Problem 2.1A

When Bob came home he found a disaster. Bill and Mary were both lying dead in a puddle on the floor. Pieces of broken glass were all over the floor. What caused the disaster?

Solution

One way to approach this type of novel problem is to set up a matrix in which the columns are possible options for solution and the rows are comprised first of the constraints followed by the criteria for evaluation. The last row would be the decision as to the best option based on the weight of evidence presented in the matrix.

If the problem is not novel, then you can recognize valid patterns of successful solution for related problems, and follow those patterns. With novel problems and lack of data, you often make assumptions that prove to be incorrect. In this problem you might assume that Bill and Mary drowned in the water from a waterbed as one option. But what about the glass? A second option might be that Bill and Mary were in a bathroom or kitchen where both water and glass exist. A third option would require a distinct stretch of the imagination, but is the simplest solution: Bill and Mary were goldfish and the glass came from the broken goldfish bowl. Various options can be evaluated in terms of the constraints in the problem: presence of broken glass, water on the floor, Bill and Mary dead, and so on. A decision based on the data would eliminate options such as Bill and Mary drowned in a swimming pool or in a river as the constraints are violated by the meager data available.

Problem 2.1B

Why are 1994 new dimes worth more than 1984 dimes that have been in circulation?

Solution

In this novel problem you must focus on what the problem is. Think about it. What is the unknown? What do the words mean? Perhaps draw a figure. Can you rephrase the problem?

If you assume that the numbers refer to the year of minting, the question seems to be nonsense as a dime is worth 10 cents no matter what the year of issue. Does the question mean that circulated dimes, being worn, have less numismatic value than new dimes? Or do the number simply refer to the number of dimes? Clarifying a problem statement is always the first step in problem solving. Why solve the wrong problem well?

Problem 2.2A

Solve the four linear material balances

$$x_1 - x_2 + x_3 - 4x_4 = 20$$
$$2x_1 + x_3 - x_4 = 15$$
$$3x_2 + x_3 = 30$$
$$2x_1 - x_3 + 2x_4 = -10$$

using a computer program

Solution

A number of standard computer programs easily handle problems of this type such as spreadsheet packages, Matlab, Mathcad, Polymath, and so on as well as symbolic manipulators such as Mathematica, Maple, Derive, etc. Most statistic packages and equation solvers will also solve linear equations and have a simple user interface.

The solution for this example was obtained via the Excel spreadsheet. The equations can be represented in the matrix notation $\mathbf{Ax} = \mathbf{b}$ where the arrays are

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 1 & -4 \\ 2 & 0 & 1 & -1 \\ 0 & 3 & 1 & 0 \\ 2 & 0 & -1 & 2 \end{bmatrix} \quad \text{4 x 4 (4 rows, 4 columns)}$$

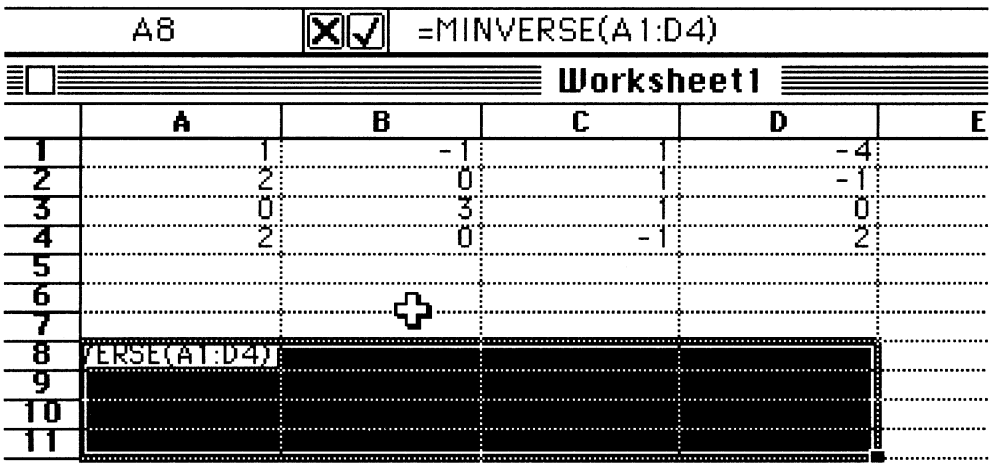
$$\mathbf{x} = \begin{bmatrix} x1 \\ x2 \\ x3 \\ x4 \end{bmatrix} \quad \text{4 x 1 (4 rows, 1 col)}$$

$$\mathbf{b} = \begin{bmatrix} 20 \\ 15 \\ 30 \\ -10 \end{bmatrix} \quad \text{4 x 1 (4 rows, 1 col)}$$

To solve for the elements in **x**, you need to form the matrices (arrays) **A**, **b**, and **x**; calculate the inverse of **A**, $\mathbf{A}^{-1}$; and multiply as follows:

$$\mathbf{A}^{-1}\mathbf{b} = \mathbf{x}$$

Excel carries out these operations transparently for you. The first screen shows the worksheet with the data entered along with the dark cells reserved for the elements of the inverse matrix $\mathbf{A}^{-1}$. The next figure show the elements of the inverse matrix $\mathbf{A}^{-1}$ and the solution (in column H).



Worksheet1

	A	B	C	D	F	G	H
1	1	-1	1	-4			
2	2	0	1	-1			
3	0	3	1	0			
4	2	0	-1	2			
5							
6							
7							
8	0.11111111	0.14814815	0.03703704	0.2962963	20		2.59259259
9	0.22222222	-0.3703704	0.40740741	0.25925926	15		8.51851852
10	-0.6666667	1.11111111	-0.2222222	-0.7777778	30		4.44444444
11	-0.4444444	0.40740741	-0.1481481	-0.1851852	-10		-5.3703704
12							

Problem 2.2B

In the back of the book is a disk containing the program Broyden that can solve sets of nonlinear equations. (It is actually a minimization program that minimize the sums of the squares of the deviations between the function values and zero). Solve the following two equations using Broyden or another program. You must compile the subroutine containing these equations via Fortran compiler on your computer before executing the program if you use Broyden.

$$25 - x_1^2 - x_2^2 = 0$$
$$x_1 + x_2 - 2 = 0$$

Solution

The output shows

ITER	NFUNCT	NDRV	FN VALUE	X-VALUES	
0	5	2	0.457000E+03	-0.200000E+01	0.000000E+00
1	21	4	0.440910E+03	0.199082E+01	-0.199541E+00
2	45	6	0.602136E+02	-0.847026E+00	-0.478122E+01
3	56	8	0.598039E+02	-0.845112E+00	-0.483793E+01
4	70	10	0.417177E+02	0.825930E+00	-0.508889E+01
5	80	12	0.365274E+02	0.157019E+01	-0.501562E+01
6	93	14	0.102013E+02	0.338699E+01	-0.393178E+01
7	103	16	0.686787E+01	0.322017E+01	-0.383872E+01
8	116	18	0.230040E+01	0.397599E+01	-0.318019E+01
9	130	20	0.167794E+00	0.427840E+01	-0.252328E+01
10	146	22	0.110081E-02	0.438222E+01	-0.241096E+01
11	156	24	0.288763E-04	0.439219E+01	-0.239034E+01
12	163	26	0.229370E-08	0.439115E+01	-0.239120E+01
13	170	28	0.933623E-12	0.439116E+01	-0.239117E+01
14	173	28	0.932459E-12	0.439116E+01	-0.239117E+01

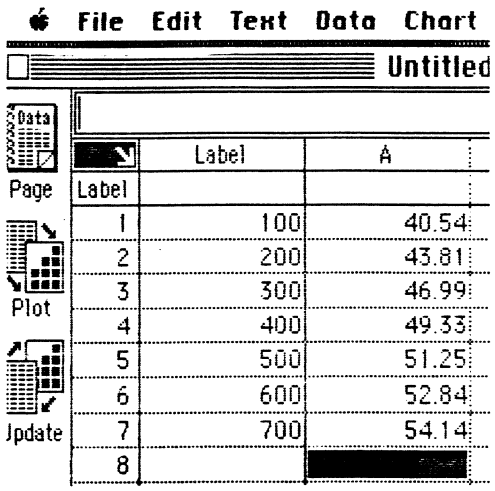
Problem 2.2C

You are asked to plot the following data and fit the best curve to it to relate C_p, the heat capacity, to the temperature.

	T(°C)		C_p[J/(g mol) (°C)]
	100		40.54
	200		43.81
X Axis	300	Y Axis	46.99
(horizontal)	400	(vertical)	49.33
	500		51.25
	600		52.84
	700		54.14

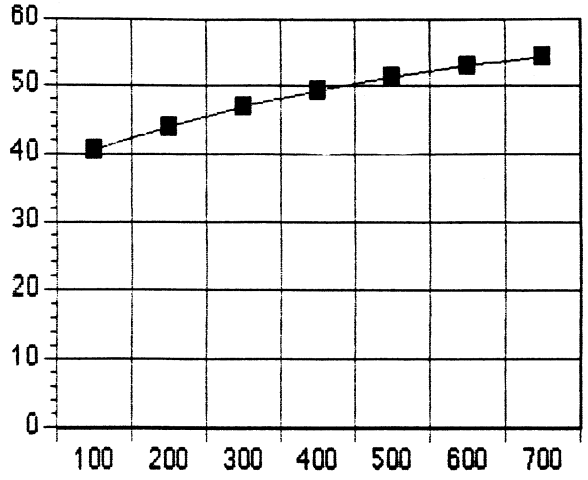
Solution

A program called DeltaGraph was used to carryout this assignment. The screen after entry of the data is



	Label	A
Label		
1	100	40.54
2	200	43.81
3	300	46.99
4	400	49.33
5	500	51.25
6	600	52.84
7	700	54.14
8		

and the graph with a superimposed plot of a third order polynominal looks as follows



CHAPTER 2 – ADDITIONAL PROBLEMS
(Answers will be found in Appendix A)

Section 2.1

2.1A The following experiment is carried out in a laboratory. A 1.45 kg copper ball (sp. gr. 8.92) is allowed to fall from 100 cm height into a tall vessel containing phenol (sp. gr. 0.789). Will the ball move more rapidly through the phenol if the material is at 25°C or at 100°C? Explain why.

2.1B Six beakers are aligned next to each other in a row on a table. The first three beakers are one-half full of ethanol. The second three beakers are empty. How can you arrange the beakers so that the half full and empty vessels alternate in the row by moving only 1 beaker?

2.1C One evening a man at work saw behind him two masked men. He turned right and ran straight ahead. Then he turned left and continued running straight ahead, and kept running while turning left twice more. As soon as he turned the last time he saw in the distance the two masked mean waiting for him. Who were they?

Section 2.2

2.2A Solve the following set of linear equations using a spread sheet program, MathCad, Matlab, or a related package:

$$2x + y + z = 7$$
$$x - y + z = 2$$
$$x + y - z = 0$$

2.2B Solve the following set of nonlinear equations using a spread sheet program, MathCad, or Matlab, or a related package:

$$y = 48 - x^2$$
$$x^5 = y^2$$

2.2C Create an x-y graph for the following data points, labeling the axes properly, via a spread sheet, MathCad, or a related program

time(s)	3	7	12	22	45	78
distance(min)	10	5	8	2	17	21