readings of the watch at the end of each element and records this reading on the observation sheet. The time for each element can later be determined by subtraction.
   - *Repetitive timing:* The hands of the watch are snapped back to zero at the end of each element. The main advantage to the repetitive method over the continuous method is that the time for each element is visible on the observation sheet and the time study analyst can see the variations in time values as the study is made.
   - *Accumulative timing:* Two stop watches are mounted close together on the observation board and are connected by a lever mechanism in such a way that when the first watch is started, the second watch is automatically stopped and when the second watch starts, first watch will be stopped, here there is no need of doing subtractions.

2.46 How do you determine the number of observations required or number of cycles to be timed?

Ans.
- The number of cycles required depends upon the accuracy and the confidence level on the reliability of the reporting of the time values. Usually an accuracy of + or – 5% and confidence level of 95% are considered reasonable.
- A preliminary study has to be made and the cycle times Xi are taken for N-number of cycles.

Then
$$N' = \left[\frac{40\sqrt{N.\sum x_i^2 - (\sum x_i)^2}}{\sum x}\right]^2$$

- Where N' is the required number of observations to predict the true time within + or – 5% precision and 95% confidence level.

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2.69 What is MTM?

Ans.
- Methods time measurement (MTM), another predetermined motion time system (PMTS) was developed by Maynard during world war -II.
- Special types of MTMs have been developed for predetermining the time in maintenance work and also for the measurement of non-repetitive indirect work
- MTM was developed from the analysis of motion pictures taken from various industrial operations i.e. the data is based on micro motion analysis.
- 10 basic motions are identified, and for each motion, predetermined time values are given in separate table for MTM-1.

2.70 What are the different basic motions identified for MTM-1?

Ans.
1. REACH (R)
2. MOVE (M)
3. TURN (T)
4. APPLY PRESSURE (AP)
5. GRASP (G)
6. RELEASE LOAD (RL)
7. POSITION (P)
8. DISENGAGE
9. EYE TIMES
10. BODY, LEG AND FOOT MOTIONS.

2.71 What is the unit of time considered for MTM?

Ans. The unit of time used in all MTM tables is TMU, time measurement unit = one hundred-thousandth of hour
= 0.00001 hours
= 0.0006 minutes.

2.72 How does the basic motion. REACH classified ?

Ans. There are five classes of REACH. The time to perform a REACH is affected by the nature of the object towards which the REACH is made i.e. whether the object is in fixed location or jumbled with other objects etc.

2.73 Explain the significance of the basic motion MOVE in MTM?

Ans. MOVE is the basic element used when the purpose is to transport an object to a destination. There are three types of MOVES called A,B and C, depending upon whether the object is to be moved to the other hand or to an approximate location or to an exact location. The time for MOVE is affected by the following variables.

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1. Nature of destination,
2. Length of the motion,
3. Type of MOVE,
4. Weight factor, static or dynamic.

2.74 Explain on what variables, the basic element TURN depend?

Ans. TURN is the basic element employed to turn the hand, wrist and forearm about the long axis of the forearm. The time for TURN depends on two variables:

1. Degrees turned,
2. Weight factor.

2.75 What is the significance of the basic element GRASP?

Ans. It is employed when the purpose is to secure sufficient control of one or more objects with the fingers or hand to permit to perform the next basic element.

2.76 How do you calculate 'EYE TRAVEL TIME' and 'EYE FOCUS TIME'?

Ans.
- EYE TRAVEL TIME = 15.2 × T/D tmu, with a maximum value of 20 TMU.
- Where T = distance between points from and to which eye travels.
- D = The perpendicular distance from the eye to the line of travel.
- EYE FOCUS TIME = 7.3 TMU

2.77 Few conventions for recording MTM?

Ans.

Examples	*Significance*
R8C	Reach, 8 inches, case C
M6A	Move 6 inches, case A object weighs less than 2.5 lbs.
MM10C	Move 10 inches, case C, Hand in motion at the beginning, object less than 2.5 lbs.
T30	Turn hand empty 30°.
T90L	Turn object weighing more than 10 lbs, by 90°.
APB	Apply pressure, includes regrasp.
PINSD	Position, class 1 fit, non symmetrical part, difficult to handle.
ET14/10	Eye travel between points 14" apart where line of control is 10" from eyes.
Fm	Foot motion
SS16C1	Side step, 16", case 1.

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2.78 Write the outline of the procedure of establishing time standard using MTM?

Ans.
- Record all the details of job.
- Describe the equipment used such as jigs, fixtures used for holding the component and the gauges used for inspection.
- Break the operation into elements.
- Identify the MTM elements.
- Record the MTM motions in the conventional symbols.
- Find the time required in TMUs for each element by using standard data tables.
- Add all the elemental times of the operation under study. This gives the basic or normal time for the operation.
- Add the required allowances to get the standard time.

2.79 What are the improved versions of MTM and other PMTS?

Ans.
- *MTM-V:* It is designed for use with machine tool operations including such items as setting machine tools, handling tills and work pieces, measuring and gauging, cleaning the work and the machine etc.
- *MTM-M:* It is a second level functional system of the original data designed for the analysis and measurement of manual assembly work performed under stereoscopic magnification of 5 to 30 power.
- *MTM-C:* It is applicable for electrical related works. It includes desktop operations and tasks such as filing, typing and data entry.
- *4D DATA (Micro-matic methods and measurement)*: It is a computer aided means of applying MTM-1. The computer takes over much of work of analyst and assists in standard data development.
- *MTM-2:* It is designed for works of low repetition, more suitable for training the supervisors and managers on the predetermined motion time analysis.
- *MTM-3:* Can be applied in the areas such as assembly, foundry, sheet metal work, welding, maintenance and office work.

2.80 What is work sampling?

Ans.
- It is a method of taking random qualitative observations of a specific work, to determine the percentage time

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period consumed by various activities of the system, and to find how much percentage of the time the system is productive and how much percentage of the time the system is unproductive.

- L.H.C. Tippet first developed this method in 1934 in British textile industries, under the name as "snap reading technique".
- This was further developed by Prof. Robert Lee Morrow of New York university under the name as "Ratio delay method". This is also called activity sampling.

2.81 Explain what is confidence level?

Ans. • It is necessary to decide what level of confidence is desired in the results. The most common confidence interval is 95%. The area under two sigma or two standard. Deviations is 95.45%, which if rounded off, gives 95%. This means that the probability is that 95% of the time the random observations will represent the facts and 5% of the time they will not. One sigma generates less confidence i.e. 68% only.

2.82 State the formula for determining the sample size or number of observations to be taken based on a pilot study?

Ans. • $S \times P = K.\sqrt{\frac{p(1-p)}{N}}$ where

$S =$ Desired relative accuracy.

$P =$ Percentage expressed as a decimal (No. of idle observations to the total observations in a pilot study).

$K =$ Number of standard deviations based on confidence level.

$N =$ Sample size required.

2.83 illustration: Assume 200 readings were taken in the preliminary or pilot study and the number of idle observations is 60. For 95% confidence, and for 5% accuracy, calculate the sample size?

Ans. $S.P = 2\sqrt{\frac{p(1-p)}{N}}$ where P = 60/200 = 0.30

$$0.05 \times 0.30 = 2\sqrt{\frac{0.3(1-0.3)}{N}}$$

$$N = 37.34 \text{ approx.}$$

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engineer all made the study simultaneously. They agreed precisely on cycle times but varied on rating the worker. The experienced rated the worker 100% and the other engineers rated the worker 80% and 110%. The firm uses 0.15 allowance fraction.

Ans.

Cycle time	25	29	30	31
No. of time observed	1	2	2	1

1. Determine the standard time using the experienced industrial engineers worker rating?
2. Fin the standards using the data of inexperienced engineers also? What is your interpolation? Are you sure the experienced engineer is correct. What could be done to enhance consistency in analyst performance ratings?

Ans. Rating the worker at 100%

Normal time = (25 (1) + 29 (2) + 30 (2) + 31(1)/1 + 2 + 2 + 1) × 100% = 29 mts.

Standard time = normal time/(1-allowance fraction) = 29/(1–0.15) = 34.12 mts.

Rating the worker at 110%

Normal time = 29 × 110% = 31.90 mts.

Standard time = 31.90/(1–0.15) = 37.53

Rating the worker at 80%

Normal time = 29 × 80% = 23.2 mts.

Standard time = 23.2/(1-0.15) = 27.29 mts.

Usually, there will be more confidence in an experienced engineer. However, there is a possibility of errors in his collection of data. Hence we should suspect the process. We could enhance the consistency by training through visual aids and short courses etc.

2.98 As a cargo loader for Indian Airlines you are responsible for setting a time standard for uploading palletized loads. The study was conducted over 300 hours with 900 up loadings performed.

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Worker rating	*Activity*	*Number of timed observed*
80	Manually check and lift the load to the trailer	100
100	Two loaded trailer with tractor to aircraft	300
120	Check electrical contacts (this time will be reduced by 5%) by an additional inspection	400
90	Correct any malfunctioning observed	100
110	Load the palletized load into plane bay with automatic lift	400
140	Return tractor and trailer to warehouse	400

Allowance fraction is 0.10 for an 8 hour work day?

Ans. Solution average cycle time = 300 hrs/900 up loadings = 1/3 hours = 20 mts.

Normal minutes/uploading

$$20 \times (100/2000) \times 0.80 = 8 \text{ mts}$$

$$20 \times (300/2000) \times 1.00 = 3.0 \text{ mts}$$

$$20 \times (400/2000) \times 1.20 \times 0.5 = 2.4 \text{ mts}$$

$$20 \times (100/2000) \times 0.90 = 0.9 \text{ mts}$$

$$20 \times (400/2000) \times 1.10 = 4.4 \text{ mts}$$

$$20 \times (400/2000) \times 1.40 = 4.2 \text{ mts}$$

$$\text{Total normal time} = 22.9 \text{ mts}$$

Standard time = 22.9/(1–0.10) = 25.4 mts/uploading.

2.99 What is a job? How is it different from function?

Ans. Job is a group of related tasks or activities that need to be performed to meet organizational objectives. Jobs are then grouped into larger units called 'Departments and departments are grouped into basic functions such as marketing, engg and production.

Example: Assembling of bolts and nuts with the help of a wrench to place the machine tool firmly is the task, if it is repeated several times it is a job in the department.

2.100 What is a 'Job design'?

Ans. In production and operations, job design follows the planning and designing of product, process and equipment. Job design specifies the content of each job and determines

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how work is distributed within the organization. Just as an architect can build (design) a house in many ways with many different materials, so can a manager build/design a job with many different tasks. A combination of creativity and adherence to basic goals is critical to both the architect and the manager.

2.101 Give the meaning of 'Specialization of labour'/'Division of labour'?

Ans. Jobs can be broken apart into tasks. Breaking apart jobs into tasks and assigning tasks to different workers according to their special skills, talents and tools.

2.102 What can be studied with 'Gang process chart'?

Ans. It is a graphical tool to trace the interaction of several workers with one machine.

2.103 What are the 'principles of motion Economy'?

Ans. A broad set of guidelines focusing on work arrangements, the use of human hands, and body and the use of tools.

Principles of Motion Economy

Using the human body the way it works best

1. The work should be arranged so that natural rhythm can become automatic.
2. The symmetry of the body should be considered
   (*a*) Simultaneous, beginning and completing their motions at the same time.
   (*b*) Opposite and symmetrical.
3. The human body is an ultimate machine and its full capabilities should be employed.
   (*a*) Neither hand should ever be idle.
   (*b*) Work should be distributed to other parts of the body in line with their ability.
   (*c*) The safe design limits of the body should be observed.
   (*d*) The human should be employed at its highest use.
4. The arms and hands as weights are subject to physical laws and energy should be conserved.
   (*a*) Momentum should be made use of.
   (*b*) The smooth and continuous arc of the ballistic is most efficient.

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(*c*) The distance of movement should be minimized.
(*d*) Tasks should be turned over to Machines.

5. Tasks should be simplified
   (*a*) Eye contacts should be few and grouped together.
   (*b*) Unnecessary actions idle time should be eliminated.
   (*c*) The degree of required precision should be minimized.

Principles to be followed in laying out work place	*Principles in using mechanical devices and their design*
1. There should be a definite place for all tools and materials.	1. Vises and clamps that hold the work precisely should be used.
2. Tools, materials and controls should be very close to the point of use.	2. Guides can assist in positioning the work without close operator attention.
3. Tools, materials and controls should be located to permit the best sequence and path of motions.	3. Mechanical devices can multiply human abilities.
4. The works place should be fitted to the task and to the human.	4. Mechanical systems should be fitted to human use.

2.104 What is the importance of job rotation and job enlargement?

Ans.
- After world war-II, Manager and Behavioural scientists workers had the blue-collar blues. Job rotation and job enlargement were the responses to an overemphasis of scientific management. Rotating employees among different jobs can reduce boredom and monotony and expose the employee to a broader perspective of the entire production process.
- *Job enlargement*: because of boredom and job dissatisfaction, many workers withdraw from the organization, or the organization suffers from high levels of tardiness, absenteeism and resignations. If the managers would reduce the ill effects of simplified, too specialized jobs. Job enlargement offers the employee four opportunities.

(*a*) Variety
(*b*) Autonomy

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Ans. • Several workers may perform as a unit, thus forming a team-assembly operation. These teams may have one standard for the teams output. By adding all the individual and team standards managers can set department standards for quantity, quality, costs and delivery dates etc.

• By combining the various departmental standards operations manager can establish plant standards in terms of a specified targets like volume of goods or services that must be produced. Labour, materials and cost standards must be maintained.

2.110 Which standards can earn more benefits in service sector?

Ans. Since the service sector is more labour intensive it could benefit most from labour time standards.

2.111 What is 'Labour efficiency variance'?

Ans. • Labour efficiency variance = standard costs –actual costs.

Ex: A manufacturing firm introducing a new product sets a preliminary labour standard at 10 units/hour. The standard labour rate is Rs. 8 per hour. During the third month of production 800 units were produced using 90 labour hours. Find the labour efficiency variance?

• Standard cost = 0.10 hours/unit × 800 units × 8 rupees/hour

= Rs. 640

Actual cost = 90 hours × 8 rupees/hour

= Rs. 720

Therefore labour efficiency variance = 640–720 = Rs. 80

Negative sign indicates actual costs dominating standard costs.

EXERCISE

1. How productivity is related to work method design?
2. Define method study?
3. What different equipment may be needed for conducting method study?
4. Explain briefly the steps involved in method study?
5. Differentiate between cyclegraph and chrono cyclegraph?
6. Explain micromotion study?
7. Explain briefly the steps involved in method study?

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8. Explain man-machine system? How to analyse such systems using man-machine chart?
9. What is memo-motion study?
10. Clearly explain 'confidence level', precision, and accuracy to rely on the collected data?
11. What is work measurement? List out various techniques available?
12. Explain what is rating? Also explain different methods of rating?
13. Explain normal time, standard time and allowances? Establish the relation between them?
14. What is work sampling?
15. Explain the procedure of work sampling?
16. How do you rate the worker doing the job?
17. What are Predetermined Motion Time Systems (PMTS)?
18. What is the result of summing up elemental times of all manual elements of a job using any PMTS?
19. Why don't we rate the machines?
20. What is ergonomics?
21. What techniques you suggest for reducing monotony of jobs?
22. What is bio-energetics?
23. Explain the applications of ergonomic equipment like cycle-ergometer, anthropometer, Pulse readers etc. in ergonomic design of job?

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3

Job-Evaluation and Merit Rating

Organization may consist of hundreds of jobs and thousands of workers. It is necessary to gratify jobs based on which pay can be fixed for various lelels of jobs. In this chapter, different methods of job evaluation are presented. The meaning of merit rating and it's methods are also explained.

3.1 What is Job evaluation?

Ans. Job evaluation is concerned with job contents or demands of the job and not the 'value' of the job to the organization. Job evaluation rates the job and not the man. Job evaluation is a disciplined judgement about the hierarchical positioning of the job.

3.2 What is the need of Job evaluation?

Ans. Due to changes in technology, methods, procedures, systems and structure of the organization, changes in the job contents take place. Hence, different jobs are to be evaluated for hierarchical positioning.

3.3 What are the basic systems/methods available for job evaluation?

Ans.
- Job evaluation can be basically of two types:
  1. Comparing a whole job against other jobs.
  2. Comparing compensible elements of the job to a predetermined yardstick.
- Under the first type, two methods are available.
  (1.a) Ranking method
  (1.b) Grading or classification system
- Under the second type, two methods are available
  (2.a) Points systems
  (2.b) Factor comparison system.

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3.4 What are the benefits associated with job evaluation?

Ans. 1. Job evaluation helps to eliminate wage inequalities.
2. It helps to establish sound wage differentials among the jobs.
3. To establish a sound wage structure.
4. To minimize labour unrest and to improve job satisfaction.

3.5 What are the steps involved in job evaluation?

Ans.
- Determining the range of jobs to be evaluated.
- Analyzing each job.
- Evaluating each job
- Fitting the job into wage structure.

3.6 After identifying range of jobs to be evaluated, each job has to be analyzed in job evaluation? What is job analysis?

Ans. Job analysis may be defined as the process of determining the information relating to the nature of a job. It is the determination of tasks which comprise the job and of the skill, knowledge, abilities and responsibilities required for the worker for successful job performance.

Job anasysis consists of two aspects:
1. Job description
2. Job analysis.

Job description is the systematic outline of the information obtained during a study. It describes the work performed, the responsibilities involved, skill and training required, the conditions under which the job is done. Job analyst can get information by questioning the works.

Job specifications refer to a summary of the personal characteristics required for a job. It describes the type of employee required informs of skill, knowledge, experience and aptitude etc., and outlines the working conditions which are necessary for performing the job.

3.7 How does the 'Job description Form' look like?

Ans.

Job Description Form	
Department	Standard Code
Sub division	Standard Title
Branch of plant	Plant Title
Date	Plant code

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because of some bias based on the general impression formed by him about an employee. It is minimum in 'check—list method'.

3.17 What is the logic behind 'Rating Scale Method' for rating employees?

Ans. It involves (*a*) Identification of merit factors or standards to rate the employees. e.g. Standard output, intelligence, Job knowledge, dependability etc. (*b*) Dividing each factor into different levels like excellent, good, fair, unsatisfactory etc. (*c*) Assign some points to each grade and total them for all his merit factors.

3.18 Difference between Job evaluation and merit rating?

Ans. Job evaluation rates the jobs not employees or workers. Merit rating rates employees.

EXERCISE

1. Explain what is job evaluation?
2. List out methods of job evaluation?
3. Explain the relative merits and demerits of different methods of job evaluation?
4. If the organization consists of large number of jobs, what methods you advice for evaluation of all jobs?
5. What is merit rating?
6. Explain relative merits and demerits of various methods of merit rating?
7. Clearly explain the differences between job evaluation and merit rating?
8. What is Halo effect in rating procedures?
9. What is the differences between merit rating and performance rating?
10. Will you consider the speed of a worker in merit rating procedure?

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PART II

Optimization Techniques

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4

Basics of Optimization and Linear Programming Techniques

4.1 INTRODUCTION

4.1 What is Optimization?

Ans. Optimization is the process of finding the conditions that give the maximum or minimum value of a function.

The ultimate goal of technological or managerial decisions is either to minimize the efforts required to maximize the desired benefit.

The optimum seeking methods are also called mathematical programming techniques.

4.2 What are the important milestones in the development of Operations Research?

Ans. The development of Simplex method by Dantzig in 1947 for linear programming problems and the annunciation of the principle of optimization in 1957 by Bellman for dynamic programming problems paved the way for the development of the methods of constrained optimization.

4.3 Write the statement or general mathematical model of an Optimization problems?

Ans. An optimization or mathematical programming problem can be stated as follows:

$$\text{Find } x = \begin{Bmatrix} x_1 \\ x_2 \\ : \\ : \\ x_0 \end{Bmatrix} \begin{matrix} \text{which minimizes } f(x) \\ \\ \\ \\ \text{Subject to the constraints} \end{matrix}$$

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$$g_j(x) \leq 0, J = 1, 2, \ldots m$$
$$l_J(x) = 0, J = 1, 2, \ldots p$$

Where x is an n-dimensional vector called the design vector, $f(x)$ is the objective function and $gj(x)$ and $lj(x)$ are known as inequality and equality constraints, respectively. The number of design variables and the number of constraints are no way related to each other.

4.4 What is a Linear programming problem?

Ans. If the objective function and the constraints are in the linear form, then the optimization problem is called an LP problem.

4.5 Some of the pioneers who have contributed different OR techniques?

Ans. (*a*) Different calculus methods: Newton and Libnotz

(*b*)	Steepest descent method for constrained minimization	: Cauchy
(*c*)	Simplex method for LP	: Dantzig
(*d*)	Dynamic programming	: Bellman
(*e*)	Kuhn – tucker conditions for constrained NLP	: Kuhn and Tucker
(*f*)	Geometric programming	: Gomery
(*g*)	Integer programming	: Gomory
(*h*)	Stochastic programming techniques	: Dantzig, Charnes and Cooper
(*i*)	Game theory	: Von Newmann
(*j*)	Goal programming for multi objectives	: Charnes and Cooper
(*k*)	Transportation algorithms	: Hitchcock and Koopman

4.2 GENERAL FRAMEWORK FOR LINEAR PROGRAMMING

4.6 What are the basic requirements of LP problems?

Ans. LP problem should:

- Consist of well defined objective function. It can be either maximizing the function or minimizing the function.
- Consist of Limited resources. If resources are not limited, there is no need of optimizing the business situation.
- Consist of large number of inter-related decision variables.

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- Consist of alternative courses of action.
- Consist of any linear objective function and a set of linear constraints.
- LP problem should consist of decision variables, which are of non-negative in nature.
- Consist of deterministic technological coefficients, profit or cost coefficients and the availability of resources.

4.7 What are the techniques available for LP problems?

Ans. If an LP problem consists of only two decision or design variables, then Graphical method can be employed to find out the solution. If an LP problem consists of more than two variables, simplex method can be applied.

4.8 What is called a 'Feasible solution'?

Ans. The values of design variables that satisfy the 'constraints set constitute a feasible solution.

4.9 Define a 'Basic feasible solution' (BFS)?

Ans. The values of the design variables that satisfy the constraints set as well as the non-negativity restriction constitute a BFS.

4.10 What is an Optimal solution?

Ans. A basic feasible solution that gives the maximum value of the objective function (Maximization case) or minimum value of the objective function (in the case of minimization problems) is called an optimal solution.

4.11 Write the Scalar form of LP problem?

Ans. Minimize $f(x_1, x_2, \ldots x_n) = c_1 x_1 + c_1 x_2 + \ldots + c_n x_n$

Subject to the constraints

$$\begin{array}{l} a_{11}x_1 + a_{12}x_2 + \ldots\ldots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots\ldots + a_{2n}x_n = b_2 \\ \vdots \qquad\qquad \vdots \qquad\qquad \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots\ldots + a_{mn}.x_n = b_m \end{array}$$

and $x1, x2, x3 + \ldots\ldots xn \geq 0$

Where Cj, bj and aij ($i = 1, 2, \ldots m$, $j = 1, 2, \ldots n$) are known as technological coefficients and xj are the decision variables. Above form is also called 'Standard form' of LPP.

4.12 Write the matrix form of LP problem?

Ans. Minimize $f(x) = c^T X$

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Subject to the constraints $AX = b$

$x_i \geq 0$ where

$$x = \begin{Bmatrix} x1 \\ x2 \\ : \\ : \\ xn \end{Bmatrix};\ B = \begin{Bmatrix} b1 \\ b2 \\ : \\ : \\ bm \end{Bmatrix};\ C = \begin{Bmatrix} c1 \\ c2 \\ : \\ : \\ cn \end{Bmatrix}$$

$$A = \begin{pmatrix} a11 & a12 & a13 & \ldots & a1n \\ a21, & a22, & a23, & \ldots & a2n \\ : \\ : \\ am1, & am2, & am3, & \ldots & amn \end{pmatrix}$$

4.13 What are the characteristics of LP problem in the standard form?

Ans. Objective functions is of the minimization type, all constraints are of equality type and all the decision variables are non-negative. Any LP problem can be expressed in the standard form by using the transformations.

4.14 What is a Convex polygon?

Ans. A convex polygon consists of a set of points having the property that the line segment joining any two points in the set is entirely in the convex polygon. In problems having more than two decision variables, the feasible region is called a convex polyhedron.

4.15 How do you define Vortex or Extreme point?

Ans. Vertex is a point in the convex set that does not form a line segment joining two other points of the set. For example, every point on the circumference of a circle and each corner point of a polygon can be called a vertex or extreme point.

4.16 State some important theorems of Linear programming?

Ans. The intersection of any number of convex sets is also convex.
The feasible region of a LP is global/convex set.
Any local minimum solution is global for a LP problem.
Every basic feasible solution is an extreme point of the convex set of feasible solutions.

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4.17 What is Hyperplane?

Ans. In n-dimensional space, the set of points whose co-ordinates satisfy a linear equation.

$a1x1 + a2x2 + \ldots + anxn = b$ is called a hyperplane.

4.18 Write two different forms of Linear Programming?

Ans. (*a*) Canonical form of LPP. If the general formulation of LP problem is expressed as Maximize

$$\Sigma c_j x_j,\ j = 1 \text{ to } n$$

Subject to

$$\Sigma a_{ij}\, x_j \le bi,\ i = 1, 2, \ldots m$$

and $$x_j \ge 0;\ j = 1, 2, \ldots n$$

Then it is called the canonical form of LP problem.

(*b*) Standard form of LPP

If the general formulation of LP problem is expressed as

Maximize $$z = \sum_{j=1}^{n} c_j x_j$$

$$s/t\ \sum_{j=1}^{n} gij\ x_j = bi\ i = 1, 2, \ldots, m$$

$$x_j \ge 0;\ j = 1, 2, 3, \ldots, n$$

4.19 What is a basic solution?

Ans. For set of m equations in n variables ($n > m$), a solution obtained by setting $(n - m)$ variables to zero and solving for remaining 'm' equations in 'm' variables is called a basic solution.

The variables whose values did not appear in the solution are called non-basic variables and the variables, which appear in the solution, are called basic variables.

4.3 ILLUSTRATION OF GRAPHICAL METHOD TO SOLVE A GIVEN LP PROBLEM

4.20 Max Z = 15 × 1 + 10 × 2 subject to

4 × 1 + 6 × 2 ≤ 360

3 × 1 + 0. × 2 ≤ 180

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All points lying on the line segment BC represent optimal solutions of the problem yielding the same optimal value, Z = 360. ∴ Either of the B (60, 20) or C (30, 40) can be chosen as the optimal solution.

4.23 Explain Infeasitbility in connection with graphical technique of LPP?

Ans. In some LP problems, there may be a situation where there is no feasible region i.e., there are no points that satisfy all the given set of constraints. Whatever is the form of the objective function, there is no feasible solution to the LP problem in this situation.

4.24 Explain the Case of unboundedness in LP problems?

Ans. In an unbounded LP problem, one or more decision variables will increase indefinitely without violating feasibility and the value of the objective function can be increased indefinetely.

Ex.: Max $Z = 3 \times 1 + 5 \times 2$ subject to

$2 \times 1 + x2 \geq 7$

$x1 + x2 \geq 6$

$x1 + 3x2 \geq 9$

$x1, x2 \geq 0$

The feasible region will be above the lines represented by the constraint lines. To increase the value of the objective function, the values of x1 and x2 can be increased indefinetely in the infinite feasible solution space. It is not possible to get the bounded solution to get the maximum value of objective function.

4.5 PROCEDURE OF SIMPLEX METHOD FOR MAXIMIZATION PROBLEMS

'Simplex Method' can be applied to solve any type of LP problem, irrespective of the number of decision variables, and irrespective of the nature of linear inequalities i.e., = or ≤ or ≥ or mixed set of constraints. Hence it is called as 'Universal Technique' to solve LP problems.

4.25 Explain the procedure taking an illustration.

Max $Z = 45 \times 1 + 80 \times 2$ subject to

$5 \times 1 + 20 \times 2 \leq 400$

$10 \times 1 + 15 \times 2 \geq 450$

$x1, x2 \geq 0$

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Ans. *Step* 1: Convert the given LP problem into standard form by introducing slack variables S1 and S2, one to each constraint. Since the slack variables physically mean the unused resources, profit co-efficients on those unused resources are treated as zeros. The problem is modified as

Max $Z = 45 \times 1 + 80 \times 2 + 0.\ S1 + 0.\ S2$ subject to

$$5 \times 1 + 20 \times 2 + S1 = 400$$

$$10 \times 1 + 15 \times 2 + S2 = 450$$

and $x1, x2, S1, S2 \geq 0.$

Step 2: Check the value of slack, variables whether they satisfy non-negativity restirction or not when the values of design variables are treated as zeros.

Step 3: Represent the information in the standard form of LP in the form of a Simplex table. The format of the Simplex table is as follows.

Table 1

		Co-efficients of					
Ci	*Basis*	*x*1	*x*2	*S*1	*S*2	*bi*	*RR*
*0	*S*1	5	20	1	0	400	$\frac{400}{20} = 20 \leftarrow$
0	*S*2	10	** 15	0	1	450	$\frac{450}{15} = 30$
	CJ	45	80	0	0		
	ZJ	0	0	0	0		
	Cj – *Zj*	45	↑ 80	0	0		

* Key Row ** Key column

In the initial solution, i.e., $x1 = 0; x2 = 0$, the values of slack variables will be positive. This is the worst solution possible. We start with this solution and improvize further. Slack variables will represent the basis here whose profit co-efficients are zeros and are shown in 'C_i' column. The co-efficients of the variables of different constraints are shown in different rows. The bi-colomn shows the availabilities of resources. The co-efficient of the variables in the objective function are shown in C_j row. The vector sum of the products of '*Ci*' with different columns are shown in *Zj* row.

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Since $Cj - Zj$ represent the net contributions corresponding to different variables $x1, x2 \ldots, S1, S2 \ldots$, The entering variable with the highest net contribution has to be chosen and identified the key column. Since $x2$ column consists of maximum, $Cj - Zj$ element, $x2$ is selected as the entering variable and the $x2$ column in marked as key column.

Step 4: To find out the departing variable, replacement ratio = (Availability)/(key column element) is to be calculated for each row. We have to choose the row (or departing variable which yields in less units of production (Replacement Ratio) if it retains in the basis or solution. The row or variable corresponding to the minimum positive replacement ratio is chosen as the key row. The intersection of key column and key row is called 'key element'. Since $S1$ row consists of least positive replacement ratio, it is chosen as the departing variable (key row). The intersection element '20' is called 'key element'.

Step 5: In the next iteration, $S1$ the departing variable is replaced by the entering variable $x2$ and corresponding profit contribution = 80 will be displayed in 'Ci' column. The elements of the previous key row will be changed. New elements of the key row will be obtained by dividing the original elements by key element.

Table 2

		Co-efficients of					
Ci	*Basis*	*x1*	*x2*	*S1*	*S2*	*bi*	*RR*
80	$x2$	1/4	1	1/20	0	20	80
0	$S2$	25/4	0	–3/4	1	150	24
	CJ	45	80	0	0		
	ZJ	20	80	4	0		
	$Cj - Zj$	25↑	0	–4	0		

The elements of other rows are to be calculated according to the formula

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New element =

$$\text{Original element} - \left[\frac{\begin{pmatrix}\text{Key column element}\\ \text{in that row}\end{pmatrix}}{(\text{Key element})} * \begin{pmatrix}\text{Vertically}\\ \text{positioned}\\ \text{key row}\\ \text{element}\end{pmatrix}\right]$$

Cj – *Zj* row is evaluated in the usual way. Again the key column and key row are selected following the criteria given in steps 3 and 4.

Step 6: Now iterations can be carned out till we get all the elements in *Cj* – *Zj* row as zeros or negative values. If we reach a stage or iteration where *Cj* – *Zj* are either zeros or negative values, it is an indication of optimality as it won't result in any positive contribution to the objective function by choosing any variable as entering variable.

Table 3

				Co-efficients of			
Ci	Basis	*x*1	*x*2	*S*1	*S*2	*bi*	RR
80	*x*2	10	1	2/25	–1/25	14	
45	*x*1	1	0	–3/25	4/25	24	
	CJ	45	80	0	0		
	ZJ	45	80	1	4	2200	
	Cj – *Zj*	0	0	–1	–4		

Since all elements in *Cj* – *Zj* row are turned out to be zeros and negative values, optimality is reached. The optimal value of the objective function is *ZJ* under availability column. The values of the decision variables in the basis can be read directly in availability column.

$$\therefore \quad x1 = 24$$

$$x2 = 14$$

$$Zj = 2200$$

4.26 Explain the procedure/criteria for selecting the entering variable and departing variable in the minimization problems?

Ans. Since the *Cj* row consists of cost co-efficients in the objective function, *Cj* – *Zj* row indicates net cost elements. The variable or column with the least or minimal or most nega-

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4.7 CHARNE'S METHOD (OR) BIG-M METHOD

4.37 Use charne's method (or) big-M method to solve the following: LP problem, Minimize

$$Z = 3X_1 + 4X_2 + X_3 + 6X_4 \text{ s/t constraints}$$

$$5X_1 - 2X_2 + X_3 - 3X_4 \geq 2$$

$$6X_1 + X_2 - 5X_3 - 3X_4 \geq 5$$

$$-X_1 + 4X_2 + 3X_3 + 7X_4 \geq 6$$

and $X_j \geq 0$

$j = 1$ to 4.

Ans. Adding slack variables to convert inequalities to equality constraints, constraints become,

$$5X_1 - 2X_2 + X_3 - 3X_4 = 2 + S_1$$

$$6X_1 + X_2 - 5X_3 - 3X_4 = 5 + S_2$$

$$-X_1 + 4X_2 + 3X_3 + 7X_4 = 6 + S_3$$

$$X_1 = X_2 = X_3 = X_4 = 0$$

$$\Rightarrow \quad S_1 = -2,\ S_2 = -5,\ S_3 = -6.$$

The values for slack variables are negative i.e. this solution violates non-negativity restriction. Hence artificial variables are to be added to the right hand side of the equations yielding negative values for slack variables. Artificial variables represent artificial resources, unit costs of that are assumed biggest positive quantity says 'M'. The objective function

Min $Z = 3X_1 + 4X_2 + X_3 + 6X_4 + O.S_1 + O.S_2 + O.S_3$ is also to be modified with the introduction of artificial variables.

Since all slack variables are *–ve*, A_1, A_2, A_3 artificial variables ae introduced left hand side of each equation and in the objective function, coefficients for artificial variables are M. Therefore the LP becomes,

Min $$Z = 3X_1 + 4X_2 + 6X_4 + O.S_1 + O.S_2 + O.S_3 + M.A_1 + M.A_2 + M.A_3$$

S/t constraints

$$5X_1 - 2X_2 + X_3 - 3X_4 - S_1 + A_1 = 2$$

$$6X_1 + X_2 - 5X_3 - 3X_4 - S_2 + A_2 = 5$$

$$-X_1 + 4X_2 + 3X_3 + 7X_4 - S_3 + A_3 = 6$$

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By putting

$X_1 = X_2 = X_3 = X_4 = S_1 = S_2 = S_3 = 0$. The values of artificial variables become $A_1 = 2$, $A_2 = 5$ and $A_3 = 6$, all are positive. Artificial variables will represent the basis in the first simplex table since they are the representatives of the resources. Representing the data in the format of simplex table. (See pages 68-69.)

Table 1: Since M is the biggest positive quantity, 3–10M is the smallest quantity. Therefore X_1 column is chosen as the key column. Minimum positive replacement ratio is associated with the first row, hence first row is chosen as the key row.

Since all the values in the net cost row in table 4 are either zeros or positive values, it is an indication of optimality. Hence optimal solution is

$$X_1 = 75/83,\ X_2 = 120/83,\ X_3 = 31/83$$

and $$X_4 = 0$$

$$\therefore \qquad \text{Min } Z = 3 * \frac{75}{83} + 4 * \frac{120}{83} + 1 * \frac{31}{83} = \frac{736}{83}.$$

4.8 TWO PHASE SIMPLEX METHOD

4.38 With the help of an illustration, explain the procedure of two phase simplex method?

Ans. In the first phase, the objective function, an expression that consists of only artificial variables is to be minimized. If the given LP problem consists of feasible solution, all artificial variables will be eliminated from the basis before reaching the optimal simplex table of the first phase.

If atleast one artificial variable exists in the basis of last table of 1 phase, it means that the LP problem doesn't consist of feasible solution and can be terminated at this stage itself. If the basis column is free from artificial variables. It is an indication that there exists an optimal solution for the LP problem.

In the optimal table of phase 1, *Cj* row is replaced by actual objective function co-efficients and the problem is iterated further in search of optimum solution in the usual simplex procedure.

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Iterations for the question 4.37

Table 1

Ci	Basis	X_1	X_2	X_3	X_4	S_1	S_2	S_3	A_1	A_2	A_3	bj	RR
M	A_1	5	–2	1	–3	–1	0	0	1	0	0	2	2/54
M	A_2	6	1	–5	–3	0	–1	0	0	1	0	5	6/5
M	A_3	–1	4	3	7	0	0	–1	0	0	1	6	–ve
	C_j	3	4	1	6	0	0	0	M	M	M		
	Z_j	10M	3M	–M	M	–M	–M	–M	M	M	M		
	Cj–Zj	3–10M ↑most negative	4–3M	1+M	6–M	M	M	M	0	0	0		

Table 2 Replace A_1 by X_1 in the basis column

Ci	Basis	X_1	X_2	X_3	X_4	S_1	S_2	S_3	A_1	A_2	A_3	bj	RR
3	X_1	1	–2/5	1/5	–3/5	–1/5	0	0	1/5	0	0	2/5	–ve
M	A_2	0	17/5	–31/5	3/5	6/5	–1	0	–6/5	1	0	13/5	13/17←
M	A_3	0	18/5	16/5	32/5	–1/5	0	–1	1/5	0	1	32/5	2
	C_j	3	4	1	6	0	0	0	M	M	M		
	Z_j	3	$\frac{35M-6}{5}$	$\frac{3-15M}{5}$	$\frac{35M-9}{5}$	$\frac{-M-3}{5}$	–M	–M	$\frac{3-M}{5}$	M	M		
	Cj–Zj	0	$\frac{26-35M}{5}$ ↑	$\frac{15M+2}{5}$	$\frac{34-35M}{5}$	$\frac{3+M}{5}$	M	M	$\frac{6M+3}{5}$	0	0		

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Table 3 Replace A_2 by X_2

Ci	*basis*	X_1	X_2	X_3	X_4	S_1	S_2	S_3	A_1	A_2	A_3	*bj*	*RR*
3	X_1	1	0	−9/17	−9/17	−1/17	−2/17	0	1/17	2/17	0	12/17	−ve
4	X_2	0	1	−31/17	3/17	6/17	−5/17	0	−6/17	5/17	0	13/17	−ve
M	A_3	0	0	166/17	98/17	$\frac{-25}{17}$	$\frac{18}{17}$	−1	25/17	$\frac{-18}{17}$	1	62/17	$\frac{31}{83}$ ←
	C_j	3	4	1	6	0	0	0	M	M	M		
	Z_j	3	4	$\frac{166M-151}{17}$	$\frac{98M-15}{17}$	$\frac{21M-25M}{17}$	$\frac{18M-26}{17}$	$-M$	$\frac{21M-8M}{17}$	M			
	$Cj-Zj$	3	$\frac{168-166M}{17}$ ↑	$\frac{177-98M}{17}$	$\frac{25M-29}{17}$	$\frac{26-18M}{17}$	$-M$	$\frac{21-8M}{17}$	$\frac{35M-26}{17}$	0			

Table 4 Replace A_3 by X_3

Ci	*basis*	X_1	X_2	X_3	X_4	S_1	S_2	S_3	A_1	A_2	A_3	*bj*	*RR*
3	X_1	1	0	0	−18/83	−23/166	−5/83	−9/166	23/166	5/83	9/166	75/83	
4	X_2	0	1	0	104/83	13/166	−8/83	−31/166	−13/116	8/33	31/166	120/83	
1	X_3	0	0	1	49/83	−25/166	9/83	17/166	25/166	−9/83	17/166	31/83	
	C_j	3	4	1	6	0	0	0	M	M	M		
	Z_j	3	4	1	411/83	−21/83	−38/83	−67/83	21/83	38/83	67/83		
	$Cj-Zj$	0	0	0	87/83	$\frac{21}{83}$	$\frac{38}{83}$	$\frac{67}{83}$	$\frac{21}{83}$	$\frac{38}{83}$	$\frac{67}{83}$		

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Sufficient conditions:

If the hessian Martix J is positive definite, then the stationery point corresponds to the minimum of $f(x)$. If the Hessian matrix is negative definite, the stationery point corresponds to the maximum of (x).

What is a saddle point?

If $f(x, y,)$ is a function in two vaiables, the Hessian matrix may be neither positive nor negative definite at a point (x^*, y^*). The point (x^*, y^*) is called saddle point'. The saddle point corresponds to relative maximum or minimum of $f(x, y)$ with respect to one variable, the other variable being kept constant.

EXERCISE

1. Give the meaning of optimization?
2. Write the general format of optimization problem?
3. Write the list of optimization techniques and their respective application enviroment?
4. How do you write the general form of Linear programming?
5. Explain iterations in simplex method?
6. Explain graphical approach and it's suitability for LP?
7. Clearly explain the differences between infeasibility, unboundedness, alternate optimal solutions in LP?
8. What is classical optimization?
9. What are the necessary and sufficient conditions for unconstrained multi variable problem?
10. What is dual LP?
11. Explain how to read the values of dual variables by solving primal?
12. Explain the procedure of reading values of primal variables from dual problem?
13. What are shadow prices in connection with duality?
14. What are the disadvantages of LP?
15. Differentiate between LP and Non linear programming?

ADDITIONAL PROBLEMS AND SOLUTIONS IN LINEAR PROGRAMMING

For Practice

General form of LP Problems:

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$$\text{Min (or) Max } Z = f(X) = \sum_{j=1}^{n} C_j X_j \text{ s/t constraints}$$

$$\Sigma a_{ij} X_j \leq \text{or } \geq$$

or $= bi$ where $i = 1$ to m; $j = 1$ *tn* n

and $X_j \geq 0$ are decision variables

C_j are cost coefficients or profit coefficients

b_i are availability of resource i.

4.10 MODELLING AND GRAPHICAL SOLUTIONS

4.44 Example: A furniture company produces tables and charirs which involve mainly manual job of rubbing, finishing and painting operations in sections. *A*, *B* and *C* respectively. The manual availability of these sections, unit manufacturing time and unit manufacturing, profit per unit are shown in the following table.

Product	*Sections*			*Profit/Unit*
	A hr/unit	*B hr/unit*	*C hr/unit*	
Tables	10	7	2	Rs.12
Chairs	2	3	4	Rs. 3
Total hours available for work	100	77	80	

Various feasible objectives are:

(*i*) To maximize weekly profit

(*ii*) To achieve maximum weekly production

(*iii*) Utilize man power resources to the fullest extent.

Formulate the problem with each of these objectives.

Ans. Let X_1 be the number of tables to be produced

Let X_2 be the number of tables to be produced

Maximize profit (objective funciton)

Max $Z = 12X_1 + 3X_2$

(i) Subject to constraints,

Section *A*, $10X_1 + 2X_2 \leq 100$

Section *B*, $7X_1 + 3X_2 \leq 77$

Section *C*, $2X_1 + 4X_2 \leq 80$

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(ii) Maximize production $Z = X_1 + X_2$

Subject to constraints $10X_1 + 2X_2 \leq 100$

$7X_1 + 3X_2 \leq 77$

$2X_1 + 4X_2 \leq 80$

and non negativity constraint $X_1, X_2 \leq 0$

(iii) Maximize man hour consumption

$Z = 10X_1 + 7X_1 + 2X_1 + 2X_2 + 3X_2 + 4X_2 = 19X_1 + 9X_2$

Subject to constraints $10X_1 + 2X_2 \leq 100$

$7X_1 + 3X_2 \leq 77$

$2X_1 + 4X_2 \leq 80$

and non negativity constraint $X_1, X_2 \leq 0$

4.45 Example: A farm is engaged in breeding pigs. The pigs are fed on vaious products grown on the farm because of the need to ensure certain nutrients control unit, necessary to buy on (or) two products which we shall call as A (or) B. The nutrients contitute in each unit of product as given below:

Nutrients type	*A*	*B*	*Minimum amount of nutrients*
1	36	6	108
2	3	12	36
3	20	10	100

Product 'A' costs 20 Rs/unit and B costs 40 Rs/unit. Obtain the LP formulation of the problem so as to decide how much of the product of A and B should be purchased in order to provide the pigs with minimum required nutrients at the lowest possible cost?

Ans. Let X_1 be the purchase quantity of *A*

let X_2 be the purchase quantity of *B*

Min Z (cost) $= 20X_1 + 40X_2$

Subjected to constraints

$36\,X_1 + 6X_2 \geq 108$

$3\,X_1 + 12X_2 \geq 36$

$20\,X_1 + 10X_2 \geq 100$

and non negativity constraint X_1, X_2 ³ 0

4.46 Example: A certain company produces tea trays and ash trays in their press shop. The following data relates to the plant capacity restriction.

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Sl.No.	Plant capacity	X_1 Ash tray	X_2 Tea tray	Available capacity per day
1.	Stamping	10 sec	20 sec	30000 sec
2.	Forming	15 sec	5 sec	30000 sec
3.	Printing	10 sec	8 sec	40000 sec
	Sales revenue/unit	Rs 1.00	Rs. 1.50	
	Variable cost/unit	Rs 0.80	Rs. 1.20	

Daily fixed cost is Rs. 450. Determine the optimum production schedule and check whether it is economical to start production.

Ans. Fixed cost is the component of cost associated with set up of equipment and facilities and is constant irrespective of production quantity.

Variable cost varies with production quantity ex. raw material consumption, labour, etc.,

Contribution/unit (Ash tray) = Rs. 1.00 – 0.80 = Rs. 0.20

Contribution/unit (Tea tray) = Rs. 1.50 – 1.20 = Rs. 1.30

Let X_1 and X_2 be the number of ash trays and no. of tea trays are produced respectively. Objective is to maximize contribution, $Z = 0.2\ X_1 + 0.3\ X_2$ subject to constraints

$10\ X_1 + 20X_2 \leq 30000$ (stamping capacity constraint)

$15\ X_1 + 5X_2 \leq 30000$ (forming capacity constraint)

$10\ X_1 + 8X_2 \leq 40000$ (printing capacity constraint)

and non-negativity constraint $X_1, X_2 \geq 0$

Graphical Method:

1. $10X_1 + 20X_2 = 30000$

   Put $X_2 = 0 \Rightarrow X_1 = 3000$ (30000,0)

   Put $X_1 = 0 \Rightarrow X_2 = 1500$ (0,1500)

2. $15X_1 + 5X_2 = 30000$

   Put $X_2 = 0 \Rightarrow X_1 = 2000$ (2000,0)

   Put $X_1 = 0 \Rightarrow X_2 = 6000$ (0,6000)

3. $10X_1 + 8X_2 = 40000$

   Put $X_2 = 0 \Rightarrow X_1 = 4000$ (4000,0)

   Put $X_1 = 0 \Rightarrow X_1 = 5000$ (0,5000)

   OABC is the convex polygon formed. All the points in this region satisfy all the 3 restrictions.

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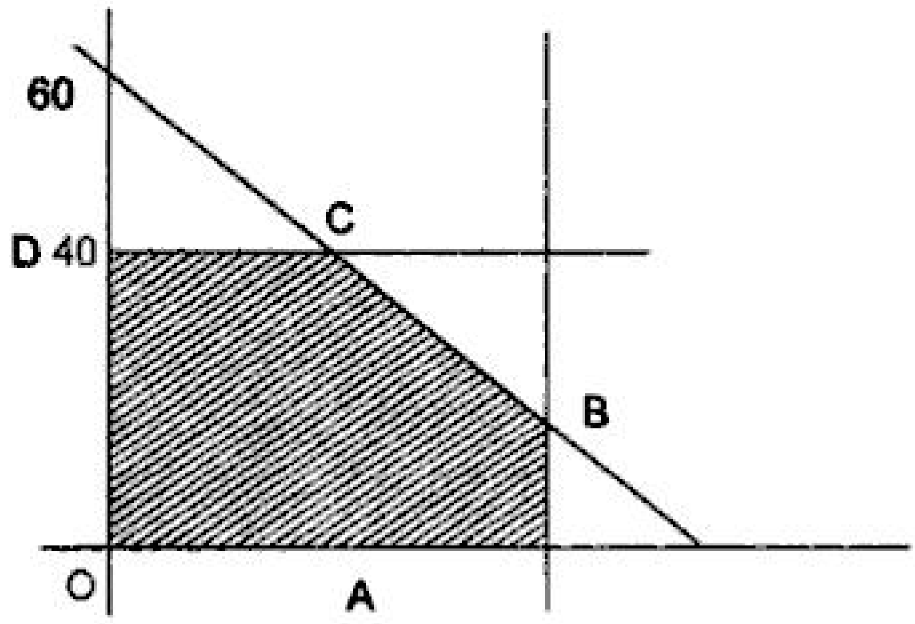

Hence (90,0) and (0,60) are extreme points of the line.

$$3X_1 + 0.X_2 = 180\} \text{ vertical line passes through (60, 0)}$$
$$X_1 = 60\}$$
$$0.X_1 + 5X_2 = 200\} \text{ horizontal line passes through (0, 40)}$$
$$X_2 = 40\}$$

0 (0, 0)	Z = 0
A (60, 0)	Z = 240
B (60, 20)	Z = 360
C (30, 40)	Z = 360
D (0, 40)	Z = 240

All the points along the *BC* line gives the same profit, so this objective function has more than one optimal point, which give the same maximum profit. Any point on the *BC* line gives the same profit. For these points profit (max) is Z = Rs. 360.

4.52 Case (2): An Unbounded Solution

Ans. If the value of the objective function increases indefinitely by moving away from the origin in the feasible region, then the LP is said to possess an unbounded solution.

Simplex Method

Procedure of Simplex Method Simplex method can be applied to solve any type of LP problems, irrespective of nature of inequalities i.e, ≤, ≥ (or) mixed set of constraints. Hence it is called an universal technique to solve LP problems.

1. Convert the given problem into standard form of LP by introducing 'Slack variables one to each constraint.

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Since the slack variables, physically, mean the unused resources, profit coefficient on these unused resources are treated as zeroes and the given model is modified as

$$\text{Max} \quad Z = 45X_1 + 80X_2 + O.S_1 + O.S_2$$

$$5X_1 + 2X_2 + S_1 = 400$$

$$10X_1 + 15X_2 + S_2 = 450$$

$$X_1, X_2, S_1, S_2 \geq 0$$

2. Check the values of slack variables whether they satisfy the non-negativity restriction or not when the values of design variables are assigned zeros.
3. If the values of all slack variables are '+ve' then solution is called initial basic feasible solution. In this case $X_1 = X_2 = 0, S_1 = 400$ and $S_2 = 450$ is called Basic Feasible Solution (BFS).

   Now represent the information in the standard form of LP in the simplex table. The format of simplex table is as shown in table (1).

   In initial solution, i.e., $X_1 = 0, X_2 = 0$ the values of slack variables are '+ve'. This is the worst feasible solution to be possible. We start with this solution and improve it further Slack variables will represent the basis in 1st table whose profit coefficients are zeros and are shown in C_j column. The coefficients of vaiables of different constraints are shown in different rows. The *bi* column shows the availability of resources. The coefficients of variables in objective funciton are shown in C_j row. The sum of the vector product of C_i with different columns are shown in Z_j row.
4. Since, C_j–Z_j gives the net contributions corresponding to different variables X_1, X_2, S_1, S_2 and so-on. The entering variable with the net highest variable has to be chosen and identify the key column. Since X_2 column consists max *CJ–ZJ* element, X_2 is selected as entering variable and X_2 column is marked as key column.
5. To find out the departing variable, replacement ratio (RR) which is equal to bi/key column is to be calculated for each row. We choose the row (or) departing variable in basis which yields in less units production (RR), If it retains in basis (or) solution. Hence the row

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or variable corresponding to the minimum '+ve' RR is chosen as the key row. The intersection of key row and key column is called key element. Since 'S_1'consists of minimum '+ ve' RR, it is chosen as departing variable [key row]. The intersectional element '20' is called key element.

6. In the table (2) 'S_1' the departing variable is replaced by the entering variable 'X_2'and corresponding profit coefficient '80' is displayed in C_i Column. Dividing the respective original elements by key element will change the elements of previous key row.

   The elements of other rows (apart from key row) are to be calculated according to the formula.

   New element = original element

   $\frac{\text{Key column element}}{\text{Key element}}$ X vertically positioned key row element.

   Cj-Zj is evaluated in the usual way. Again the key column and key rows are selected following the steps 4 and 5.

7. Now iterations can be carried out till we get all the elements of *CJ-Zj* as zeros (or) – 've' elements. If we reach the stage in any table where *CJ–ZJ* contains all zero's and –'ve' values, it is an indication of optimality as it will not result in any +'ve' contribution to the objective function by choosing any element. Since all Cj–Zj in the 3rd table are turned out to be zero's (or) –'ve' values, Optimality is reached. The optimal value of objective function ZJ is available under availability column. The values of decision variables in basis can be read directly in availability column.

4.12 A CLASS OF LP PROBLEMS WITH MAXIMIZATION AS OBJECTIVE FUNCTION AND ALL CONSTRAINTS ARE '≤ TYPE'.

4.53 Example: Max $Z = 45\,X_1 + 80\,X_2$ s/t constraints

$5X_1 + 20\,X_2 \leq 400$, $10X_1 + 15\,X_2 \leq 450$

and non negativity constraint $X_1, X_2 \leq 0$

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Ans. The constraint inequalities are converted to equalities by introducing slack variables as follows.

$5X_1 + 20X_2 + S_1 = 400$; $10X_1 + 15X_2 + S_2 = 450$ where S_1, S_2 are the slack variables which represent unused resources. coefficients of unused resources (slack variables) are zeroes. Therefore objective function is modified as

Max $Z = 45X_1 + 80X_2 + O.S_1 + O.S_2$

Represent the information in the standard form of the simplex format.

Iteration 1

Co-efficients of variables							*Availability*
Ci	*Basis*	X_1	X_2	S_1	S_2	B_i	*Replacement Ratio*
0	S_1	5	20	1	0	400	Bi/Key Column
0	S_2	10	15	0	1	450	400/20 =20
	C_j	45	80	0	0		Minimum + Ve
	Z_j	0	0	0	0		450/15 = 30
	C_j-Z_j	45	80	0	0		
			Highest				

Iteration 2

For second iteration, S_1 is replaced, by X_2 and the elements are of the key row can be obtained by dividing every element by key element.

New element} = Old element} – Key column in that row X in other row} in original position} Key element

{vertically positioned key row element for the element under consideration}

Ci	*Basis*	X_1	X_2	S_1	S_2	b_i	RR	
80	X_2	1/4	1	1/20	0	20	80	
0	S_2	25/4	0	–3/4	1	150	24	←Low
	C_j	45	80	0	0			
	Z_j	20	80	4	0			
	C_j-Z_j	25	0	–4	0			
		↑ Max						

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Table 2 Replace A_1 by X_1 in the basis column

Ci	Basis	X_1	X_2	X_3	X_4	S_1	S_2	S_3	A_1	A_2	A_3	bj	RR	
3	X_1	1	–2/5	1/5	–3/5	–1/5	0	0	1/5	0	0	2/5	-ve	
M	A_2	0	17/5	–31/5	3/5	6/5	–1	0	–6/5	1	0	13/5	13/17	←
M	A_3	0	18/5	16/5	32/5	–1/5	0	–1	1/5	0	1	32/5	2	
	C_j	3	4	1	6	0	0	0	M	M	M			
	Z_j	3	$\frac{35M-6}{5}$	$\frac{3-15M}{5}$	$\frac{35M-9}{5}$	$\frac{-M-3}{5}$	–M	–M	$\frac{3-M}{5}$	M	M			
	C_j–Z_j	0	$\frac{26-35M}{5}$ ↑	$\frac{15M+2}{5}$	$\frac{34-35M}{5}$	$\frac{3+M}{5}$	M	M	$\frac{6M+3}{5}$	0	0			

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Table 3 Replace A_2 by X_2

Ci	Basis	X_1	X_2	X_3	X_4	S_1	S_2	S_3	A_1	A_2	A_3	bj	RR
3	X_1	1	0	–9/17	–9/17	–1/17	–2/17	0	1/17	1/17	0	12/17	–ve
4	X_2	0	1	–31/17	3/17	6/17	–1	0	–6/5	1	0	13/5	–ve
M	A_3	0	0	166/17	98/17	$\frac{-25}{17}$	$\frac{18}{17}$	–1	25/17	$\frac{-18}{17}$	1	62/17	$\frac{31}{83}$ ←
	C_j	3	4	1	6	0	0	0	M	M	M		
	Z_j	3	4	$\frac{166M-151}{17}$	$\frac{98M-15}{17}$	$\frac{21-25M}{17}$		$\frac{18M-26}{17}$	M	$\frac{25M-21}{17}$	$\frac{26-18M}{17}$	M	
	C_j–Z_j	0	0	$\frac{168-166M}{17}$ ↑	$\frac{177-98M}{17}$	$\frac{25M-29}{17}$		$\frac{26-18M}{17}$	–M	$\frac{21-8M}{17}$	$\frac{35M-26}{17}$	0	

Table 4 Replace A_3 by X_3

Ci	Basis	X_1	X_2	X_3	X_4	S_1	S_2	S_3	A_1	A_2	A_3	bj	RR
3	X_1	1	0	0	–18/83	–23/166	–5/83	–9/166	23/166	5/83	9/166	75/83	
4	X_2	0	1	0	104/83	13/166	–8/83	–31/166	–13/166	8/33	31/166	120/83	
1	X_3	0	0	1	49/83	–25/166	9/83	17/166	25/166	–9/83	17/166	31/83	
	C_j	3	4	1	6	0	0	0	M	M	M		
	Z_j	3	4	1	411/83	–21/83	–38/83	–67/83	21/83	38/83	67/83		
	C_j–Z_j	0	0	0	87/83	$\frac{21}{83}$	$\frac{38}{83}$	$\frac{67}{83}$	$\frac{M-21}{83}$	$\frac{M-38}{83}$	$\frac{M-67}{83}$		

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S/t constraints:

$$6X_1 + 12X_2 \le 120$$
$$-8X_1 - 5X_2 \le -60$$
$$3X_1 + 4X_2 \le 15$$
$$-3X_1 - 4X_2 \le -15$$

Dual:

$$\min Z' = 120Y_1 - 60Y_2 + 15Y'_3 - 15Y''_3$$

S/t constraints:

$$6Y_1 - 8X_2 + 3Y'_3 - 3Y''_3 \ge 30$$
$$12Y_1 - 5X_2 + 4Y'_3 - 4Y''_3 \ge 40$$
$$\text{and } Y_1, Y_2, Y'_3, Y''_3 \ge 0$$

Table 1

Ci	*Basis*	Y_1	Y_2	Y'_3	Y''_3	S_1	S_2	A_1	A_2	*bj*	*RR*
M	A_1	6	–8	3	–3	–1	0	1	0	30	5
M	A_2	12	–5	4	–4	0	–1	0	1	40	10/3 ←min
	C_j	120	–60	15	–15	0	0	M	M		
	Z_j	18M	–13M	7M	–7M	–M	–M	M	M		
	C_j-Z_j	120–18M ↑min	13M–60	15–7M	7M–15M	M	0	0			

Table 2

Ci	*Basis*	Y_1	Y_2	Y'_3	Y''_3	S_1	S_2	A_1	A_2	*bj*	*RR*	S_1/ *Keycol*
M	A_1	0	–11/12	1	–1	–1	1/2	1	–1/2	10	10	–1 ← min
120	Y_1	0	–5/12	1/3	–1/3	0	–1/12	0	1/12	10/3	10	0
	C_j	120	–60	15	–15	0	0	M	M			
	Z_j	120	$\frac{-11M}{2}$	–50*M*+40–*M*	–40	–*MM*–10	$\frac{-M}{2}$	+10				
	C_j-Z_j	0	11M+10	–M	–25M+25 ↑min	M10–*M*/20	3*M*	–10				

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Table 3

C_i	Basis	Y_1	Y_2	Y_3	Y_4	S_1	S_2	A_1	A_2	bj	RR
15	Y_3	0	$\frac{-11}{2}$	1	–1	–1	$\frac{1}{2}$	1	$\frac{-1}{2}$	10	–ve
120	Y_1	1	$\frac{17}{2}$	0	0	$\frac{1}{3}$	$\frac{-1}{4}$	$\frac{-1}{3}$	$\frac{1}{4}$	0	←min+ve
	C_j	120	–60	15	–15	0	0	M	M		
	Z_j	120	$\frac{395}{2}$	15	–15	+25	$\frac{-45}{2}$	–25	$\frac{+45}{2}$		
	$C_j - Z_j$	0	$\frac{-515}{2}$ ↑min	0	0	–25	$\frac{45}{2}$	25+M	$\frac{-45}{2+M}$		

Table 4

C_i	Basis	Y_1	Y_2	Y_3	Y_4	S_1	S_2	A_1	A_2	bj	RR
15	Y_3	$\frac{66}{17}$	0	1	–1	$\frac{5}{17}$	$\frac{-8}{17}$	$\frac{-5}{17}$	$\frac{8}{17}$	10	
–60	Y_2	$\frac{12}{17}$	1	0	0	$\frac{4}{17}$	$\frac{-3}{17}$	$\frac{-4}{17}$	$\frac{3}{17}$	0	
	C_j	120	–60	15	–15	0	0	M	M		
	Z_j	$\frac{270}{17}$	–60	15	–15	$\frac{-165}{17}$	$\frac{-60}{17}$	$\frac{165}{17}$	$\frac{-60}{17}$		
	$C_j - Z_j$	$\frac{1770}{17}$ ↑min	0	0	0	$\frac{165}{17}$	$\frac{+60}{17}$	$\frac{M-165}{17}$		$\frac{M+60}{17}$	

$$Y' = 10 \qquad X_1 = \frac{165}{17}$$

$$Y_2 = 0, Y_1 = 0, Y''_3 = 0 \qquad X_2 = \frac{60}{17}$$

Min Z′ = 150

mix Z = 150.

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3. Delphi.
4. Market surveys.
5. Time series Ex: exponential smoothing.
6. Regression and correlation (causal models).
7. Econometric models.

6.6 Discuss the qualitative techniques of forecasting?

Ans. The techniques, which do not quantify the data and use mathematical models are called qualitative techniques. Some of the qualitative techniques are:

(*a*) Opinions and judgements
(*b*) Historical analogy
(*c*) Delphi
(*d*) Market survey etc.

(*a*) *Opinions and judgements:* The opinions and judgements of individuals who are expected to have best knowledge of current activities and future plans are collected, and used to plan the future. The cost is very less and used for situations difficult to model. **Example:** To know the size of contracts we obtain from the government next year etc.

(*b*) *Historical analogy:* This is based on the assumption that what happened in past will happen to repeat in future also. It compares the product with different stages in life cycle of comparable and competitive products and assumed to follow similar patterns. The cost is relatively medium and used for long range planning. **Example:** To predict sales of a new product.

(*c*) *Delphi Technique:* Series of questionnaires are prepared and issued to a panel of experts. Each expert has access to all information. Here it assumes that experts are knowledgeable. This technique also helps for long range planning. **Example:** To predict the requirements of new faculties for new products.

(*d*) *Market surveys:* It makes use of questionnaires, surveys, market panels for collecting data and testing the hypothesis about the consumer behaviour, assuming that surveys are reliable and representative. This is used for both short range and large range plans. **Example:** political forecasts before polls.

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7.7 Write a list of few Time-series models (Naive models) and their applications?

Ans. A time—series is a set of observations of some variable over time. The forecaster is intersted in determining how the series is dependent on time and in developing a means of predicting future levels with some degree of reliability. There are different methods to estimate the trend of a time series.

(*a*) Last period demand
(*b*) Arithmetic average
(*c*) Moving average
(*d*) Weighted moving average
(*e*) Simple exponential smoothing
(*f*) Free hand curves
(*g*) Method of least squares etc.

(*a*) *Last period demand:* Mathematically this can be expressed as

$F_t = D_{t-1}$ where F_t = forecast for period 't'

$D\,t - 1$ = actual demand in the period '$t - 1$'

i.e. last period's actual demand is the forecast for the subsequent period.

(*b*) *Arithmetic average:* In this method, forecast for tth period can be estimated as

$$F_{n+1} = \sum_{i=1}^{n} \frac{D_i}{n} = \frac{D1 + D2 + \ldots + Dn}{n}$$

Where $Fn + 1$ = Forecast for (n+1)th period
Di = Demand in ith period.
n = number of time periods.

Example: The following table represents the demad of a product for the last 6 months of a year.

Month	*Jan*	*Feb*	*Mar*	*Apr*	*May*	*June*
Demand	400	450	500	600	650	500

Use arithmetic average to forecast the sales for July

$$F7 = \frac{D1 + D2 + D3 + \ldots + D6}{6} = \frac{400 + 450 + \ldots + 500}{6}$$

$$= 517 \text{ units}$$

Forecast of July = 517 units.

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(*c*) *Moving average:* A moving averages are obtained by summing and averaging the values from a given number of peiods respectively each time deleting the oldest value and adding a new value.

$$MA_t = \sum_{t=1}^{n} \frac{D_{t-1}}{n}$$

Where MA_t = Moving average for *t*th period by considering the average of *t*–1 periods.

$D\,t-1$ = Demand in (*t*–1)th period

n = no. of periods for calculating averages

Example: A company uses a 5 months moving average to forecast next period's demand, past actual demand (in units) is shown in the tabel below:

Month	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
Actual Demand	100	110	130	100	105	120	135	95	115

1. Computer a simple 5-month moving average to forecast demand for month 10?
2. If the actual demand in the month of 10 is 120 units, what shall be the forecast for the month of 11 ?

Solution:

$F10$ = forecast for month 10 = $\frac{D5 + D6 + D7 + D8 + D9}{5}$

5 months moving average = $\frac{105 + 120 + 135 + 95 + +115}{5}$

= 114 units

ii. given demand for month 10 is 120 units.

$F11$ = forecast for 11th month = 5 month moving average up to 10th month.

$= \frac{120 + 135 + 95 + 115 + 120}{5}$ = 117 units

(*c*) *Weighted moving average:* Instead of considering the most recent moving average only to forecast, weightages will be given to few past moving averages.

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4. Accessibility to the markets is poor.
5. Educational and amusement centres are not available.
6. Expert/consultancy services are not available.
7. ancillary concerns cannot be found nearby.

7.11 Plant location case using Break Even Analysis: Location A would result in annual cost of Rs.300,000, variable costs of Rs. 63 per unit, and revenues of Rs. 68 per unit. Annual fixed costs at location B are Rs. 800,000, variable costs are Rs. 32 per unit, and revenues are Rs 68 per unit. Sales volume is estimated to be 25000 units per year. Which location is most attractive?

Ans. A Break Even Analysis is more helpful as the cost figures are given in terms of fixed costs, variable costs and revenue costs.

Break Even Quantity, V = Fixed costs (revenues per unit-variable cost per unit)

BEQ at location A, $V_A = 300{,}000/(68 - 63) = 60{,}000$ units

BEQ at location B, $V_B = 800{,}000/(68 - 32) = 22{,}222$ units

At the expected demand of 25000 units, profits (or losses) for two different locations are:

Profit = sales revenue–total costs

= Revenue per unit × production quantity – (fixed costs + variable costs per unit × production quantity)

Profit for location $A = 25000 \times 68 - (300{,}000 + 25000 \times 63)$

= – Rs 1,75,000 (loss).

Profit location $B = 25000 \times 68 - (800{,}000 + 25000 \times 32)$

= Rs 100,000.

Since the profit of Rs. 100,000 is evident for the sales volume of 25000 units at location *B*, location *B* is attractive and to be chosen.

7.12 If the distances are the main criteria for location problem i.e., instead of cost information, only distances and loads to be moved from location to location are given, then what are the techniques available for solving plant location problems?

Ans. (a) Simple median model and (b) Transportation algorithms can be applied to solve these types of location problems. Any new unit if to be located, it should be tested for economic feasibility. If an organization already has few production units and few ware houses then based on the

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supply capacities of vendors and destination requirements of warehouses, new location can be added as the last row considering transportation costs and capacity of that unit. The transportation problem is solved as usual by first finding initial solution in any of the methods viz, North-West corner rule or Least cost method or Vogel's approximation method and determining the optimal solution either by Modified Distribution method or Stepping Stone Method. All feasible locations can be tested by formulating transportation problem each time with one feasible location. The feasible location with minimum total shipping costs should be considered as an attractive one.

7.13 A sight is sought for a temporary plant to supply cement to three existing construction sites: *A*, *B* and *C*. The locations of the existing sites and the loads to be delivered to each are as follows:

Delivery site	*Coordinate locations* (x_i, y_i)	*Loads to new plant*	*Costs to move one load one mile Rs.*
A	(20,10)	22	10
B	(10, 40)	43	10
C	(40, 20)	36	10

Find the best site for the cement plant. What shipping cost will result?

Ans. Since the coordinates of the existing delivery sites are given, it is convenient to apply the median model to arrive at the coordinates of the optimal site.

Using the median model, the median load is 51. To find the coordinates of the new plant, we begin at zero on the grid and move in the x-direction to the location that receives the 51 st load. (see Fig.). At the first location B $(x = 10)$, 43 loads go to the location B. At the next location A $(x = 20)$, 22 loads go to the location A, and this location will recieve the median load of 51 st load. Hence optimal x-coordinate is 20. In a similar fashion, the median load in the y-direction is delivered to the location C $(y = 20)$. The resulting shipment costs are shown in the table and calculated to be Rs. 22, 300.

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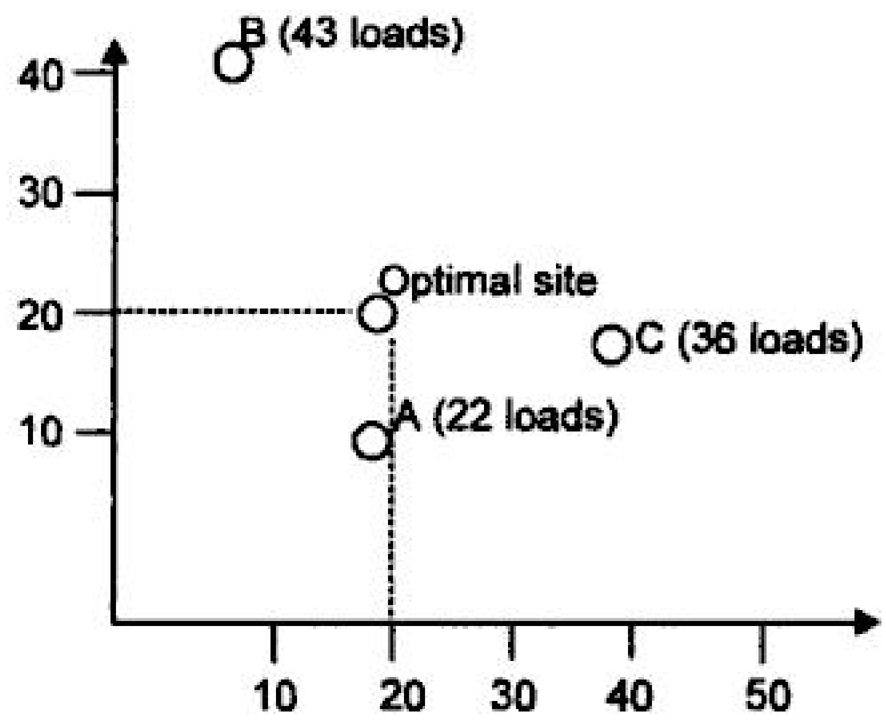

Location Grid

PLANT LAYOUT

7.1 What is plant layout problem? List out the objective of plant layout?

Ans. The arrangement of facilities and equipment within a plant or department is known as plant layout. The objectives of best layout include but not limited to the following:

1 Maximise the use of machine tools.
2 To minimise the labour requirements.
3 Provide smooth flow of materials.
4 Minimise accidents and health hazards.
5 Minimise material handling costs.
6 Maximise the output.
7 Maximise the utility of available space.
8 Provide maximum flexibility to avoid bottleneck operations and congestion troubles.

7.2 List out different types of layouts? Explain in brief the features of each?

Ans. (a) Process oriented layout (b) Product layout or line oriented layout (c) Fixed position layout (d) Combination layout (e) Group technology layout.

(a) *Process oriented layout*: Process oriented layouts are arrangements that group the people and equipment performing similar functions. These are also known as job shop layouts, because specific functions such as painting, X-raying, inspection, machining etc. are done

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8.7 What are Cycle Inventories?

Ans. *Cycle Inventories*: These are also called lot-size inventories as the item is purchased in lots instead of procuring it as and when it is required. This is being done to achieve economics in the inventory system.

8.8 What are the parameters to be evaluated for any inventory problem?

Ans. The operating doctrine/inventory decisions to be found out or to be taken are:

(*a*) Order quantity i.e., how much to be purchased/ordered.
(*b*) Re-order level (when should the order be placed) and
(*c*) Buffer stock to be maintained i.e., how much safety stock has to be maintained.

8.9 Explain the terminology used in inventory situation?

Ans. General Frame Work For Inventory Models:

Manufacture and/or order: The item may be manufactured inside the firm or may be purchased from out side agency. Both of the above is referred as on order.

Demand: We make the decision regarding order quantity, keeping in view the future demand of the item. A demand may be probabilistic or deterministic. In deterministic demand situation, one knows in advance with certainty the level of demand. **Ex.** 400 tons of steel will be needed for the month of August. On the other hand, a demand may be probabilistic i.e., is known in advance that there is a 20 per cent chance that demand for an items will be 500 tons and 80 per cent chance that demand for an item will be 600 tons.

Lead Time or Delivery Lag: It is the time between placement of an order and the receipt of materials. Lead time may be deterministic or probabilistic.

Static or Dynamic Inventory Problem: If the inventory problem is such that it is sufficient to take the decision regarding how much to order only once, then the problem is called a static problem. In dynamic situation, orders will be placed continually (separate decisions) in this case, one order directly affects the subsequent orders.

Ordering Cycle: The time span between placement of two successive orders. There are two cases in this cycle

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(*i*) *Continuous Review*: In this type a close watch is kept on the inventory, the moment the stock level touches a certain lower limit, the order is placed.

(*ii*) *Periodic Review*: In this type orders are placed at equally spaced in intervals.

8.10 Outline the different costs associated with stocking of items?

Ans. *Cost Associated with Inventory Systems*: There are different cost components associated with the inventory of items:

(*a*) *Manufacturing Cost/Production Cost/Purchase Cost*: It is the unit manufacturing cost or purchase cost of the item, denoted by 'p'. If purchased in bulk quantity, unit rate way be reduced on earning discounts.

(*b*) *Ordering Cost or Setup Cost*: This is the fixed cost associated with placement of an order. This includes typing of purchase order, posting the order, telephone charges, transport costs for the materials to bring to the firm etc. All the expenditures that do not change with the quantity ordered will come under this category. It is denoted by Co.

(*c*) *Carrying Cost/Holding Cost*: This includes storage costs, interest on the capital invested in material, description costs etc. it is usually expressed as Rs/Unit/Period and denoted by Ch.

(*d*) *Shortage/Stock out Cost*: If an item is out of stock (unavailable), it costs money. There are two cases namely (i) Back orders not Permitted (ii) Back orders not Permitted. In the former case, unfilled demand will be met at a later date. In this case, firm has to negotiate with customers and some extra expenditure will be incurred to maintain communication with customers waiting for service. In the later case, unfilled demand will not be met. The profit on shortage number of items will be lost in addition to the loss of goodwill.

(*e*) *TimeHorizon*: This is the period over which the inventory is controlled. This may be finite say one year or infinite.

8.11 Based on the integration of materials with the finished products, classify the materials and explain?

Ans. Inventory can be broadly defined as the stock of goods, commodities of resources that are stored at any given period

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for future production or for meeting future demand. Inventory can be divided into two classes:

(*i*) *Direct Inventories*: Direct Materials include those items which play a direct role in the manufacture and become an integral part of finished products. There are four sub-categories (a) Raw materials (b) Work-in-process (c) Finished goods and (d) Spare parts.

(*ii*) *Indirect Inventories*: Indirect stock include those items which do not form the part of finished products but aids in manufacturing process. **Ex.** Oils, Lubricants, petrol, office materials, maintenance materials etc.

8.12 What are the objectives of inventory management in any organisation?

Ans. *Objectives of Inventory Control*:

(*i*) Inventories Constitute substantial portion of total current assets of a firm and control of such stock results in greater saving in costs.

(*ii*) Inventory Control ensures that the value of material consumed in minimum.

(*iii*) Inventory control helps to maintain timely records of inventories of all items and to maintain the stock within the desired limits.

(*iv*) It helps in timely replenishment (refill) of stock of various items.

(*v*) It also allows flexibility in production scheduling.

(*vi*) It reduces over stocking and/or under stocking.

(*vii*) It helps to absorb shocks due to demand and lead time uncertainties.

8.13 What is Economic Order Quantity? List out the assumptions to be followed in developing a mathematical model for purchase inventory model for a material with uniform demand?

Ans. Deterministic Inventory Model

Model: Economic Order Quantity Model With Uniform Demand.

EOQ: Economic Order Quantity is the size of the order quantity for which the total cost of ordering and holding the stock is at minimum.

Assumptions:

(*i*) Demand rate is deterministic (Known with certainty) and is constant over time.

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(*ii*) Inventory is replenished when inventory is exactly equal to zero (or equal to safety stock if safety stock is to be maintained).

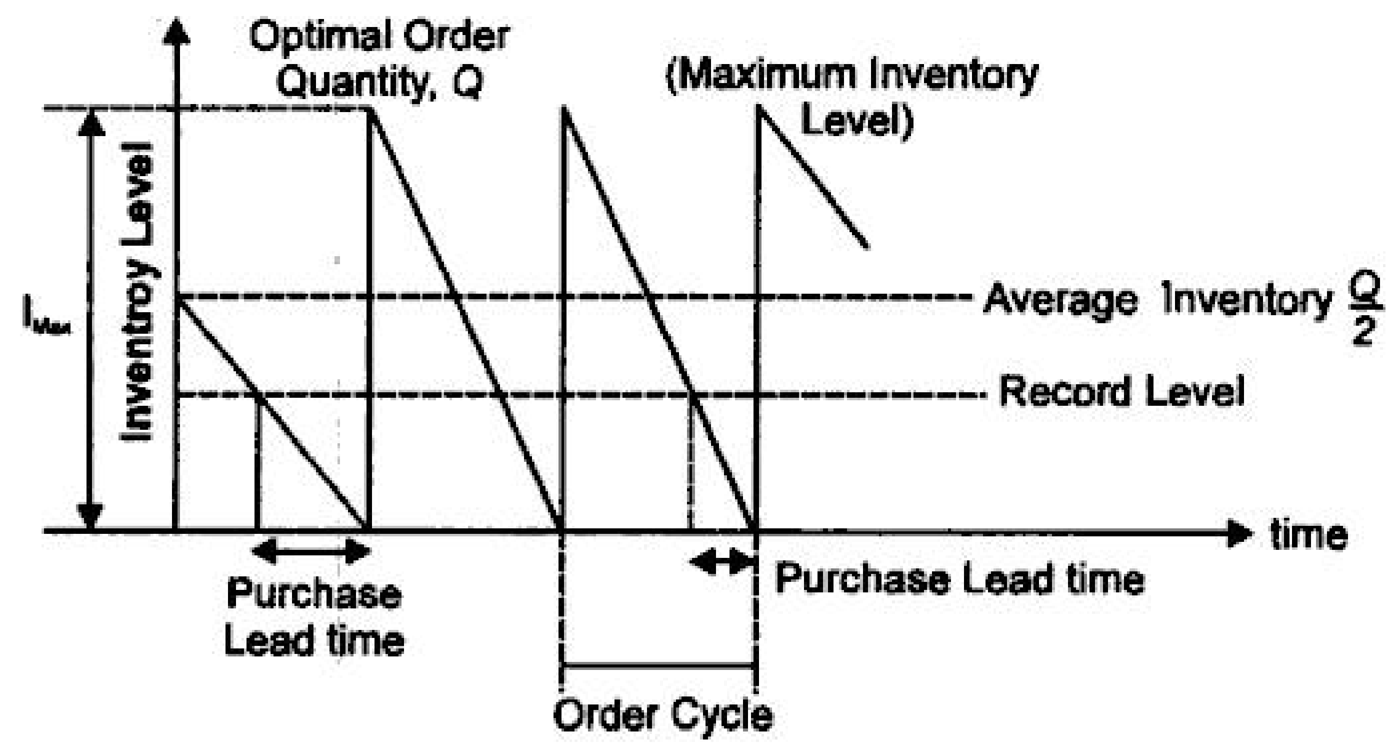
(*iii*) Purchase lead time (time between placement of order and receipt of materials) is known and constant.

(*iv*) Ordering cost is linear with the number of orders per year.

(*v*) Quantity discounts are not allowed i.e., purchase price per unit is constant.

8.14 Draw the inventory profile for purchase inventory situation and use calculus methods to calculate *EOQ* and other parameters of the inventory system?

Ans. Inventory Profile of EOQ Model:



Notations: Q = Order Quantity per order

Q^* = Economic Order Quantity

D = Annual demand (Units per year)

P = Unit cost of an item

Co = Order cost per order

C_h = Stock Holding Cost per unit per period of time

T = Time between two successive orders, or during cycle.

N = Number of orders or manufacturing runs per year.

TC = Total Cost.

Annual Ordering Cost = (No. of Orders in a year)* Order cost per order.

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For **Ex,** Bill of materials for an end-item Z shown in the Fig. 1 can be written as shown below:

Indented bill of materials for model Z (level '0')

Level	*Quality*	*ID*	*Description of component*
1	1	*E*	
1	1	*F*	
2	1	X	
2	2	*Y*	

9.13 Explain the procedure to form BOM matrix?

Ans. If the from is involved in the production or assembly of different end-items, then the need of formulation of BOM matrix arise. The major input for this task is the manufacturing file, shows that each individual item consists of what sibling items and in how many numbers that sibling items are used to form one parent item. The file shows the break up of end product into different sub-assemblies then break up of subassemblies into different components generally purchased parts.

If the bill of material of end-item be defined as the row vector, $Bi = (bi1, bi2, ...bij...)$ where bij is the no. of units of item j required to make one unit of item 'i'.

If the company fabricates n-different end products then

$$\text{BOM} = \begin{Bmatrix} B1 \\ B2 \\ B3 \\ . \\ . \\ Bn \end{Bmatrix}$$

BOM matrix can be written in the form of an upper triangular matrix as

$$B = \begin{array}{c|ccc} & e & s & p \\ \hline e & 0 & Bes & Bep \\ s & 0 & Bss & Bsp \\ p & 0 & 0 & 0 \end{array}$$

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Where e = end point, s = subassemblies and p-parts.

9.14 For the product structures shown in Fig. 1. Write BOM matrix?

Ans.

		e		S	P		
		W	*Z*	*F*	*X*	*Y*	*E*
e	end product *W*	0	0	0	1	1	0
	Z	0	0	1	1	2	1
s	Sub assemblies *F*	0	0	0	1	2	0
p	Parts *X*	0	0	0	0	0	0
	Y	0	0	0	0	0	0
	E	0	0	0	0	0	0

9.15 What are the techniques available to arrive at the planned order releases in MRP?

Ans. There are two techniques available to calculate:

(*i*) Matrix inversion technique

(*ii*) List processing technique (the processing logic is shown in Q. No. 10)

9.16 How gross requirements are computed in matrix inversion technique?

Ans. Requirements, *R* for each one of all end times =

$R = (I\text{-}B)^{-1}$ where

I = unit matrix of the size equal to that of B

B = BOM matrix

Gross requirements = (d) (R) where

d = row vector of demands for all end items.

Depending upon on-hand inventory and open orders, net requirements can be computed as gross requirements minus on-hand stock. Using this approach, for each period, requirements can be computed separately.

9.17 What is pegging in MRP?

Ans. Pegging is the process of tracing through MRP records for all levels in the product structure to identify how changes in the records of one component will affect the records of other component.

9.18 What is the significance of cycle counting?

Ans. Accurate records are to be maintained in MRP, otherwise, production schedules cannot be maintained, deliveries cannot be met. Cycle-counting is the counting of on-hand

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(*d*) Fixed period requirements
(*e*) Period order quantity
(*f*) Least unit cost
(*g*) Least total cost
(*h*) Past period balancing
(*i*) Wagner-whiten algorithm.

9.22 Explain the lot sizing techniques, lot for lot, EOQ and fixed order quantity approaches?

Ans. *Lot for lot*: In this approach, planned order receipts are exactly equal to the net requirements of that period. Orders will be released in periods off set in time by the item's purchase lead time or item's manufacturing lead time if fabricated internally. Inventory costs are almost negligible as the order quantity is exactly equal to the net requirements.

Economic order quantity: EOQ can be introduced in the MRP system if the user wishes so.

EOQ = sqrt (2. U. C/IC)

Where, u = total usage during in time units fence
c = set up cost per order
l = inventory carrying per cent
C = unit lost

If the demand is continuous and stable, it is well applicable.

Fixed order quantity approach: This is followed for the items for which order costs are sufficiently high and total net requirements during the time horizon would be met by one or two fixed order quantities. Number of periods of coverage will vary.

9.23 Explain the following lot sizing techniques:
(*a*) Fixed period requirements
(*b*) Period order quantity
(*c*) Least unit cost

Ans. *Fixed period requirements*: This technique is equivalent to the rule of ordering 'X-months supply', used in some stock replenishment systems, except that here the supply is determined not by forecasting but by adding up future net requirements. The span of coverage can be determined arbitrarily or intuitively. The users specify the number of periods to be covered by each other.

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Period order quantity (POQ): This is modified EOQ approach used in discrete period demand environment. The EOQ is calculated by using the standard formula to determine the number of orders per year to be placed. The planning horizon is then divided by this quantity to determine the ordering interval. Fixed period requirements and period order quantity approaches are same except that in POQ ordering interval is computed.

EOQ = sqrt (2DC/IC) where D = annual demand

$$\text{Number of orders per year} = \frac{\text{annual demand}}{\text{EOQ}}$$

$$\text{Ordering interval} = \frac{\text{No. of periods in the horizon}}{\text{No. of orders}}$$

Least unit cost: In determining the order quantity, least unit technique asks, whether this quantity should equal the first periods net requirements or whether it should be increased also to cover next period's requirements, or one after that etc. The decision is based on unit cost (setup cost per unit + inventory cost per unit) for each of successive order quantities, the one with the least unit cost is chosen to be the lot size.

9.24 Case let: Pioneer Assemblies has a gear assembly that requires component Mx400 with production requirements scheduled as shown below. Average demand is 84 units/week. The cost of placing an order (setup) is Rs. 300, and the inventory carrying charge is Rs 2 per unit per week. Stocking costs are calculated on the basis of average inventory each week.

Ans.

	Week								
	0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*
Requirements	-	28	120	80	0	160	194	20	70
	-								
Initial inventory	0								
Ending inventory	0								

Using the lot-for-lot method, and fixed period requirements approach, complete the MRP record and compare the costs in each of the mentioned lot sizing techniques?

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(*ii*) Only one job can be processed on a given machine at a time.

(*iii*) The time taken by the jobs in moving from one machine to another (move time) is very negligible and is assumed zero.

(*iv*) Once the processing of a job on a machine is started, operation should be completed.

(*v*) Processing machines are of different types (for different operations).

(*vi*) The order of completion of jobs has no significance.

11.3 How many models are available for sequencing problems to classify? What are they?

Ans. There are basically five sequencing models viz., n-jobs through two machines, n-jobs through three machines, n-jobs through m-machines, two jobs through m-machines and n-jobs through one work centre.

11.4 Explain n-jobs two machines problem and Johnson's algorithm for solving such problems?

Ans. *Model 1*: Processing n-jobs through two machines: In this model, only two machines A and B are involved and each Job is processed in the order AB. The processing times A_1, $A_2, \ldots, A_n, B_1, B_2, B_3, \ldots B_n$ are known with certainty.

Job, i	1	2	3	.	.	.	n
A_i	A_1	A_2	A_3	.	.	.	A_n
B_i	B_1	B_2	B_3	.	.	.	B_n

Johnson's Algorithm:

Step 1: Select the smallest processing time occurring in the list $A_1, A_2, A_3, \ldots A_n, B_1, B_2, B_3, \ldots B_n$. If tie exists, select either of the smallest processing time.

Step 2: If the smallest processing time is Ar, do the rth Job first and place it at the beginning of the sequence. If the smallest processing time is B_s, (because of the given order AB) place (s) at the end of the sequence. This concept is applied to both machines A and B.

Step 3: If there is a tie for minimum $A_r = B_s$, Process r^{th} job first and s^{th} job in the last.
If there is tie for minimum among A_r's, then do any one of these jobs first (Choose Arbitrarily).

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If there is tie for minimum among B_s's, then do any of these jobs in the last.

Step 4: There are now $(n - 1)$ jobs left to be sequenced as one job is already decided Repeat the procedure in steps 1,2,3 to the reduced set of processing times obtained by discarding the processing time of a job on both the machines corresponding to the Job already assigned.

Step 5: Continue the placing of Jobs next to first or next to last and so on till all the jobs are assigned their respective positions in what is called 'Optimal Sequence'.

Step 6: After Optimal Sequence is determined, the total elapsed time, idle times on machines are to be computed.

Total Elapsed Time: The time between starting the first job in the optimal sequence on machine A and the completing the last job in Optimal sequnce on machine B.

Idle time on Machine A: (Time when the last Job in the optimal sequence is completed on Machine B) – (Time when the last job in the optimal sequence is completed on Machine A).

Idle Time on Machine B: (Duration of first job in the optimal sequence on Machine $A + \sum^{n}$ [(Time when kth Job starts on machine $k = 2B$) – (Time $(K–1)$th Job finished on Machine B))]

11.5 Example: Six jobs are to be processed through machines A and B in the order AB. The processing times of Jobs on different machines are given below. Find the optimal sequence of Jobs, Total elapsed time and Idle times on Machines?

Job No.	1	2	3	4	5	6
Machine A	8	12	7	10	11	9
Machine B	10	7	11	6	12	8

Ans. On applying the procedure to the processing times data, the smallest processing time is '6' and is for 4th Job on the machine B. As it is occurring on machine B (and since the processing order is AB), Place 4th Job at the end of the optimal sequence.

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Discarding the time data for 4th Job as it is already assigned, in the remaining processing times, minimum time is '7' i,e., $A_3 = B_2 = 7$. Since a tie exists as $A_r = B_s$, first place 3rd job first, 2nd Job next to last.

3			2	4

Deleting the processing time data of jobs already assigned in the remaining processing times, minimum is 8. Here is once again tie of the fashion $A_r = B_s$ i.e., $A_1 = B_6$. Hence place 1st job in the vacant position from the beginning and 6th job in the vacant cell coming from the end.

3	1		6	2	4

There is only one job left for assignment and can be placed in the vacant cell to complete the optimal sequence.

3	1	5	6	2	4

Optimal Sequence.

The minimum elapsed time, Idle times on machines can be found by completing this Table.

Optimal	*Machine A*		*Machine B*		*Idle time for B*
Sequence	*Time In*	*Time Out*	*Time In*	*Time Out*	
3	0	7	7	18	7
1	7	15	18	28	–
5	15	26	28	34	–
6	26	35	35	43	1
2	35	47	47	54	4
4	47	57	57	63	3

Total Elapsed Time = 63 Hours

Idle time on machine $A = 63 - 57 = 6$ Hours

Idle time on machine $B = 7 + 1 + 4 + 3 = 15$ Hours

11.6 What are the assumptions followed for solving n-job through three machines sequencing problem? Also give the procedure to solve such class of sequencing problems?

Model 2: Processing n-Jobs through Three Machines:

Ans. *Assumptions:*

1. All n-Jobs are required to pass through machines A, B, and C in the Order ABC.
2. The same sequence of Jobs over each machine has to be followed.

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3. The expected processing time on three machines are known as given below:

Job	1	2	3	.	.	.	*n*
Machine *A*	A_1	A_2	A_3	.	.	.	A_n
Machine *B*	B_1	B_2	B_3	.	.	.	B_n
Machine C	C_1	C_2	C_3	.	.	.	C_n

The objective is to find the optimal sequence of jobs, which minimises the total elapsed time.
Johnson & Bellman's algorithm can be extended to solve this model. To apply Johnson's algorithm, one of the following conditions is to be satisfied.

(*a*) The smallest processing time on machine $A \geq$ the largest processing time on Machine *B*.

(*b*) The smallest processing time on machine $C \geq$ the largest processing time on Machine *B*.

If any one of the above conditions is satisfied, then this three machines problem can be converted to an equivalent two machines problem. These imaginary/fictitious machines are denoted by *G* and *H* and the processing times for *G* and *H* can be computed as

$$G_1 := A_i + B_i$$

$$H_i := B_i + C_i$$

Now this two machines—*n* jobs problem is usually solved by an algorithm discussed in the previous model to find out the optimal sequence. Once the best sequence is determined, to find out the total elapsed time, idle time on different machines *A*, *B* and *C* we use the processing time data of the given machines *A*, *B* and *C*.

11.7 Example: Find the Sequence that minimizes the total elapsed time in hours required to complete the following Jobs on three machines *A, B* and *C* in the order *ABC*.

Ans.

Job	1	2	3	4	5
Machine *A*	4	9	8	6	5
Machine *B*	5	6	2	3	4
Machine C	8	10	6	7	11

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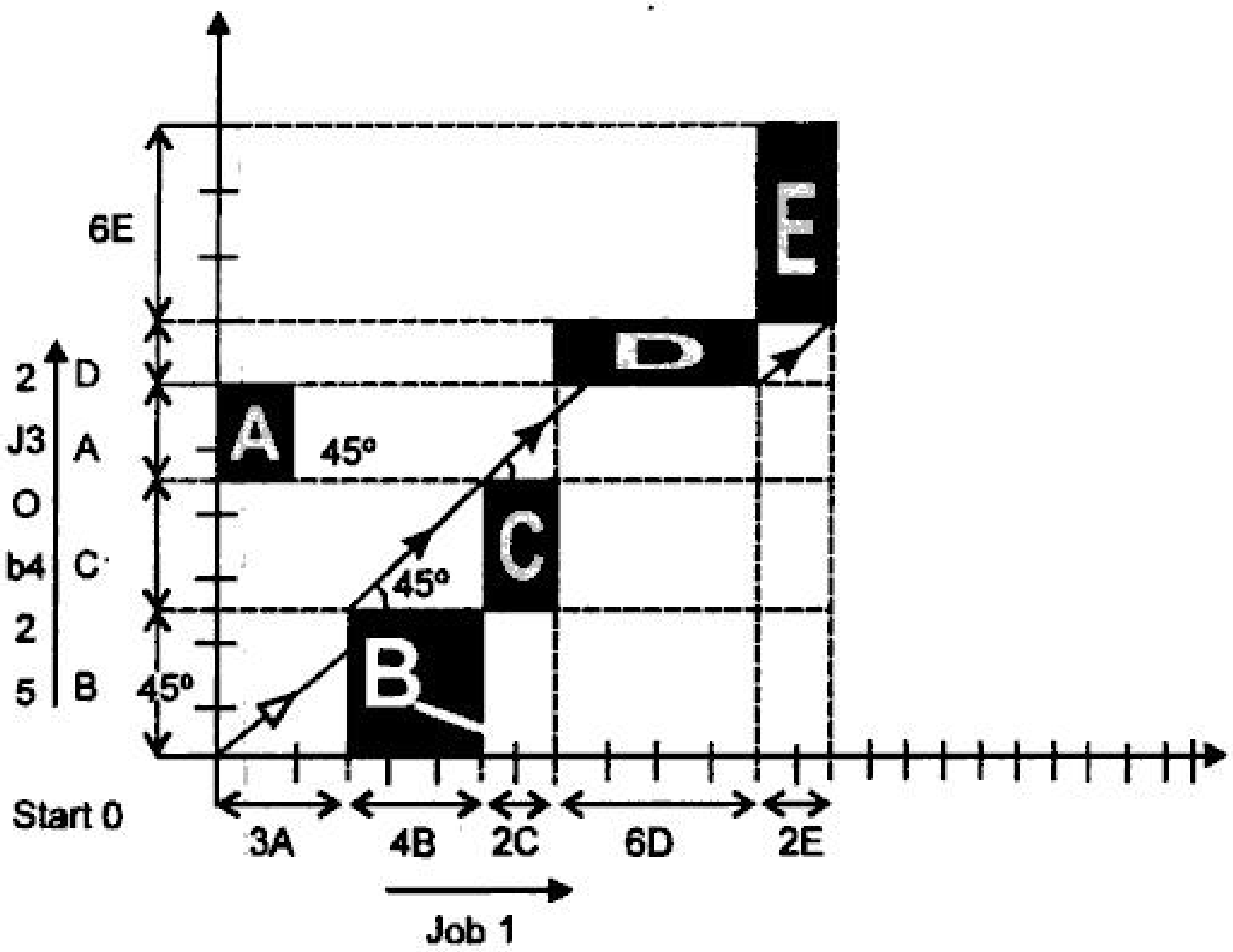

Graphical Solution for two Jobs — 5 Machines sequencing Problem.

Step 3: Machine A requires 3 hours for Job 1, 3 hours for Job 2. So we construct a rectangle for machine A. Similar rectangles are constructed for other machines.

Step 4: Starting from the origin, draw 45° line segments, or vertical lines or horizontal lines to reach the finish point without passing through the interior of rectangles drawn. If a 45° line drawn touches the vertical side of a rectangle, move along the vertical edge to the top corner and again draw 45° line with the objective of reaching the finish point. If a 45° line touches the bottom edge of a rectangle, then move along the bottom edge to the right bottom corner and again 45° line has to be tried. The path is drawn following the above procedure.

When a path is at 45° it indicates that both jobs are being processed on two different machines simultaneously. When the path is a vertical line, it indicates that job 1 is idle and only job2 is being processed. Similarly a horizontal line is an indication that job 2 is idle and only job 1 is being processed.

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Total elapsed time = Total processing times for a job + Idle times for that job.

Total elapsed time = (3 + 4 + 2 + 6 + 2) + (2 + 6)
Considering Job 1 Idle times for Job 1
= 25 hours

Total elapsed time = (5 + 4 + 3 + 2 + 6) + 5
Considering Job 2 Idle times for job 2
= 25 hours

11.10 Describe *n*-jobs through *m*-machines problem? What are the conditions to be satisfied for converting this into n-jobs through two machines problem for applying Johnson's procedure?

Ans. *Model 4*: Processing *n*-Jobs through *m*-Machines: Here the assumption is more than two jobs are to be processed through more than three machines. The order for a job over the machines will be given and will not be changed for other jobs. The expected processing times will be given. Johnson's algorithm is applicable if any one of the following conditions is valid.

(*a*) The minimum processing time of first machine ≥ maximum processing time of all the processing times on the remaining machines (leaving first and last machines).

(*b*) The minimum processing time of last machine ≥ maximum processing time of all the processing times on the remaining machines. (leaving first and last machines).

If at least one condition is valid, convert this problem to n jobs—two machines problem by considering two fictitious machines *G* and *H*.

G_i = sum of processing times of job *i* for all machines except last one.

H_i = sum of processing times of jobs *i* for all machines except first machine.

Determine the optimal sequence and total elapsed time using Johnson's algorithm.

11.11 Example: Determine the Optimal sequence for the following sequencing problem of four jobs and five machines when passing is not allowed of which processing times are given below:

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Machines

Job	*A*	*B*	*C*	*D*	*E*
1	8	6	2	3	9
2	6	6	4	5	10
3	5	4	5	6	8
4	8	3	3	2	6

Also find the elapsed time, idle times on different machines?

Ans. (*a*) Minimum time on machine $A = 5$ is not $\geq$ maximum time on machine $B, C, D = 6$.

(*b*) Minimum processing time on machine $E = 6 \geq$ maximum processing time on B, C, D.

Second condition is valid. Hence this problem is converted to 4 jobs two machines problem. Processing times for two fictitious machines G and H are computed as

$G_i = A_i + B_i + C_i + D_i$

$H_i = B_i + C_i + D_i + E_i$

Job	*1*	*2*	*3*	*4*
Gi	19	22	20	16
Hi	20	25	23	14

Applying Johnson's algorithm, optimal sequence can be obtained as:

1	3	2	4

Optimal sequence is 1-3-2-4. To determine total elapsed time, following table is completed.

Optimal	*Machine A*		*Machine B*		*Machine C*		*Machine D*		*Machine E*	
Sequence	*In*	*Out*	*In*	*Out*	*In*	*Out*	*In*	*Out*	*In*	*Out*
1	0	8	8	14	14	16	16	19	19	28
3	8	13	14	18	18	23	23	29	29	37
2	13	19	19	25	25	29	29	34	34	47
4	19	27	27	30	30	33	33	36	36	53

Total Elapsed time = 53 hours

Idle time of machine $A = 53 - 27 = 26$ hours

Idle time on machine $B = 8 + 1 + 2 + (53 - 30) = 34$ hours

Idle time on machine $C = 14 + 2 + 2 + 1 + (53 - 33) = 39$ hours

Idle time on machine $D = 16 + 4 + (53 - 36) = 37$ hours.

Idle time on machine $E = 19 + 1 = 20$ hours.

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11.12 What is priority sequencing? What are the different priority rules followed at one work centre for sequencing n-jobs?

Ans. *Priority Sequencing*: When jobs compete for work centre's capacity, priority-sequencing rules are applied to determine the sequence in which jobs are processed.

Few priority rules:

(*i*) *First Come First Served (FCFS)*: As the name indicates, incoming arrivals/jobs/customers will be processed in the order of their arrival.

(*ii*) *Earliest Due Date (EDD)*: Top priority is given to the job whose due date is earliest.

(*iii*) *Shortest Processing Time Rule (SPT)*: In this rule, a job of least processing time will be processed first and then job of next higher processing time and so on.

(*iv*) *Least Slack Rule*: Here in this rule, top priority will be given to the job whose slack time is least. Slack time is the difference between the length of time remaining until the job is due it's length of operation time.

11.13 Explain the terms Flow time, Job lateness, Tardiness?

Ans. The total time that a job is in the system is called flow time.

Lateness = completion date – due date.

Tardiness = max [0, lateness].

11.14 Example: The processing times for Jobs A, B, C, D and E at a work centre, due dates for different jobs are applied here with

Ans. FCFS Rule

Arrival (FCFS) Job Sequence	*Processing Time (Days)*	*Flow Time*	*Due Date*
A	4	4	6
B	17	21	20
C	14	35	18
D	9	44	12
E	11	55	12

Average Flow time = (4 + 21 + 35 + 44 + 55) ÷ 5 = 31.8 days

Completion time for all jobs = 55 days

Average no. of jobs in the system

Each day = Average no. of Jobs flowing in the system waiting from the beginning of sequence through the time when the last job is finished.

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Ans. *Step 1*: First of all, find an initial basic feasible solution which consists of $m + n - 1$ occupied cells in any one of the methods northwest corner Rule, least cost or Vogel's method.

Step 2: Determine a set of numbers (implied cost components) for each row and each column. Ui designates implied cost component for ith row. Vj designates the cost component for jth column. $Cij = ui + vj$. For $m + n - 1$ occupied cells, $m + n - 1$ equations will be obtained. But we need to determine $(m + n)$ different values of ui and vj ($i = 1, 2, \ldots m; j = 1, 2, \ldots n$). Hence, we can assign an arbitrary value (preferably zero) to one of the variables ui or vj without violating the equations. Usually zero is assigned for that ui or vj, the row or column for which maximum number of occupied cells exists.

Step 3: For the unoccupied cells, calculate the opportunity cost (improvement factor) by using the equation $D_{ij} = Cij - (Ui + Vj)$; $i = 1, 2, \ldots m; j = 1, 2, \ldots n$. For the unoccupied cell $(Ui + Vj)$ represents implied cost. The opportunity index can be obtained by subtracting implied cost from unit shipping cost in that cell.

Step 4: If the opportunity cost of all unoccupied cells is either positive or zero, then it indicates the current solution as optimal. If any one or more of the unoccupied cells is associated with negative opportunity cost, then the solution is not optimal and is the indication that further improvement in the solution is possible.

Step 5: choose the unoccupied cell with the largest negative opportunity cost and include that cell in the next solution by forming a loop or closed path.

Step 6: with the cell selected in step 5, loop can be formed with the occupied cells with right angle turns in the path i.e., only horizontal and vertical movements are permitted for loop formation using unoccupied cell at one corner and only occupied cells in the remaining corners. If more unoccupied cells are with negative opportunity, then select the cell with the most negative opportunity cost and form a loop with

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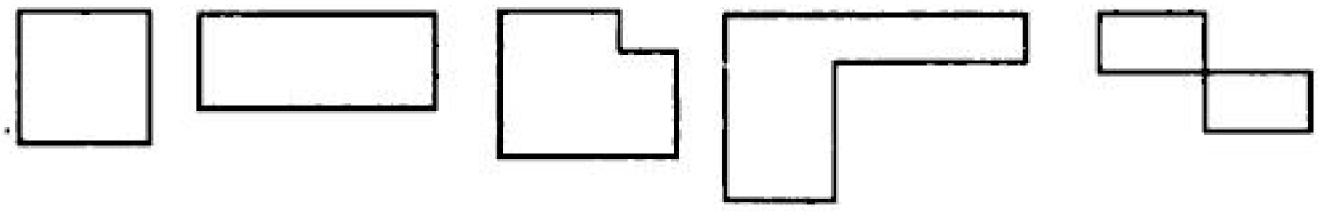
occupied cells. A loop can take any of the following forms with unoccupied cell of most negative opportunity at one corner and the occupied cells at the remaining corners.



Step 7: Give the positive sign for the unoccupied cell as it has to be included in the next solution and alternate signs (+ and –) for the corner cells in a loop.

Step 8: The smallest assigned occupied cell having negative sign in the closed loop is identified and this assigned quantity should be added to the unoccupied cell and cells with positive sign and the same quantity has to be subtracted from the occupied cells in the loop having negative signs to get the new solution. In this procedure, unoccupied cell under consideration becomes occupied cell and one or more cells will become unoccupied.

Step 9: Above procedure is to be repeated till we get optimality indication i.e., the opportunity costs of all unoccupied cells should be either positive or zero. Calculate the transportation cost associated with the optimal solution.

Note: (1) If two or more unoccupied cells have the same maximum negative opportunity cost, an unoccupied cell should be chosen which when included results in the largest decrease in transportation cost.

11.8 Can the shipment problem has alternative optimal solutions (multiple optimal solutions)?

Sol. If the optimal solution to a given transportation problem contains one or more cells with zero opportunity costs (in the unoccupied cells), it is an indication of multiple optimal solutions case i.e., there are different shipping schedules and all the alternative solutions give us the same optimum transportation cost.

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11.9 Solve the following transportation problem?

Find out the minimum cost solution for the following transportation problem, which has cost structure as:

		W1	W2	W3	Availability
	F1	16	19	12	14
From	F2	22	13	19	16
	F3	14	28	8	12
Requirements		10	15	17	42 / 42

Sol. Initial solution for the problem is determined in least cost method as shown below. *Xij*-indicates the number of units to be moved from *i*th source to *j*th destination, quantity inscribed in a circle.

From \ To	W1	W2	W3	Supply
F1	(9) [16]	[19]	(5) [12]	~~14~~ 9
F2	(1) [22]	(15) [13]	[19]	~~16~~ 1
F3	[14]	[28]	(12) [8]	~~12~~
Demand	~~10~~ 1	~~15~~	~~17~~ 5	42 / 42

Since there are 5 occupied cells which is equal to $m + n - 1$ $= 3 + 3 + - 1 = 5$ (no of rows + no. of columns-1), the solution obtained is called initial basic feasible solution. The cost of this schedule = $16 \times 9 + 12 \times 5 + 22 \times 1 + 13 \times 15 + 8 \times 12 =$ Rs. 517.

Optimality is tested and improved in the MODI Method in the following steps. First calculate *Ui*-row numbers and *Vj*-column numbers that represent implied cost components. Considering $Cij = Ui + Vj$ for occupied cells, the values of *Ui* and *Vj* are evaluated. Since there are $3 + 3 = 6$ variables (*Ui* and *Vj*) and there are only five equations for five occupied cells the row/column with highest number of

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For the unoccupied cells, now the opportunity cost Dij are computed as follows:

$Dij = Cij - (Ui + Vj) = 2 - (-1 + 3) = 0$

$D12 = C12 - (U1 + V2) = 2 - (-1 + 3) = 0$

$D14 = C14 - (U1 + V4) = 4 - (-1 + 1) = 4$

$D21 = C21 - (U2 + V1) = 3 - (0 + 2) = 1$

$D31 = C31 - (U3 + V1) = 4 - (-1 + 2) = 3$

$D33 = C33 - (U3 + V3) = 5 - (-1 + 2) = 4$

$D34 = C34 - (U3 + V4) = 9 - (-1 + 1) = 9$

Since D_{ij} for all unoccupied cells are either zero or positive the current solution is optimal.

The optimal transportation cost $1 \times 20 + 1 \times 10 + 3 \times 20 + 2 \times 20 + 1 \times 10 + 2 \times 20 =$ Rs. 180

11.11 Explain the maximisation case in transportation problems?

Ans. Usually transportation problems are modelled and algorithms are available to minimise the transportation cost. If the data regarding contribution per unit of shipment is given, then the objective will be to maximise the total contribution/unit of individual cell from the highest contribution in the matrix. Then the elements will be relative loss elements, and the objective becomes minimisation of total loss. Then the problem can be solved in the usual procedure.

11.12 Explain the Unbalancedness in transportation problems?

Ans. When the total supply is equal to the demand, then the problem is called balanced transportation problem. In actual practice, company may be unable to fulfil the demand or sometimes it's available inventory exceeds the demand requirements. Such type of transportation problems are called unbalanced transportation problems.

When supply capacities, of factories exceed the demand requirements of warehouses/distribution centres, then to absorb additional supply, dummy warehouse (dummy column) has to be added to the table for which unit shipping costs for all the cells in the dummy column are zeros. This is because there is no such warehouse in reality, and by considering such column, unbalanced problem can be converted to balanced problem.

When demand requirements of warehouses exceed the supply capacities of factories, then to satisfy extra

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requirements, a dummy factory (dummy row) has to be added which is assumed to supply the shortage quantity. But in reality, there is no such factory and as for as the transportation problem is concerned, the row is just added to satisfy the rim condition.

11.13 A case of maximisation: A transport company has four factories *F*1, *F*2, *F*3 and F4 which produces the same product, processing and raw material costs differ from factory to factory and are given in the table below. The unit shipment costs from the factories to warehouses *S*1, *S*2, *S*3 are also given. Sales price at warehouses *S*1, *S*2, *S*3 are different and given.

		*F*1	*F*2	*F*3	*F*4	*Sales price Per unit*	*Demand*
Production cost		15	18	14	13		
Raw material/unit		10	9	12	9		
Transportation							
Cost/unit	*S*1	3	9	5	4	34	80
	*S*2	1	7	4	5	32	120
	*S*3	5	8	3	6	31	150
Supply		10	150	50	100		

Determine the most profitable shipment schedule and the corresponding profit. Excess production is assumed to yield zero profit?

Sol. The profit elements are evaluated for different routes.
Profit = sales price–production cost–raw material cost–transportation cost.

Table 1

	*S*1	*S*2	*S*3	*Supply*
*F*1	6	6	6	10
*F*2	–2	2	–4	150
*F*3	3	2	2	50
*F*4	8	5	3	100
Demand	80	120	150	310 350

Total demand exceeds the total supply, hence dummy factory F5 is considered for which profit elements are zeros. The capacity for dummy factory = 350 – 310 = 40 units is assumed to convert the problem into balanced problem.

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Table 2

	S1	*S2*	*S3*	*Supply*
F1	6	6	1	10
F2	–2	–2	–4	150
F3	3	2	2	50
F4	8	5	3	100
F5	0	0	0	40
Dummy				
Demand	80	120	150	350
				350

Relative loss matrix is prepared from Table-2, by substracting individual profits from maximum profit element. Relative loss matrix is shown below.

	S1	*S2*	*S3*	*Supply*
F1	8 – 6 = 2	8 – 6 = 2	8 – 1 = 7	10
F2	8 – (–2) = 10	10	12	150
F3	5	6	6	50
F4	0	3	5	100
F5	8	8	8	210
Dummy				350
Demand	80	120	150	350

Least cost method is applied to get initial solution shown below:

	S1	*S2*	*S3*	*Supply*
F1	2	2 (10)	7	10
F2	10	10 (40)	12 (110)	150
F3	5	6 (50)	6	50
F4	0 (80)	3 (20)	5	100
F5 Dummy	8	8	8 (40)	40
Demand	80	120	150	350 / 350

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Since there are $m + n - 1 = 7$ occupied cells, it is initial basic-feasible solution. Now MODI method is applied to check the optimality and to improve the solution.

	S1	S2	S3	Ui's
F1	2′	2 (10)	7	U1 = 2
F2	10	+ 10 (40)	– 12 (110)	U2 = 10
F3	5	– 6 (50)	6 (+)	U3 = 6
F4	0 (80)	3 (20)	5	U4 = 3
F5 Dummy	8	8	8 (40)	U5 = 6
Vj's	V1 = –3	V2 = 0	V3 = 2	

V2 = 0 is assumed as 2nd column consists of maximum number of allocations. Applying *Cij* = *Ui* + *Vj* for occupied cells, other *Ui*'s and Vj's computed.

D11 = C11 – (*u*1 + *v*1 = 2 – (2 – 3) = 3

D13 = C13 – (*u*1 + *v*3 = 7 – (2 + 2) = 3

D21 = C21– (*u*2 + *v*1 = 10 – (10 – 3) = 3

D31 = C31 – (U3 + V1 = 5 – (6 – 3) = 2

D33 = C33 – (U3 + V3 = 6 – (6 + 3) = – 2

D43 = C43 – (U4 + V3 = 5 – (3 + 2) = 0

D51 = C51 – (U5 + V1 = 8 – (6 – 3) = 5

D52 = C52 – (U5 + V2 = 8 – (6 + 0) = 2

Current solution is not optimal as (*F*3 *S*3) cell is associated with negative opportunity cost. In the next solution, this cell has to be included and a loop is formed as shown in the table, starting with + sign for (*F*3 *S*3), signs are alternatively used. Minimum assignment in the loop having negative sign is 50 units, hence 50 units added to (*F*3 *S*3) cell, 50 units are subtracted from 110 units in (*F*2 *S*3) cell, 50 units added to (*F*2 *S*2) cell, 50 units subtracted from (*F*3 *S*2) cell, to obtain the next solution. While doing this (*F*3 *S*3) has become occupied cell and (*F*3 *S*2) has become unoccupied. New solution is shown below:

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	S1	S2	S3	Ui's
F1	2	2 (10)	7	U1 = 2
F2	10′	10 (90)	12 (60)	U2 = 10
F3	5	6	6 (50)	U3 = 4
F4	0 (80)	3 (20)	5	U4 = 3
F5 Dummy	8	8	8 (40)	U5 = 6
Vj's	V1 = –3	V2 = 0	V3 = 2	

Now optimality is checked again by first assuming V2 = 0 and remaining *Ui's* and *Vj's* are computed. For unoccupied cells, opportunity costs are:
$D11 = 2 - (2 - 3) = 3$
$D13 = 7 - (2 + 2) = 3$
$D21 = 10 - (10 - 3) = 3$
$D31 = 5 - (4 - 3) = 4$
$D32 = 6 - (4 + 0) = 2$
$D43 = 5 - (3 + 2) = 0$
$D51 = 8 - (6 - 3) = 5$
$D52 = 8 - (6 + 0) = 2$
All *Dij* are either positive or zero, hence the solution is optimal. Shipping schedule is
(*F*1 *S*2) = 10; (*F*2 *S*2) = 90; (*F*2 *S*3) = 60
(*F*3 *S*3) = 50; (*F*4 *S*1) = 80; (*F*4 *S*2) = 20;
The profit $10 \times 6 + 900\ (-2) + 20 \times 5 + 60\ (-4) + 50 \times 2$ = Rs. 480.

11.14 What is degeneracy in transportation problems?

Ans. If a feasible solution obtained consists of $m + n - 1$ occupied cells, it is called basic feasible solution. Otherwise if an initial solution exists with less number of occupied cells than $m + n - 1$, it is called a degeneracy case. There may be two situations where degeneracy can be spotted. While finding the initial solution itself, this may arise. In that case, e, negligibly a small positive quantity is added to the lowest

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	D1	D2	D3	D4	Ui's
P1	19 (5)	30	50	12 (2)	$U1 = 0$
P2	70	30 (+)	40 (7)	60 (3)	$U2 = 48$
P3	40	10 (8)	60	+ 20 (10) +	$U3 = 8$
Vj's	$V1 = 19$	$V2 = +2$	$V3 = -8$	$V4 = 12$	

Implied cost components are determined by first assuming $U1 = 0$. Then $19 = U1 + V1$

$V1 = 19 \Rightarrow V1 = 19$

$C14 = 12 = U1 + V4 \quad \Rightarrow V4 = 12$

$C32 = 40\ V3 + U2 \quad \Rightarrow V3 = -8$

$C42 = 60\ V4 + U2 \quad \Rightarrow U2 = 48$

$C23 = 10\ V2 + U3 \quad \Rightarrow V2 = 2$

$C43 = 20\ V4 + U3 \quad \Rightarrow U3 = 8$

Opportunity cost for unoccupied cells

For $P1D2 = 30 - (2 + 0) = 28$

For $PD3 = 50 - (8 + 0) = 58$

For $P2D1 = 70 - (19 + 48) = 3$

For $P2D2 = 30 - (2 + 48) = -20$

For $P3D1 = 40 - (19 + 8) = 13$

For $P3\ D3 = 60 - (-8 + 8) = 60$. Since the ($P2$. $D2$) cell contains negative opportunity cost, solution can be improved further. Loop is formed with this cell to form one corner and with the three occupied cells at the remaining corners as shown in the table above. Smallest quantity associated with the negative sign in the loop is 3 units. Add this to the unoccupied cell connected in the loop and to the occupied cells of

	D1	D2	D3	D4	Ui's
P1	19 (5)	30	50	12 (2)	$U1 = 0$
P2	70	30 (3)	40 (7)	60	$U2 = 48$
P3	40	10 (5)	60	20 (13)	$U3. = 8$
Vj's	$V1 = 19$	$V2 = -2$	$V3 = -8$	$V4 = 12$	

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positive sign in the loop and substract the same quantity from the cells of negative sign in the loop. We get the following solution. Again optimality is cheked as shown below:

Assume $U1 = 0$. then

$19 = V1 + U1 \Rightarrow V1 = 19;$

$C_{14} = 12 = V4 + U1 \Rightarrow V4 = 12$

$20 = V4 + U3 \Rightarrow U3 = 8$

$10 = V2 + U3 \Rightarrow V2 = 2$

$30 = V2 + U2 \Rightarrow U2 = 28$

$40 = V3 + U2 \Rightarrow V3 = 12$

Now the opportunity cost for the unoccupied cells are computed below:

For $P1D2 = 30 - 2 = 28$

For $P2D1 = 70 - (28 + 19) = 23$

For $P3D1 = 40 - (19 + 8) = 13$

For $P1D3 = 50 - 12 = 38$

For $P2D4 = 60 - (12 + 28) = 20$

For $P3D3 = 60 - (12 + 8) = 40$

The signs of all opptunity costs for rthe unoccupied cells are positive, hence the solution is optimal.

Cost $19 \times 5 + 12 \times 2 + 30 \times 3 + 7 \times 40 + 5 \times 10 + 20 \times 13$

Cost = Rs. 799

Saving = 1000– 799 = Rs. 201.

11.16 Example: solve the following transportation problem. A company has three plants P1, P2, P3 each producing 50,100 and 150 units of a similar products. There are 5 warehouses W1, W2, W3, W4, W5 having demand 100,70,50,40,40 units respectively, the cost of sending a unit from various plants to warehouses differs as given by the cost martix below:

	W1	*W2*	*W3*	*W4*	*W5*
P1	20	28	32	55	70
P2	48	36	40	44	25
P3	35	55	22	45	48

Ans. The cost data along with the demand and supply of warehouses and factories are put in the transportation table. Vogel's method is employed to get initial basic feasible solution (shown below).

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U3 = 0 then
45 = V4+U3⇒ V4 = 45   20 = U1+V2 ⇒ = −15
28 = V3+U3⇒ V3 = 28   25 = U2+V5; 36 = V2+U2⇒U2= −7
V1 = 35   V2 = 28 +15 = 43

Then opportunity costs for the unoccupied cells are shown below.

P1W3 = 32 − (28 − 15) = 19   P2W4 = 44 − (45 − 7) = 6
P1W4 = 55 − (45 − 15) = 25   P3W2 = 55 − 43 = 12
P1W5 = 70 − (32 − 15) = 53   P3W5 = 48 − 32 = 16
P2W1 = 48 − (35 − 7) = 20   P2W3 = 40 − (28 − 7) = 19

Since all are positive, the solution obtained is optimal and the total cost.

Cost = Rs. 9240.

11.17 Example: ABC Ltd. has 3 production shops supplying a product shops supplying a product to 5 ware houses. The cost of production varies from shop to shop and costt of transportation from one shop to a warehouse also varies. Each shop has a specific produntion capacity and each warehouse has certain demand. The cost of transportation, the cost of manufacture of the product are given below:

Find the optimum shipment schedule.

	I	*II*	*III*	*IV*	*V*	*Capacity*	*Variable cost*	*Fixed cost*
A	6	4	4	7	5	100	14	7000
B	5	6	7	4	8	125	16	4000
C	3	4	6	3	4	175	15	5000
Requirement	60	80	85	105	70			

Ans. Initial basic feasible solution is obtained in Vogel's method below:

	I	*II*	*III*	*IV*	*V*	*Supply*
A	[20]	[18]	85 [18]	[21]	15 [19]	100
B	[21]	80 [22]	[23]	[20]	45 [24]	125
C	60 [18]	[19]	[21]	105 [45]	10 [19]	175
Demand	60	80	85	105	70	400 400

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Row Penalties					*Column penalties* Col I	Col II	Col III	Col IV	Col V
Row I :	0	1	1	1	2	1	3	2	0
Row II :	1	1	1	2	2	1	1	2	0
Row III :	0	0	1	0	2	1	–	–	0
					–	1	–	–	0

(1) $m + n - 1 = 3 + 5 = 7$.

no. of occupied cells = 7. Optimality is checked by first assuming $U3 = 0$ as it contains maximum number of occupied cells. Then apply $Cij = Ui + Vj$ for occupied cells to obtain remaining *Ui's* and *Vj's*.

	I	*II*	*III*	*IV*	*V*	*Ui's*
A	[20]	[18]	85 [18]	[21]	15 [19]	*U*1 = 0
B	[21]	80 [22]	[23]	+ [20]	– 45 [24]	*U*2 = 5
C	60 [18]	[19]	[21]	– 105 [18]	+ 10 [19]	*U*3 = 0
Vj's	*V*1 = 18	*V*2 = 17	*V*3 = 18	*V*4 = 18	*V*5 = 19	

The opportunity costs are computed for unoccupied cells below:

For *AI* = 20–(18) = 2 for *A*II = 18 – (17) = 1

For *A*IV 21 – 18 = 3

For *BI* = 21 – (18 + 5) = –2

For *B*III = 23 – (18 + 5) = 0

For *B*IV = 20 – (18 + 5) = –3 Most negative. Form a loop with this cell, assign + sign to this cell.

For CII = 19 – (17) = 2.

For CIII = 21 – (18) = 3. By including the (B, IV) cell, next solution is obtained as shown below:

	I	II	III	IV	V	*Ui's*
A	[20]	+ [18]	85 [18]	[21]	– 15 [19]	*U*1 = 0
B	[21]	– 80 [22]	[23]	+ 45 [20]	[24]	*U*2 = 2
C	60 [18]	[19]	[21]	– 60 [18]	+ 55 [19]	*U*3 = 0
Vj's	*V*1 = 18	*V*2 = 20	*V*3 = 18	*V*4 = 18	*V*5 = 19	

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	1	2	*Dummy*	*Supply*
Month 1 RT	800 [1]	120 [1.5]	[0]	920
OT	[2.25]	580 [2.75]	340 [0]	920
Month 2 RT	[∞]	250 [1]	[0]	250
OT	[∞]	250 [2.25]	[0]	250
Demand	800	1200	340	2340

Row Penalties					*Column penalties* *Col I*	*Col 2*	*Col 3*
Row 1:	1	0.5	0.5	0.5	1.25	0.5	0
Row 2:	2.25	0.5	0.5	0.5	1.25	0.5	–
Row 3:	1	∞	–	–	1.25	0.5	–
Row 4:	2.25	∞	∞		1.25	2.25	–

Vogel's method is applied to find the optimal solution. Modi method is applied to check the optimality and found optimal as all unoccupied cells are of positive opportunity costs.
Cost = 800 + 180 + 1595 + 0 + 250 + 562.5
= Rs 3387.5

	1	2	*Dummy*	*Ui's*
Month 1 RT	800 [1]	120 [1.5]	[0]	*U*1 = 0
OT	[2.25]	580 [2.75]	340 [0]	*U*2 = 1.22
Month 2 RT	[∞]	250 [1]	[0]	*U*3 = – 0.5
OT	[∞]	250 [2.25]	[0]	*U*4 = 0.75
Vj's	*V*1=1	*V*2 = 1.5	*V*3=1.25	

Total Cost = Rs 3387.5

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Industrial Engineering and Management

C. Nadha Muni Reddy

The book explains the subject through a series of graded questions and answers and thus helps the students in a better preparation for their examinations. Some questions are of short answer type for which answers are presented in a paragraph. Some questions are of subjective type for which answers are presented at length.

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C. Nadha Muni Reddy is Professor and Head, Dept. of Mechanical Engineering, Annamacharya Institute of Technology & Sciences, Cuddapah, Andhra Pradesh. He obtained his B.Sc. (Electronics), B. Tech. (Mechanical Engg.) and M. Tech. (Industrial Engg.) degrees from S.V. University, Tirupathi. He obtained his Ph.D. degree (1999) in Industrial Engineering from S.K. University, Kurnool. He had earlier served as Associate Lecturer in K.L. College of Engineering, Vijayawada and as Lecturer and subsequently Associate Professor in G.P.R. Engg. College, Kurnool. He has published several papers in international and national journals of repute.

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