CHAPTER

18

Design for Torsion

Introduction

In structural design, occasionally, torsional moment may be significant and hence it may be necessary to check for torsional stresses. Frequently, torsion may be secondary and its effect must be considered in combination with the action of other forces, such as axial compression and bending. The shapes that are very good for columns and beams, i.e., those which have their material distributed as far from their centroid as practicable, are not equally efficient in resisting torsion. Thin walled circular and square or rectangular hollow sections are torsionally stronger than sections with the same area but having cross sections such as channel, angle, I-, T- or Z-shapes. The code (IS 800 : 2007) does not have any provisions for design of members subjected to torsion. Hence, in this chapter, the behaviour of members subjected to torsion is described. The difference between uniform and non-uniform torsion is explained and some approximate design methods, to take into account the torsional stresses, are also suggested.

18.1 Torsional Loading in Practice

In actual practice, there are only a few occasions where torsional loading will result in significant twisting. Mostly these arise during the construction stage, when the members are not braced. During the functional stage, the members will be laterally restrained along their length and hence will not be allowed to twist freely. Therefore, the rotation due to torsional loading will be (at the maximum) limited to the end slope of the transversely attached members.

The effect of torsion may be classified based on whether it is due to the transfer of load (*primary torsion)* or due to some secondary action. *Secondary torsion* may arise as a result of differential twist rotations compatible with the joint rotations of the primary frames (see Fig. 18.1). The magnitude of the secondary torsion can be predicted by a three-dimensional analysis program. Secondary torques are usually small, when there are alternative load paths of high stiffness, and are often ignored in the design (Trahair et al. 2001).

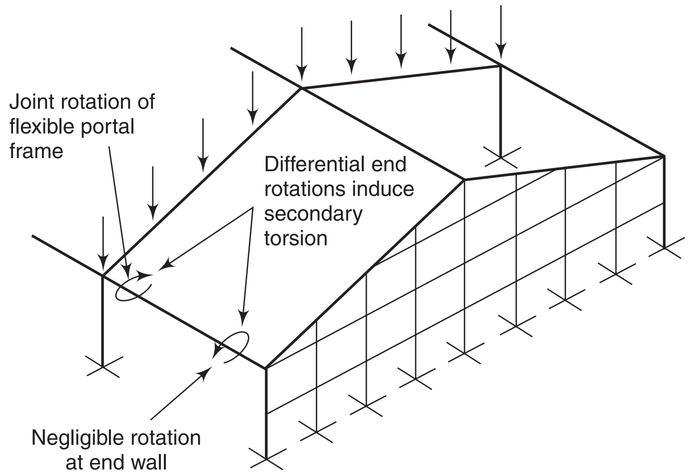


Fig. 18.1 Secondary torsion in an industrial building

Primary torsion is classified as free (uniform), restrained (warping or non-uniform), and destabilizing. (see Fig. 18.2). As shown in Fig. 18.2, when the member *ABC* is rigidly connected to the member *DCE*, it is not only applying torsion on member *DCE*, but also applying a restraining action called the *restraining torsion*. In this case, the compatibility between the members must be satisfied in the analysis. When the member *ABC* is connected to the member *DCE* with simple connections [as in Fig. 18.2(c)], *ABC* will not restrain the twisting of the member *DCE* and

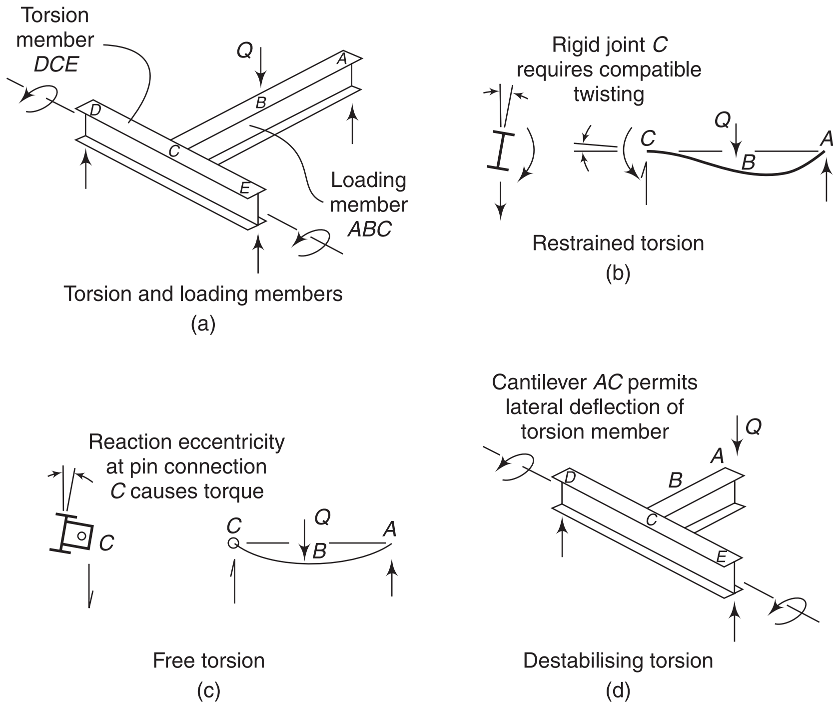


Fig. 18.2 Classification of primary torsion (Trahair et al. 2001)

hence there is *free torsion*. However, member *ABC* will prevent the lateral deflection. When the member *ABC* acts as a cantilever as shown in Fig. 18.2(d), *destabilizing torsion* occurs. Now, member the *ABC* restrains neither the twisting nor the lateral deflection of the member *DCE*.

Torsion exists on spandrel beams, where the loading may be uniformly distributed, unlike the cases discussed in Fig. 18.2, where a beam frames into a girder on one side only. On many occasions, torsion occurs in combination with bending actions. Any situation where the loading or reaction acts eccentrically to the shear centre gives rise to torsion. Channel purlins should be placed in correct orientation on the rafters. Among the two cases shown in Fig. 18.3, torsion is likely to occur in case (b), since the load application point and the shear centre of the purlin are too far apart. To avoid this situation, the purlins may be arranged in alternate orientations. An interacting case study on the failure of purlins due to torsion is presented by Subramanian (1999). The design of crane runway girders involves the combination of biaxial bending and torsion. For the details of determining torsional moment in a framing system, the reader may refer to Chen and Joleissaint (1983).

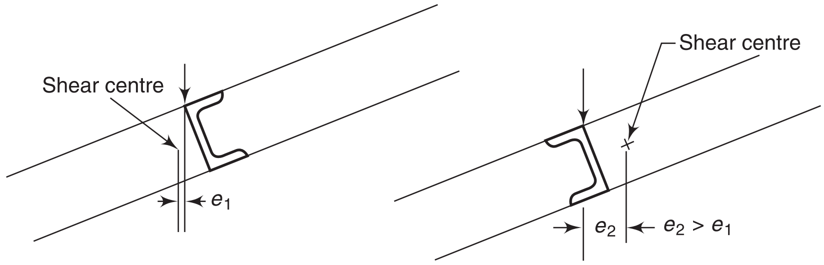


Fig. 18.3 Torsion induced in purlins

18.2 Behaviour of Members Due to Torsion

The resistance of a structural member due to torsional loading may be considered to be the sum of two components: (1) uniform torsion, i.e., when the rate of change of the angle of twist rotation is constant along the member. The longitudinal warping deflections are also constant along the member in this case and (2) non-uniform torsion, i.e., when the rate of change of the angle of twist varies along the member. In this case, the warping deflections vary along the member. Both these types of torsions are discussed in this section.

18.2.1 Saint Venant's (uniform) Torsion

When a simple circular tube is twisted, shearing stress will result as shown in Fig. 18.4. The cross section, initially planar remains a plane and rotates only about the axis of the member. This kind of 'pure' torsion is usually called St Venant torsion, after St Venant who was the first to develop the theory for the general case. The variation in stress is linear, if the proportional limit of the material is not exceeded. The angle of twist ϕ per unit length is

$$\phi = T/(G I_p) \tag{18.1}$$

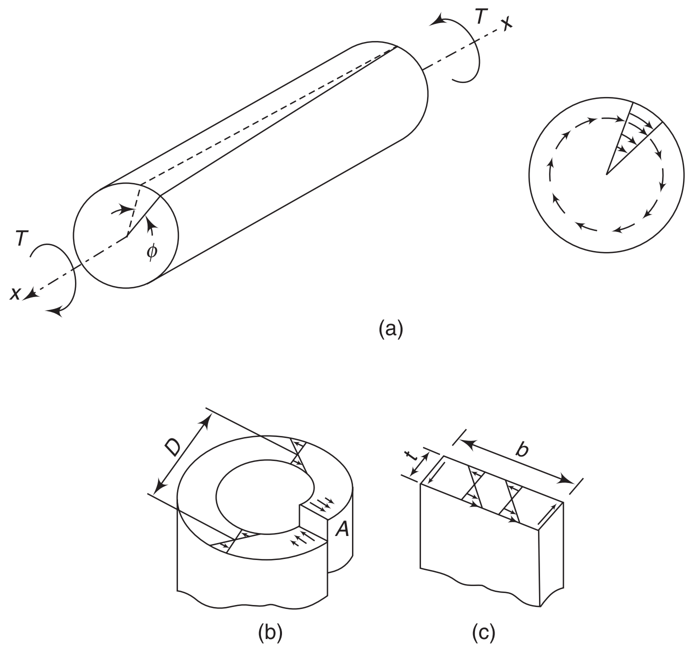


Fig. 18.4 Shear stresses due to St Venant's torsion

where T is the applied torsional moment, G is the shearing modulus of elasticity = $E/[2(1 + \mu)]$, and I_p is the polar moment of inertia.

This equation is valid for a solid circular cross section also. In this case, the shear stress due to torsion varies linearly from zero, at the centroid, to a maximum value, at the extreme fibre, and is given by

$$f_{st} = Tr/I_p \tag{18.2}$$

where f_{st} is the shear stress due to torsion and r is the distance from centroid of the section.

For the specific case of a circular section of diameter d, no warping of the cross section occurs and $I_p = \pi d^4/32$. Hence, the maximum shear stress at $r = d/2$ equals

$$\text{Max. } f_{st} = 16T/(\pi d^3) \tag{18.3}$$

for the hollow section, with outer radius R_o and inner radius R_i, $I_p = \pi(R_o^4 - R_i^4)/2$

$$\text{Max. } f_{st} = 2TR_o/[\pi(R_o^4 - R_i^4)] \tag{18.4}$$

The French engineer, Adhemar Jean Barre de St Venant, in 1853 showed that when torsion is applied to other solid or hollow sections, including the rectangular hollow sections of constant wall thickness, the assumption that plane sections remain plane after deformation (twisting) is not valid. The original cross section plane surface becomes a warped surface. That is, when the bar twists, each section rotates about its torsional centre and the radial lines through the torsional centre do not remain straight. The distribution of shear stress on the section is not linear and the direction of shear stress is not normal to a radius, though essentially the angle of twist is unaffected.

When a torque is applied to a non-circular cross section (e.g., a rectangular cross section), the transverse sections which are plane prior to twisting, warp in the axial direction, as described previously, so that a plane cross section no longer remains plane after twisting. However, as long as warping is allowed to take place freely, the applied load is still resisted by shearing stresses similar to those in the circular bar. St Venant's torsional stress (f_{st}) can be computed by an equation similar to Eqn (18.1) but by replacing I_p with I_t, the torsional constant.

Thus, the angle of twist *per unit length* of a non-circular cross section, solid or tubular section is given by the equation

$$\phi = T/(GI_t) \tag{18.5}$$

where I_t is the St Venant's torsional constant (in mm^4). Note that in literature I_t is often referred to as J.

The St Venant's torsional shear stress of thin walled box sections, in which the thickness t_i is small compared to the transverse dimension is given approximately by (Bredt's formula)

$$f_{st} = T/(2At) \tag{18.6a}$$

where A is the area contained in the mean line of the wall.

The St Venant's torsional shear stress of thin walled open section is given by

$$f_{st} = Tt_i/I_t \tag{18.6b}$$

The St Venant's torsional constant of the split tube shown in Fig. 18.4(b) is given by

$$I_t = bt^3/3 \tag{18.7}$$

where b is equal to the circumference and t is the thickness of the tube. Thus,

$$I_t = \pi D t^3/3$$

If we compare the polar moment of inertia of the circular tube with the torsional constant of the split tube with $t = (R_o - R_i)$ and $D = R_o + R_i$

$$(I_p/I_t) = [\pi Dt(R_o^2 + R_i)^2/2]/[\pi Dt^3/3]$$

$$= (3/2)(R_o^2 + R_i)^2/t^2$$

Thus, a 250-mm NB tube with 25-mm wall thickness is 75 times as stiff as a split tube with the same dimensions.

The shearing stress result, when a solid rectangular section is twisted, is shown in Fig. 18.4(c). The value of torsional constant in this case is given by

$$I_t = [bt^3/3]\,(1 - 0.630t/b) \text{ for } b > t$$

where b and t are the dimensions shown in the figure. If b/t is large (greater than 10), this equation may be written as

$$I_t = bt^3/3 \tag{18.8}$$

The torsional constant of any shape, composed of rectangular and/or curved elements for which b/t is sufficiently large, can be determined by adding the quantities $bt^3/3$ for all the elements, provided no part of the cross section is closed. Such a section is called an open section (e.g., angles, channels, I-sections, T-sections). RHS, SHS, tubes, box sections, etc. are called closed sections. Compound sections may be considered as closed, if the open sides are adequately laced [see Figs 9.24(i)-(m) and 9.24(p)]. The torsion constant of a single cell closed section is given by

$$I_t = (4A^2)/(\int ds/t) \qquad (18.9)$$

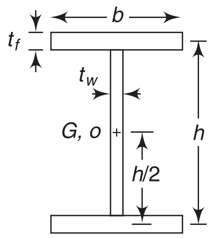
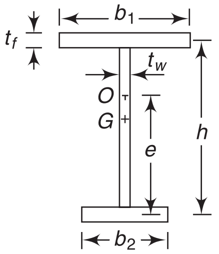
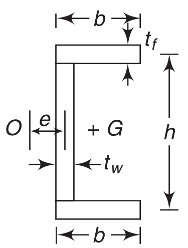
where A is the area enclosed by the midline of wall, ds is the element of the circumference of the wall, and t is the thickness of the wall.

The integration is around the entire periphery. Hence, for a square box section with 100 mm × 100 mm outside dimension and 12-mm thick wall,

$$I_t = [4(88 \times 88)^2]/(4 \times 88/12) = 8{,}177{,}664 \text{ mm}^4$$

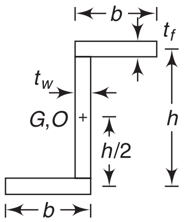
Values of I_t for multi-cell cross sections can be determined by the methods given in McGuire (1968). More accurate expressions for various structural shapes have been developed by EI Darwish and Johnston (1965). The tabulated values for St Venant torsional constant have been provided by Young and Budynas (2001). Some of these values for the most often used sections is given in Table 18.1.

Table 18.1 Properties of sections

	$I_t = \dfrac{2bt_f^3 + ht_w^3}{3}$ $I_w = \dfrac{t_f h^2 b^3}{24} = h^2 I_y/4$	If $t_f = t_w = t$, $I_t = \dfrac{t^3}{3}(2b + h)$
	$e = h\dfrac{b_1^3}{b_1^3 + b_2^3}$ $I_t = \dfrac{(b_1 + b_2)t_f^3 + ht_w^3}{3}$ $I_w = \dfrac{t_f h^2}{12}\dfrac{b_1^3 b_2^3}{(b_1^3 + b_2^3)}$	If $t_f = t_w = t$, $I_t = \dfrac{t^3}{3}(b_1 + b_2 + h)$
	$e = \dfrac{3b^2 t_f}{(6bt_f + ht_w)}$ $I_t = \dfrac{2bt_w^3 + ht_w^3}{3}$ $I_w = \dfrac{t_f b^3 h^2}{12}\dfrac{(3bt_f + 2ht_w)}{(6bt_f + ht_w)}$	If $t_f = t_w = t$, $e = \dfrac{3b^2 t_f}{6b + h}$ $I_t = \dfrac{t^3}{3}(2b + h)$ $I_w = \dfrac{tb^3h^2}{12}\dfrac{(3b + 2h)}{(6b + h)}$

(*contd*)

(*contd*)



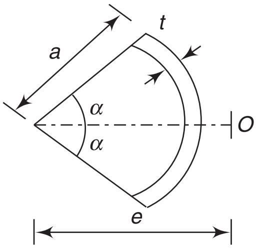
$$I_t = \frac{2bt_f^3 + ht_w^3}{3}$$

$$I_w = \frac{b^3h^2}{12(2b+h)^2} \times [2t_f(b^2 + bh + h^2) + 3t_w bh]$$

If $t_f = t_w = t$,

$$I_t = \frac{t^3}{3}(2b + h)$$

$$I_w = \frac{tb^3h^2}{12}\frac{(b + 2h)}{(2b + h)}$$



$$e = 2a\frac{\sin\alpha - \alpha\cos\alpha}{\alpha - \sin\alpha\cos\alpha}$$

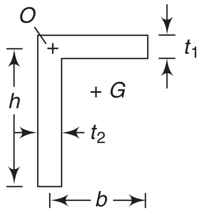
$$I_t = \frac{2a\alpha t^3}{3}$$

$$I_w = \frac{2ta^5}{3} \times \left[\alpha^3 - \frac{6(\sin\alpha - \alpha\cos\alpha)^2}{\alpha - \sin\alpha\cos\alpha}\right]$$

If $2\alpha = \pi$,

$$e = \frac{4\alpha}{\pi} \quad I_t = \frac{\pi a t^3}{3}$$

$$I_w = \frac{2ta^5}{3}\left(\frac{\pi^3}{8} - \frac{12}{\pi}\right) = 0.0374ta^5$$



$$I_t = (bt_1^3 + ht_2^3)/3$$

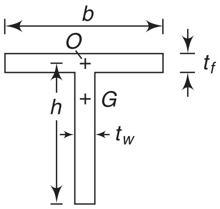
$$I_w = (b^3t_1^3 + h^3t_2^3)/36$$

≈ zero for small t

If $t = t_1 = t_2$,

$$I_t = t^3(b + h)/3$$

$$I_w = t^3(b^3 + h^3)/36$$



$$I_t = (bt_f^3 + ht_w^3)/3$$

$$I_w = (b^3t_f^3/4 + h^3t_w^3)/36$$

≈ zero for small t

$$e = (t_f + h)/2[1/(1 + b^3t_f/(t_w^3 h)]$$

If $t = t_f = t_w$

$$I_t = t^3(b + h)/3$$

$$I_w = t^3(b^3/4 + h^3)/36$$

It is to be noted that the stress distribution as shown in Figs 18.4(b) and (c) will be uniform throughout the length of a bar, if warping is not restrained and is uniform throughout its length. Torsion with uniform warping is usually called St Venant's torsion.

18.2.2 Non-uniform Warping Torsion

Uniform warping of an I-section [see Fig. 18.5(a)] is shown in Fig. 18.5(b). However, most of the structural members will be supported in such a way as to prevent uniform warping. Figure 18.5(c) shows the non-uniform warping of a cantilevered I-section, where the warping is prevented at its left end. Such non-uniform warping

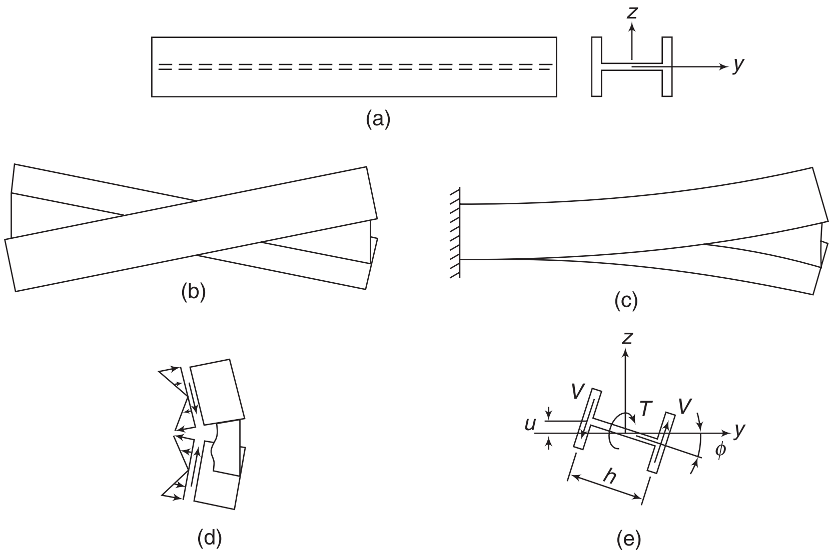


Fig. 18.5 Warping of I-sections

results in additional shear stresses and an increase in the torsional stiffness. In this case, the oppositely directed bending of the flanges of the I-sections produces shears (V) as shown in Fig. 18.5(d), which constitutes a couple opposing the applied torque T as shown in Fig. 18.5(d). In addition, there are bending stresses. Figure 18.6 gives an outline of all these stresses produced by warping torsion in I-sections and channels. Warping deflections due to the displacement of the flanges vary along the length of the member. The torsional and warping deformations of I- and box-sections are shown in Fig. 18.7.

Available literature on the elastic analysis of I-beams subject to torsion is based on fairly complex analytical techniques. They consider the total torsional moment on I- or channel sections as composed of the sum of the St Venant's torsion and warping torsion as

$$T = T_v + T_w \tag{18.10}$$

The solution of the torsion problem may be obtained by solving the differential equation

$$T = GI_t\,(d\phi/dx) - EI_w(d^3\phi/dx^3) \tag{18.11}$$

where ϕ is the angle of twist, I_t is the torsional constant of the cross section, I_w is the warping constant of the cross section, and E and G are the elastic and shear modulus of rigidity.

The torsional moment T depends on the loading and in usual situations will be a polynomial in x. For I-sections torsionally simply supported at ends and with a concentrated torsional moment applied at mid span, the solution to the differential equation may be obtained as (Nethercot et al. 1989; Salmon & Johnson 1996)

$$\phi = [T/(2GI_t\lambda)]\ [\lambda x - (\sinh(\lambda x)/\cosh(\lambda L/2))] \tag{18.12a}$$

and

$$d\phi/dx = \phi' = [T/(2GI_t)]\ [1 - (\cosh(\lambda x)/\cosh(\lambda L/2))] \tag{18.12b}$$

$$d^2\phi/dx^2 = \phi'' = [T\lambda/(2GI_t)]\ [-\sinh(\lambda x)/\cosh(\lambda L/2)] \tag{18.12c}$$

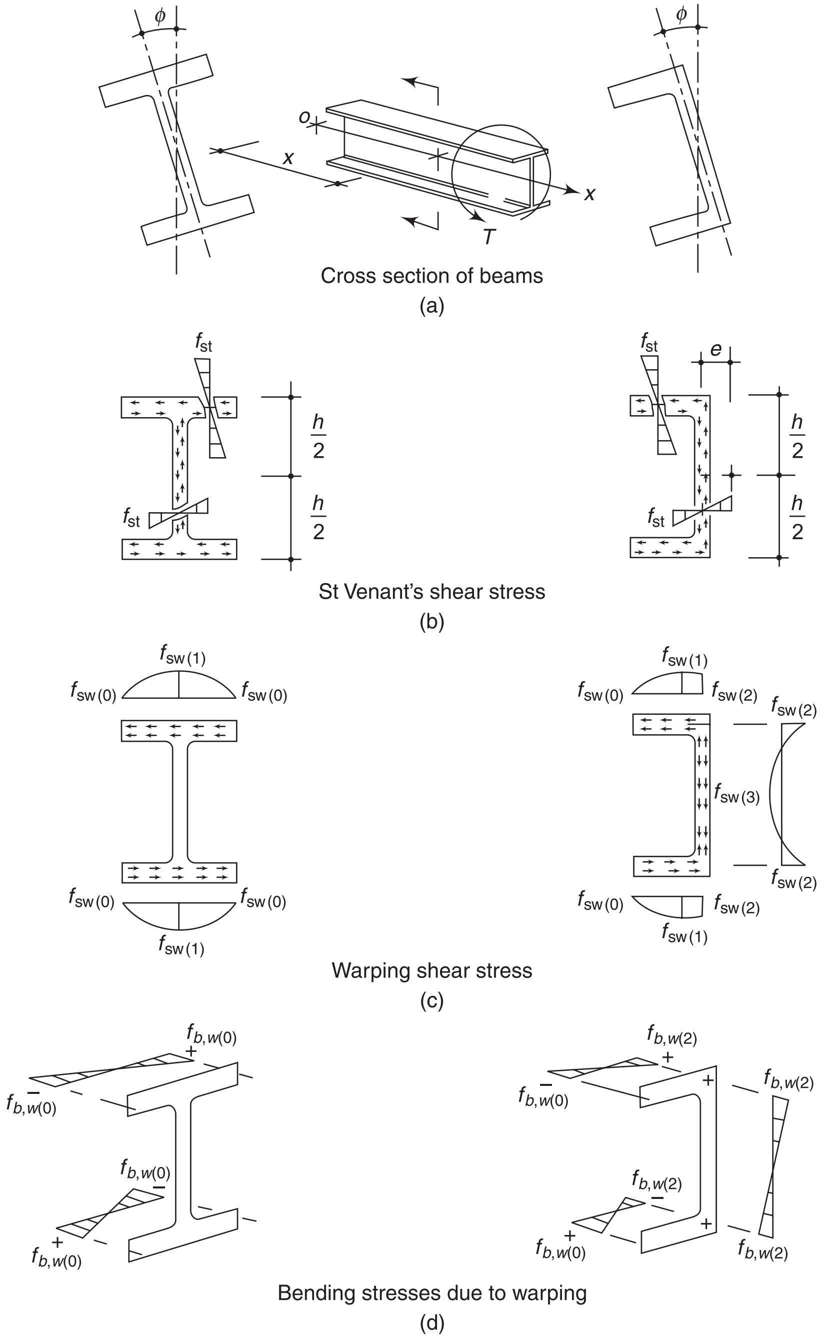


Fig. 18.6 Stresses in I-section and channels due to warping torsion (Ballio & Mazzolani 1983)

$$d^3\phi/dx^3 = \phi''' = [T\lambda^2/(2GI_t)]\,[-\cosh(\lambda x)/\cosh(\lambda L/2)] \tag{18.12d}$$

where $\lambda^2 = (GI_t/EI_w)$ (18.12e)

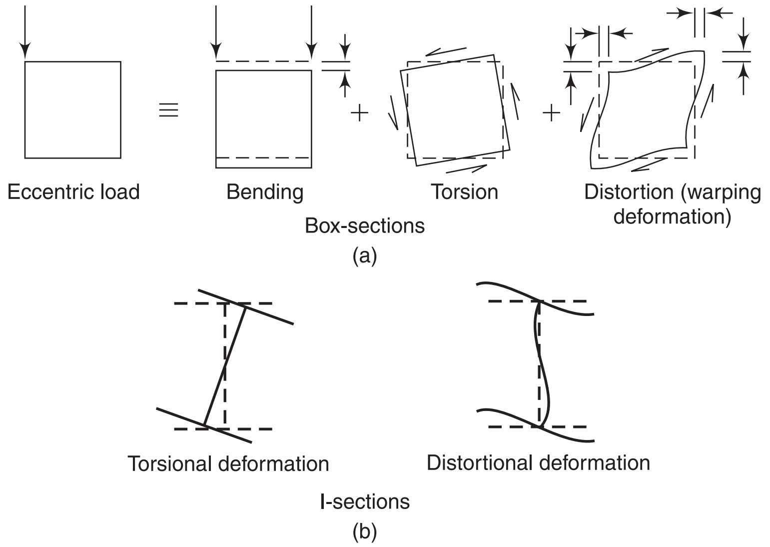


Fig. 18.7 Deformation of cross sections due to torsion and warping

From these equations, the shear stress f_{st} due to St Venant's torsion may be computed as

$$f_{st} = Gt\phi' \tag{18.13}$$

where t is the thickness of the element under consideration. Though this stress is shown as uniform across the flange of an I-section in Fig. 18.6, the stress drops sharply to zero at the flange tips (Salmon & Johnson 1996). Similarly, the warping shear stress f_{sw}, which varies parabolically across the width of the rectangular flange {see Fig. 18.6(c)} is given by

$$f_{sw} = -(Eh/2t_f)Q_f\phi''' \tag{18.14a}$$

where Q_f is the statical moment of the area about the y-axis.

For maximum shear stress, which actually acts at the face of the web but may be approximated as acting at the mid width of the flange, Q_f may be taken as

$$Q_f = A\bar{Z} = (bt_f/2)(b/4) = b^2t_f/8 \tag{18.14b}$$

Substituting Q_f in Eqn (18.14a), we get the maximum absolute value of warping stress, which is

$$f_{sw} = (Eb^2h/16)\,\phi''' \tag{18.14c}$$

where b is the breadth of the beam and h is the distance between the flange centroids.

The tension or compressive stress due to the lateral bending of flanges caused because of warping, which varies linearly across the flange width {see Fig. 18.6(d)} is given by

$$f_{b,w} = (EI_w/h)Z/(I_f\phi'') \tag{18.15a}$$

where I_f is the moment of inertia for one flange about the y-axis of the beam. The maximum stress occurs at $Z = b/2$. Noting that $I_w = I_f h^2/2$ and using Eqn (18.15a), we get the maximum value for bending stress due to warping as

$$f_{b,w} = (Ebh/4)\phi'' \tag{18.15b}$$

Note that the solutions for ϕ given in Eqn (18.12), is applicable only for the case of torque applied at the middle of the beam and for simply supported end conditions. The methods of evaluating ϕ, ϕ', ϕ'', and ϕ''' for the various conditions of loading and boundary conditions are given by Nethercot et al. (1989) and AISC (1983). Some approximate solutions for both elastic and plastic collapse of beams subjected to torsion for various loading and boundary conditions are given by Trahair et al. (2001). The values of torsional constant I_t and warping constant I_w are given in Table 18.1 for various sections.

Note that the torsional behaviour of hollow and open sections are substantially different. The value of the coefficient $k_t = L\sqrt{[GI_t/(EI_w)]}$ may be used to indicate whether pure torsion or warping effects are predominant. An approximate classification of the torsional behaviour of various types of thin walled sections, in terms of the coefficient k_t, has been provided by Ballio and Mazzolani (1983) as follows (see Fig. 18.8):

- *Torsion due to warping only ($0 < k_t < 0.5$)* Thin walled cold formed sections and open section bridge girders made of orthotropic plates
- *Warping prevalent ($0.5 < k_t < 2$)* Cylindrical thin shells and composite steel/concrete bridges having an open section
- *Warping and uniform torsion ($2 < k_t < 5$)* Hot rolled sections (I- and channel sections)
- *Uniform torsion prevalent ($5 < k_t < 20$)* Stocky sections (profiles for rails and jumbo shapes) and hollow sections (tubes and box sections)
- *Uniform torsion of St Venant type only ($20 < k_t < \infty$)* Solid compact sections

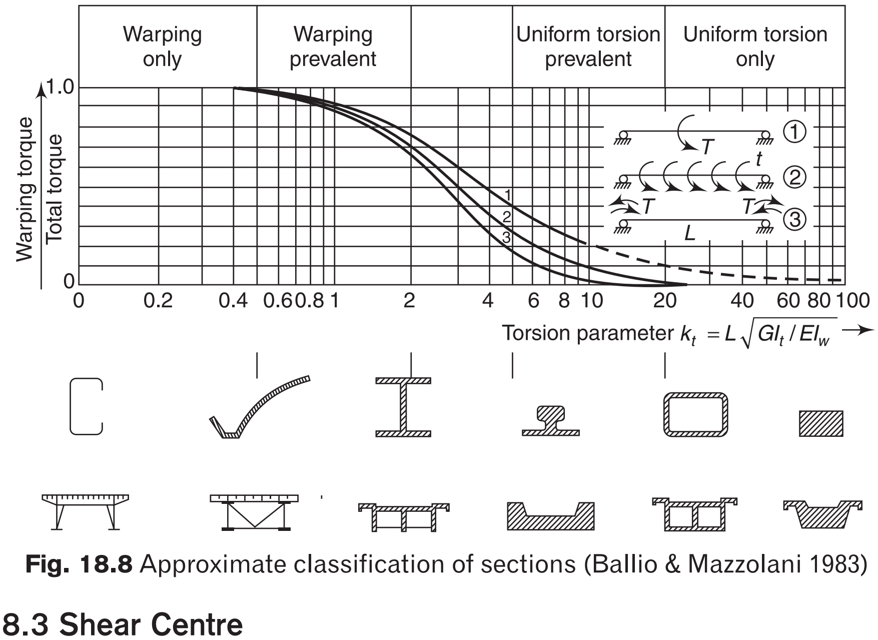


Fig. 18.8 Approximate classification of sections (Ballio & Mazzolani 1983)

18.3 Shear Centre

The *shear centre* is defined as a point in the cross section through which the lateral (or transverse) loads must pass in order to produce bending without twisting. The bending stresses are given by $f_b = M_z y/I_z$ and the shear stresses $f_s = V_y A\bar{y}/I_z$ (see Chapter 10). In other words, forces acting through the shear centre will not cause any torsional stresses to develop. For loads not applied through the shear centre, there will be additional stresses due to twisting because of torsional moments. These torsional moments are shown in Fig. 18.9. Since the shear centre (also called the centre of rotation) does not necessarily coincide with the centroid of the section, the location of the shear centre must be determined in order to evaluate the torsional stresses. For I- and Z-sections, the centroid and shear centre coincide but for the other sections such as channels and angles, they do not coincide (see Fig. 18.10). The location of shear centre is dependent only on the cross-sectional configuration of a member.

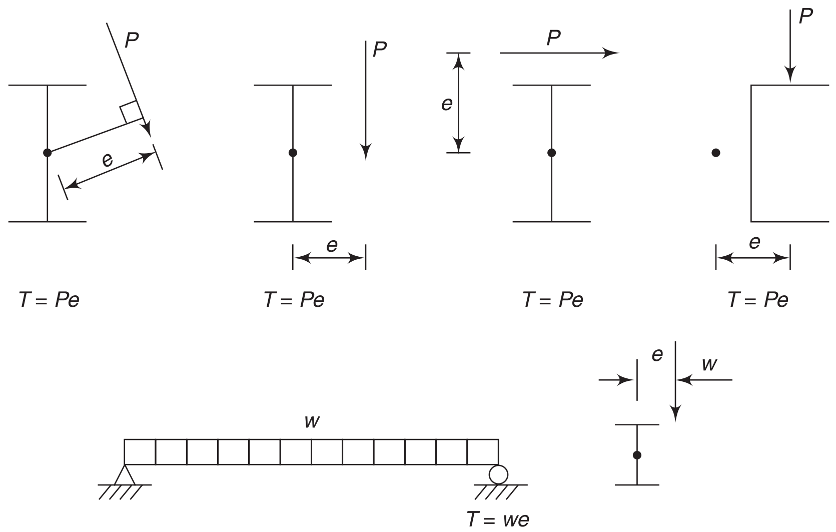


Fig. 18.9 Torsional moments due to load not acting through shear centre

The computation of the position of the shear centre is complicated for all but the simplest shapes and readers can refer to Salmon and Johnson (1996) and Trahair et al. (2001) for the calculation procedures. However, the following may be observed (see Fig. 18.10).

- For sections having an axis of symmetry, the shear centre and centroid lie on this axis.
- For sections having symmetry about both the axes, the shear centre is on the intersection of the two axes, i.e., shear centre and centroid coincide (for I- and Z-sections)
- For all sections consisting of two intersecting plate elements (angles, Ts, and cruciforms), the shear centre is at the plate intersection point.
- For channel sections, the shear centre lies outside the web and the centroid inside it.

The distance of shear centre for various sections along with the values of I_t and I_w are given in Table 18.1.

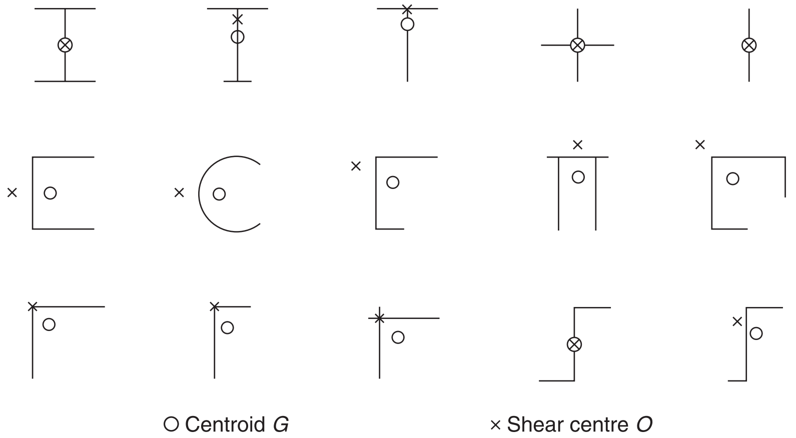


Fig. 18.10 Centroids and shear centres of various cross sections

18.4 Approximate Design Procedure for Torsion

In most practical design situations, when it is desirable to include the effect of torsion, the compressive stress due to the warping component is the quantity of most importance. The shear stress contributions are normally not significant (Salmon & Johnson 1996). Since the design for torsion as discussed in the previous sections is complicated, an approximate method was suggested by Galambos et al. (1996) and Salmon and Johnson (1996), which considers only the compressive stress due to torsion. This method may be used for the preliminary design of sections. It is limited to sections capable of developing plastic hinges and is based on the analogy between torsion and ordinary bending. This method is best demonstrated by considering a cantilever beam shown in Fig. 18.11, which is subjected to a transverse load P acting at an eccentricity e from the centroid. In this case, the applied moment on the cantilever $M_z = PL$ and torsional moment $T = Pe$. Assuming that the individual flanges will resist the torsional moment, the corresponding applied bending moment of the flanges M_f is given by (Galambos et al. 1996)

$$M_f = PeL/h \tag{18.16}$$

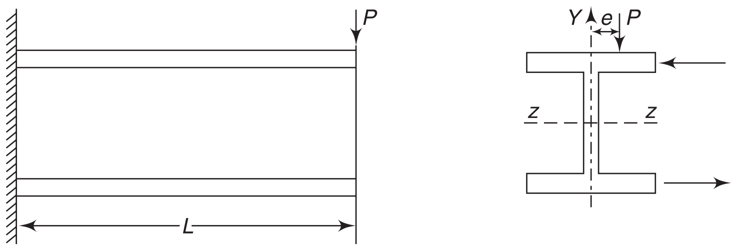


Fig. 18.11 Cantilever beam subjected to eccentric point load at free end

Hence, the required plastic section modulus $Z_{p,z}$, to resist the major axis bending, can be calculated from

$$(M_z/Z_{p,z}) + (2M_f/Z_{p,y}) \le f_y/\gamma_m \tag{18.17}$$

From the above,

$$Z_{p,z} \geq (M_z + 2C_n M_f)/(f_y/\gamma_m) \tag{18.18a}$$

where $C_n = (Z_{p,z}/Z_{p,y})$. (18.18b)

Similarly, for applied loads producing bending about the weak axis, the required plastic section modulus $Z_{p,y}$ about y-axis may be obtained from

$$Z_{p,y} \geq (M_y + 2M_f)/(f_y/\gamma_m) \tag{18.19}$$

Galambos et al. (1996) also provide simple approximate formulae for calculating the maximum flange moment M_f and maximum angle of twist ϕ. The main advantage of these formulae is that they do not involve the evaluation of hyperbolic or exponential functions. Since the method is based on plastic analysis, the rotations calculated based on the method may be large in some cases.

Methods for the design of I-beams subjected to bending and torsion are provided by Johnston (1982) and Nethercot et al. (1989). Simplified torsion design of compact I-beam is provided by Pi and Trahair (1996) and inelastic torsion and bending by Pi and Trahair (1994) and Pi and Trahair (1995). Plastic collapse analysis and design of warping torsion and non-uniform torsion (sections in which both uniform and warping torsion are prevalent), including interaction of local buckling and torsion and serviceability design are discussed by Trahair et al. (2001).

18.4.1 Buckling Check

Whenever lateral torsional buckling governs the design (i.e., when f_b is less than f_y) the values of f_w and f_{byt} will be amplified. Nethercot et al. (1989) have suggested a simple 'buckling check' along lines similar to the British code BS 5950, Part 1

$$\frac{\overline{M}_z}{M_b} + \frac{(f_{\text{byt}} + f_w)}{(f_y/\gamma_m)}\left[1 + 0.5\frac{\overline{M}_z}{M_b}\right] \leq 1 \tag{18.20}$$

f_w = warping normal stress

$f_{\text{byt}} = M_{\text{yt}}/Z_y$

$M_x = \phi M_z$

where equivalent uniform moment $\overline{M}_z = m_z M_z$

and

the buckling resistance moment $M_b = \dfrac{M_E M_p}{\phi_B + (\phi_B^2 - M_E M_P)^{1/2}}$ (18.20a)

In Eqn (18.20a),

$$\phi_B = \frac{M_p + (\eta_{\text{LT}} + 1)M_E}{2} \tag{18.20b}$$

the plastic moment capacity $M_P = f_y \cdot Z_p/\gamma_m$

Z_p = the plastic section modulus

the elastic critical moment $M_E = \dfrac{M_p \pi^2 E}{\lambda_{\text{LT}}^2 (f_y/\gamma_m)}$ (18.20c)

where λ_{LT} is the equivalent slenderness.

In Eqn (18.20), the second term allows for the amplification of the stresses due to warping and minor axis bending caused by twisting due to lateral torsional buckling.

18.4.2 Applied Loading Having Both Major Axis and Minor Axis Moments

When the applied loading produces both major axis and minor axis moments, the ‘capacity checks’ and the ‘buckling checks’ are modified as follows:

Capacity check

$$f_{bz} + f_{byt} + f_w + f_{by} \leq f_y/\gamma_m \tag{18.21}$$

This equation is a straightforward capacity check involving the summation of stresses due to major and minor axis bending and warping, these stresses being coincident values. It is likely to be conservative due to the use of the material yield stress as the limit, i.e., it does not even allow for the development of limited plasticity within the cross section.

Buckling check

$$\frac{\overline{M}_z}{M_b} + \frac{\overline{M}_y}{f_y Z_y/\gamma_m} + \frac{(f_{byt} + f_w)}{(f_y/\gamma_m)}\left[1 + 0.5\frac{\overline{M}_z}{M_b}\right] \leq 1 \tag{18.22}$$

where $\overline{M}_y = m_y M_y$ and $f_{byt} = M_{yt}/Z_y$.

18.4.3 Torsional Shear Stress

Torsional shear stresses and warping shear stresses should also be amplified in a similar manner

$$f_s = (f_{st} + f_{sw})\left(1 + 0.5\frac{\overline{M}_z}{M_b}\right) \tag{18.23}$$

This shear stress should be added to the shear stresses due to bending while checking the adequacy of the section.

18.5 Distortion of Thin Walled Members

Twisting and distortion of flexural members, as shown in Fig. 18.7, may be caused by the local distribution of the forces. If the member responds significantly to these actions, then the bending stress distribution may be much different from those calculated in the usual way, due to the additional distortional stresses induced by out-of-plane bending as shown in Fig. 18.7. To avoid possible failure, the designer may have to increase the strength of the member or limit both twisting and *distortion*. The resistance to distortion of thin walled members depends on the type of cross section. While open cross sections are more flexible, members having rectangular and trapezoidal cells offer little resistance and members with closed triangular cells have high resistance to distortion. Methods to account for distortion effects

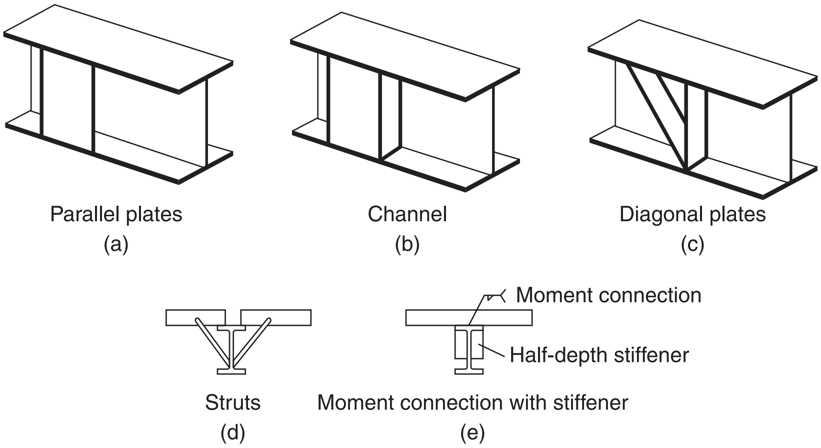


Fig. 18.12 Stiffening systems to reduce torsional effects in beams and columns

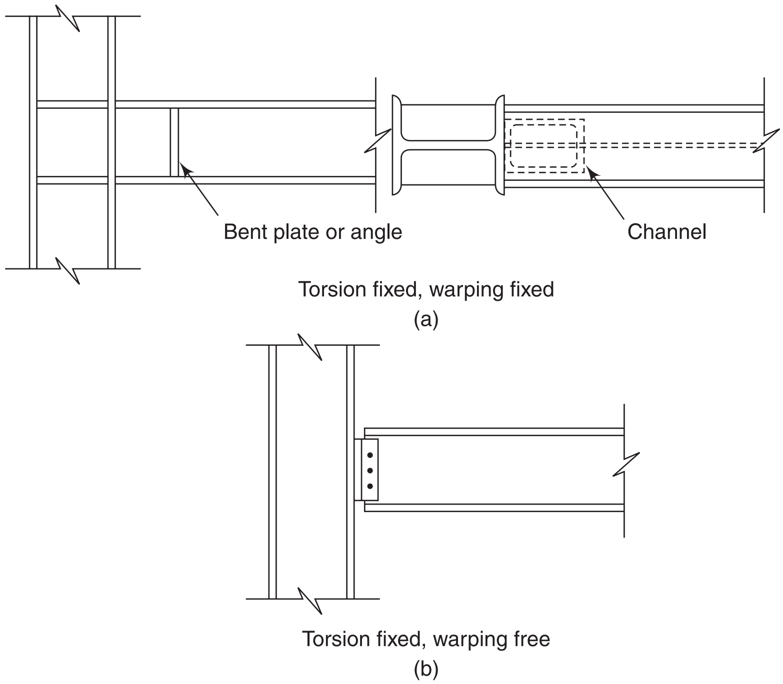


Fig. 18.13 End conditions (Narayanan et al. 2003)

have been discussed by Vacharajittiphan and Trahair (1974), Bradford (1992), and Subramanian (1982 and 1995).

18.6 Torsional Stiffening and End Restraints

As seen in the previous sections, the design for torsion is complex, especially when warping is involved. Careful detailing and selection of sections will result in minimizing or even eliminating the design complexities associated with torsion. Let us discuss some of these methods in this section.

When torsion is unavoidable, the designer should consider using box sections, since they are more effective in resisting torsion. When local concentration of torque is introduced, triangular closed box sections may be used to prevent local distortion of the cross section. Alternatively, external bracings can be used (see Section 10.4.3.2). If external bracings are not feasible, then internal stiffening can be used to reduce stresses and displacements. Commonly used internal stiffening systems are as follows.

- Longitudinal plates may be welded at the ends of the I-section beam to prevent warping at the ends. The stiffening plates are to be placed parallel to the web and attached to the flange tips by welding as shown in Fig. 18.12(a). This will result in $\phi = 0$ and $\phi' = 0$ as per Hotchkiss (1966). The length of the 'boxing' plate should be equal to the depth of the beam.
- Welding a length of channel, angle, or bent plate between the flanges as shown in Figs18.13(a) and 18.12(b) to form a stiff box section will be more efficient than the parallel plates shown in Fig. 18.12(a) to obtain $\phi = 0$ and $\phi' = 0$ at the ends.
- Diagonal plates, consisting of two plates perpendicular to the beam web; one oriented at 45° to the longitudinal axis and the other perpendicular to the longitudinal axis as shown in Fig. 18.12(c) have also been suggested (Narayanan et al. 2003).

The designer should remember that if no torsional stiffening is done at the ends, the available torsional restraint is neither simple ($\phi'' = 0$) nor fixed ($\phi' = 0$) but such that the end twist is nearly zero ($\phi = 0$) (Salmon & Johnson 1996; Szewczak et al. 1983). The designer should also ensure the following.

- Beams subjected to torsion should have sufficient stiffness (in the minor axis) and strength to resist the torsional moments in addition to other moments and forces. Note that wide flange beams resist torsion better than ISMB beams.
- The connections and bracings of such members should be idealized properly in the analysis and carefully designed to ensure that the reactions are transferred to the supports (see Fig. 18.13).
- Stresses and deflections due to combined effects should be within the specified limits.
- Whenever lateral torsional buckling governs the design, a simple buckling check must be performed (Narayanan et al. 2003).

18.7 Torsional Buckling

Some thin walled sections such as angles, Ts, Zs, cruciform sections and channels having relatively low torsional stiffness may, under axial compression, buckle torsionally while the longitudinal axis remains straight (Timoshenko & Gere 1961; Bleich 1952). Consider the buckling of a doubly symmetrical cruciform section (whose shear centre and centroid coincide) as shown in Fig. 18.14. Cruciform sections (made of four angles) are often used in tower legs. When a compressive

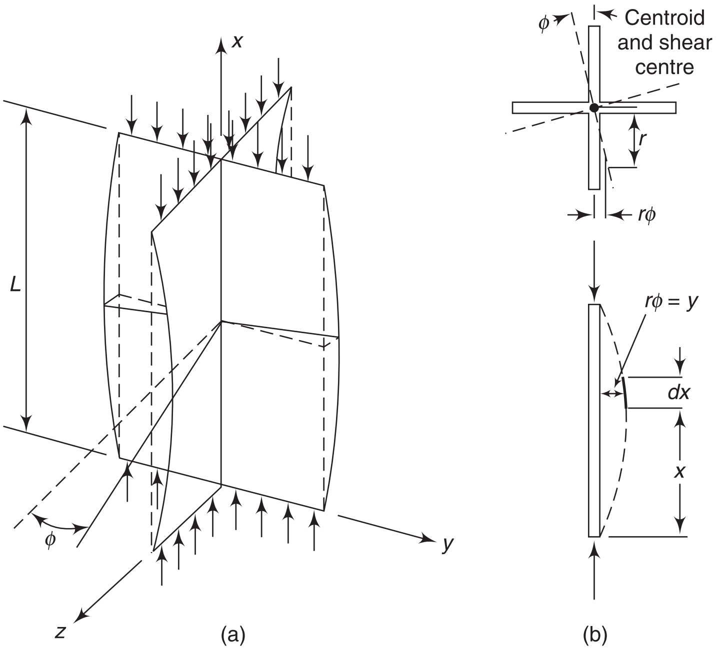


Fig. 18.14 Torsional buckling of a cruciform strut

load acts on this strut, a form of buckling occurs in which the axis remains straight but sections of the member rotate (i.e., the member twists). This kind of buckling is termed as *torsional buckling*. The analysis of torsional buckling is quite complex. The mode of buckling clearly depends on the restraints provided at the ends. The critical stress depends on the torsional stiffness of the member as well as on the resistance to warping deformations provided by the member and by the restraints at the ends.

The differential equation for equilibrium for the section shown in Fig. 18.14 is given by (Timoshenko & Gere 1961)

$$d^4\phi/dx^4 + p^2 d^2\phi/dx^2 = 0 \tag{18.24a}$$

in which

$$p^2 = (f_x I_p - GI_t)/(EI_w) \tag{18.24b}$$

where f_x is the axial stress = P/A, ϕ is the angle of twist, I_p is the polar second moment of the area of the section about the shear centre = $I_z + I_y$, I_t is St Venant's torsional constant of the section, I_w is the warping constant of the section, and G and E are shear and Young's modulus of rigidity of the material.

The general solutions to the above differential equation is given by

$$\phi = A_1 \sin px + A_2 \cos px + A_3 x + A_4$$

A pin-ended column, with rotation about x-axis prevented at each end but with warping not restricted at the ends, results in the following solution for the smallest critical stress (Bleich 1952)

$$f_{cr} = GI_t/I_p + (4\pi^2/L^2)\,(EI_w/I_p) \tag{18.25a}$$

This equation gives the stress f_{cr}, at which torsional buckling begins, provided the strut is perfectly straight, free of residual stress, etc. Although this equation was derived for the cruciform cross section shown in Fig. 18.14, it holds for any cross section for which the shear centre and centroid coincide. Equation (18.25a) can be expressed in terms of the effective length as

$$f_{cr} = [(GI_t)/I_p] + [\pi^2/(KL)^2]\,[(EI_w)/I_p] \qquad (18.25b)$$

where $K = 1$, if the ends are free to warp, and $K = 0.5$, if warping at the ends are completely restrained.

It should be noted that Eqn (18.25) is applicable only for buckling, which begins when the stress f_{cr} is less than the proportional limit stress of steel. Hence, the most probable buckling mode may involve the tangent modulus or double modulus of lateral bending about the z and y axes. Thus, the problem involves three critical values of axial load; bending about either principal axis and twisting about the longitudinal axis. In wide flange sections, with extra wide flanges and having short lengths, torsional buckling may be important.

It is important to observe that pure torsional buckling is possible only if the centroid and shear centre of the cross section are coincident. Thus, the critical value of buckling stress is strictly applicable to sections having either double or point symmetry. However, for common single angle struts, since the distance between the centroid and the shear centre of the cross section is small, Eqn (18.25b) will provide a reasonable approximation of the torsional buckling stress.

For doubly or point symmetric sections (such as built-up I-sections having thin elements, cruciform sections, and Z-sections), when the flexural torsional limit state is evaluated, an equivalent radius of gyration r_e may be compared with r_z and r_y, to reduce the computations. To develop the r_e equation, set Eqn (18.25b) equal to the Euler equation {Eqn (9.9)}.

$$(\pi^2 E)/(KL/r_e)^2 = (GI_t/I_p) + EI_w\pi^2/[I_p(KL)^2]$$

Thus,

$$r_e = \sqrt{[I_w/I_p + GI_t(K_xL)^2/(EI_p\pi^2)]}$$

Substituting $E/G = 2.6$, we get

$$r_e = \sqrt{[(I_w/I_p) + 0.04(I_t(K_xL^2)I_p)]} \qquad (18.26)$$

for doubly symmetric sections. Note that only for short lengths, r_e will be lower than r_z or r_y for I-sections.

Using the value of r_e from Eqn (18.26), the slenderness ratio (KL/r_e) can be calculated and the design compressive stress about z-z and y-y axes can be computed using Tables 9.3 and 9.4 and Fig. 9.23 given in Chapter 9 (Tables 7.4, 7.2, and Fig. 7.1 of the code). Thus, the column strength Eqns (9.38) and (9.39) are generic equations that can be used for torsional flexural or flexural torsional buckling. In Eqn (18.26), the torsional buckling effective length factor K_x may be conservatively taken as 1.0. For greater accuracy, if both ends of the column have a connection that restraints warping (say by boxing the end over a length at least equal to the depth of the member), we can take $K_x = 0.5$. If one end of the member is restrained from warping and the other end is free to warp, then we can take $K_x = 0.7$.

18.8 Torsional Deformations

The angle of rotation in radians, for all types of cross sections is

$$\phi = Tx/GI_t \quad (18.27)$$

where T is the applied torsional moment (due to working load), in kN m, x is the distance from support, in mm, G is the shear modulus of rigidity (for steel $= E/2.6$), in N/mm^2, and I_t is the torsional constant, in mm^4.

When bending moments are applied along with torsion, it will be usually sufficient to determine the deformations at working loads due to

(a) major axis bending,
(b) minor axis bending, and
(c) torsion

separately, and if required combine them vectorially to determine the actual movement.

Examples

Example 18.1 *A twisting moment of 15 kN m is applied at the end of a 1.5-m long shaft as shown in Fig. 18.15(a). Determine (a) the maximum shear stress, and (b) the maximum angle of rotation assuming the sections as shown in Figs 18.15(b), (c) and (d). Assume G = E/2.6 = 76923 N/mm² and St Venant's torsion.*

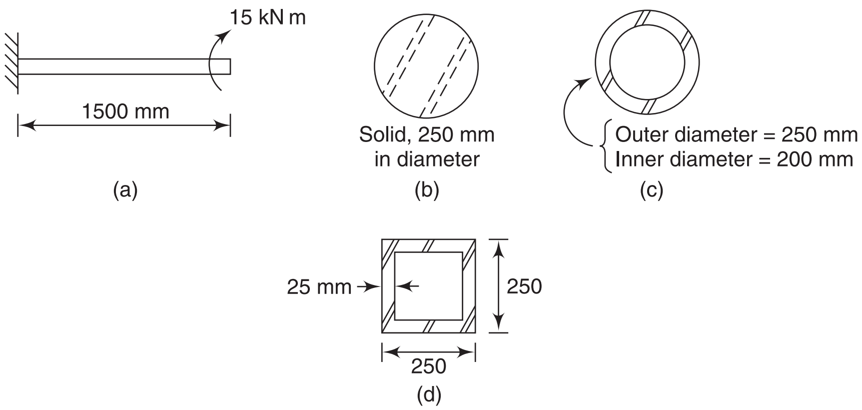


Fig. 18.15

Solution

(a) Solid circular cross section

The only force is the torsional moment and no axial forces and bending moments are acting on the shaft. Because round bars do not warp, this is the case of St Venant torsion. Normal stresses are zero throughout the shaft.

Torsional stress,

$$f_{st} = Tr/I_p$$

where $I_p = I_t = \pi R^4/2$

The maximum stresses are at the outer edge, where $r = R = 250/2 = 125$ mm

Thus,

$$I_t = \pi \times 125^4/2 = 383.5 \times 10^6 \text{ mm}^4$$

The maximum shear stress,

$$f_{st} = TR/I_t = 15 \times 10^6 \times 125/383.5 \times 10^6 = 4.9 \text{ N/mm}^2$$

The maximum angle of rotation occurs at the free end, thus,

$$\phi = TL/GI_t = 15 \times 10^6 \times 1500/(76923 \times 383.5 \times 10^6)$$
$$= 7.627 \times 10^{-4} \text{ radians}$$

To convert it into degrees, recall that $360° = 2\pi$ radians. Hence,

$$\phi = 7.627 \times 10^{-4} \times 180/\pi = 0.044°$$

(b) The torsional behaviour of a hollow circular shaft is similar to that of a round bar. Thus for St Venant torsion there is no warping and there are no normal stresses. The torsional shear stress is given by

$$f_{st} = Tr/I_t \text{ and } I_t = \pi/2\,(R_0^4 - R_i^4)$$

Thus,

$$I_t = \pi/2\,(125^4 - 100^4) = 226.42 \times 10^6 \text{ mm}^4$$

The maximum shear stress is at the outer edge, where $r = R_0 = 125$ mm. Hence,

$$f_{st} = 15 \times 10^6 \times 125/(226.42 \times 10^6) = 8.28 \text{ N/mm}^2$$

The angle of twist or rotation in radians

$$\phi = (180/\pi)(TL/GI_t) = (180/\pi)(15 \times 10^6 \times 1500/(226.42 \times 10^6 \times 76923)) = 0.074°$$

(c) When a square tube is twisted, warping is minor. Hence, the normal and shear stresses due to warping may be neglected. For a square tube, the St Venant torsional stresses are

$$f_{st} = T/(2At)$$

where A is the area contained in the mean line of walls.

$$A = (250 - 25)^2 = 50{,}625 \text{ mm}^2$$

$$f_{st} = 15 \times 10^6/(2 \times 50625 \times 25) = 5.97 \text{ N/mm}^2$$

$$I_t = 4A^2/(\textstyle\int ds/t) = 4 \times 50625^2/[(225/25) \times 4]$$
$$= 284.77 \times 10^6 \text{ mm}^4$$

The maximum angle of rotation will be at the free end.

$$\phi = (180/\pi)TL/GI_t$$
$$= (180/\pi) \times 15 \times 10^6 \times 1500/(76923 \times 284.77 \times 10^6)$$
$$= 0.0589°$$

Example 18.2 *A wide flange section W 460 × 190 × 106 beam used for a 7 m simply supported span is loaded with a concentrated load of 80 kN at mid-span. The ends of the beam are simply supported with respect to torsional restraint (i.e., $\phi = 0$) and the concentrated load acts with a 50-mm eccentricity from the plane of the web (see Fig. 18.16). Compute the combined bending and torsional stresses.*

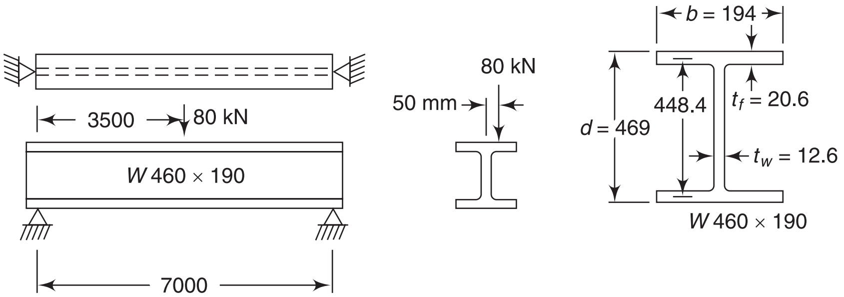


Fig. 18.16

Solution

The solution of the differential equation for this type of loading and end condition is

$$\phi = (T/2G\lambda I_t)\,[\lambda x - (\sinh\lambda x/\cosh \lambda L/2)]$$

Applied torsional moment,

$$T = 80 \times 50 = 4000 \text{ kN mm}$$

$$I_t = \Sigma bt^3/3 = 1/3[2 \times 194 \times 20.6^3 + 427.8 \times 12.6^3] = 1{,}415{,}861 \text{ mm}^4$$

for $\mu = 0.3$,

$$E/G = 2E(1 + \mu)/E = 2.6$$

$$I_w = I_f h^2/2 = (194^3 \times 20.6/12) \times (469 - 20.6)^2/2 = 1.26 \times 10^{12} \text{ mm}^6$$

Hence $\lambda = \sqrt{[GI_t/(EI_w)]} = \sqrt{[1{,}415{,}861/(2.6 \times 1.26 \times 10^{12})]}$

$$= 1/1521 = 6.574 \times 10^{-4}$$

The function values required are $k_t = \lambda L = 7000 \times 6.574 \times 10^{-4} = 4.6$.

Since k_t is greater than 2 and less than 5, both warping and uniform torsion effects will be predominant (see Section 18.2.2 and Fig. 18.8).

x	λx	$\sinh\lambda x$	$\cosh\lambda x$
0.1L	0.46	0.476	1.108
0.2L	0.92	1.055	1.454
0.3L	1.38	1.862	2.113
0.4L	1.84	3.069	3.228
0.5L	2.3	4.937	5.037

(a) Stresses due to St Venant's torsion (pure torsion):

$$f_{st} = Gt\,\phi'$$

where $\phi' = T/(2GI_t)\,[1 - (\cosh(\lambda x)/\cosh(\lambda L/2)]$

Hence,

$f_{st} = Tt/(2I_t)[1 - (\cosh(\lambda x)/5.037)]$

$= 4000 \times 10^3 t/(2 \times 1{,}415{,}861)\ [1 - (\cosh(\lambda x)/5.037)]$

The shear stress is maximum at $x = 0$ and zero at $x = L/2$.

Hence f_{st} (flange at $x = 0$) $= (4000 \times 10^3 \times 20.6)/(2 \times 1{,}415{,}861)$ $[1 - (1/5.037)]$

$= 23.32\ \text{N/mm}^2$

f_{st} (web at $x = 0$) $= 23.32 \times (12.6/20.6) = 14.26\ \text{N/mm}^2$

(b) Stresses due to warping torsion (due to bending of flanges):

$f_{sw} = (Eb^2h/16)\phi'''$

$\phi''' = T\lambda^2/(2GI_T)[-\cosh(\lambda x)/\cosh(\lambda L/2)]$; with $\lambda^2 = GI_t/EI_w$

$f_{sw} = T/(2I_w)(b^2h/16)\ [-\cosh(\lambda x)/\cosh(\lambda L/2)]$

This shear stress acts at the mid-width of the flange, and the maximum and minimum values occur at $x = L/2$ and $x = 0$, respectively.

f_{sw} (flange at $x = L/2$) $= [(4000 \times 10^3)/(2 \times 1.26 \times 10^{12})]$ $[(194^2 \times 427.8)/16]$

$= 1.60\ \text{N/mm}^2$

f_{sw} (flange at $x = 0$) $= 1.6 \times (1/5.037) = 0.32\ \text{N/mm}^2$.

For normal stress in flanges due to warping,

$f_{b,w} = (Ebh/4)\phi''$

$\phi'' = T\lambda/(2GI_t)[-\sinh(\lambda x)/\cosh(\lambda L/2)]$

$f_{b,w} = [(T \times 2.6\lambda bh)/8I_t]\ [\sinh(\lambda x)/\cosh(\lambda L/2)]$

which is maximum at $x = L/2$ and zero at $x = 0$. Hence,

$f_{b,w}$(flange at $x = L/2$) $= [(4000 \times 10^3 \times 2.6 \times 194 \times 427.8)/$ $(8 \times 1{,}415{,}861 \times 1521)]\ [4.937/5.037]$

$= 49.1\ \text{N/mm}^2$

(c) Stresses due to ordinary flexure:

Max. normal stress f_b (at $x = L/2$) $= PL/4Z_p$

$Z_p = 2Bt_f(D - t_f)/2 + td^2/4$

$= 2 \times 194 \times 20.6\ (469 - 20.6)/2 + (469 - 2 \times 20.6)^2 \times 12.6/4$

$= 2{,}368{,}476\ \text{mm}^3$

Thus, maximum normal stress (at $x = L/2$) $= 80 \times 10^3 \times 7000/(4 \times 2{,}368{,}476)$

$= 59.1\ \text{N/mm}^2$

The shear stresses due to flexure are constant for $x = 0$ to $L/2$ and are computed by

$f_s = VQ/I_z t$ with $I_z = 48790 \times 10^4\ \text{mm}^4$ and $Q = A\bar{y}$

Maximum flange shear stress, occurs at the face of the web, thus,

$Q = [(194 - 12.6)/2]\ 20.6 \times 448.4/2 = 418{,}899\ \text{mm}^3$

Shear stress f_{sb} (flange at $x = 0$) $= (418{,}899 \times 40 \times 10^3)/(48{,}790 \times 10^4 \times 20.6)$

$= 1.66\ \text{N/mm}^2$

For maximum web shear stress,

$$Q = 194 \times 20.6 \times (448.4/2) + (427.8/2)\ 12.6 \times (427.8/4)$$
$$= 11{,}84{,}238\ \text{mm}^3$$

Shear stress f_{sb} (web at $x = 0$) = $(1{,}184{,}238 \times 40 \times 10^3)/(48790 \times 10^4 \times 12.6)$
$= 7.70\ \text{N/mm}^2$

Compared to the average shear stress = $V/(Dt_w)$
$= 40 \times 10^3/(469 \times 12.6) = 6.77\text{N/mm}^2$

A summary of stresses showing combinations is given in the following table.

Type of Stress	Support, $x = 0$	Mid-span, $x = L/2$
Compression or tension, maximum stresses:		
Vertical bending, f_b	0	59.1
Torsional bending, $f_{b,w}$	0	49.1
		108.2 MPa
Shear stress, web:		
St Venant's torsion, f_{st}	14.26	0
Vertical bending, f_{sb}	7.70	7.70
	21.96 MPa	7.70 MPa
Shear stress, flange:		
St Venant's torsion, f_{st}	23.32	0
Warping torsion, f_{sw}	0.32	1.60
Vertical bending, f_{sb}	1.66	1.66
	25.30 MPa	3.26 MPa

Example 18.3 *Compute the stresses on a wide flange section W 460 × 190 × 106 beam of Example 18.2, using the flexural analogy rather than the differential equation solutions.*

Solution

The substitute system is shown in Fig. 18.17.

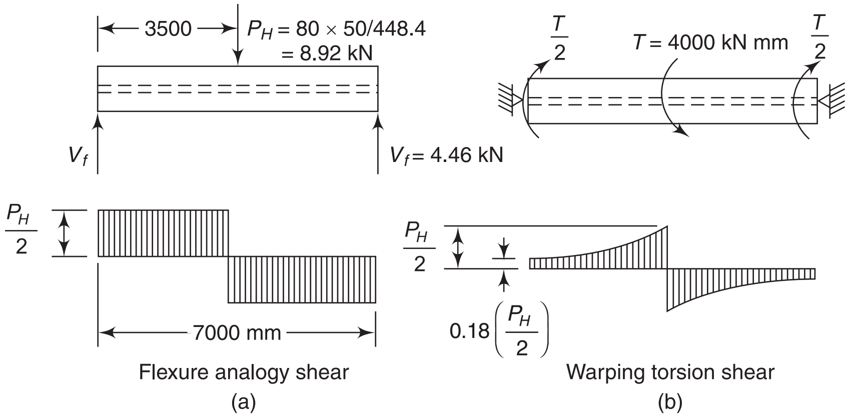


Fig. 18.17

Calculation of plastic moduli of W 460 × 190 × 106 section

$$Z_{pz} = 2Bt_f(D - t_f)/2 + td^2/4$$
$$= 2 \times 194 \times 20.6(469 - 20.6)/2 + (469 - 2 \times 20.6)^2 \times 12.6/4$$
$$= 2{,}368{,}476 \text{ mm}^3$$
$$Z_{py} = 2t_f B^2/4 + dt^2/4$$
$$= 2 \times 20.6 \times 194^2/4 + (469 - 2 \times 20.6) \times 12.6^2/4$$
$$= 404{,}630 \text{ mm}^3$$
$$I_t = 1{,}415{,}861 \text{ mm}^4 \text{ (from Example 18.2)}$$

The lateral bending moment acting on one flange

$$M_f = V_f(L/2) = 4.46 \times 3.5 = 15.61 \text{ kN m}$$

Twice the moment acting on the entire section gives

$$f_{bw} = 2M_f/Z_{py} = 2 \times 15.61 \times 10^6/404{,}630 = 77.2 \text{ N/mm}^2$$

For torsional shear stress, since $M_x = T/2 = 2000$ kN mm,

$$f_{st} = M_x t/I_t = 2000 \times 10^3 \times 20.6/1{,}415{,}861 = 29.1 \text{ N/mm}^2 \text{ (flange)}$$
$$f_{st} = 29.1(12.6/20.6) = 17.8 \text{ N/mm}^2 \text{ (web)}$$

For lateral bending, flange shear stress,

$$f_{sw} = (V_f Q_f/I_f t_f) = (4.46 \times 10^3 \times 96912.7/1253.4 \times 10^4 \times 20.6)$$
$$= 1.67 \text{ N/mm}^3$$

where $Q_f = (bt_f/2)\,(b/4) = (194 \times 20.6/2)\,(194/4) = 96{,}912.7 \text{ mm}^3$

$I_f = b^3 t_f/12 = 194^3 \times 20.6/12 = 1253.4 \times 10^4 \text{ mm}^4$

The results of the two methods (Examples 18.2 and 18.3) are compared as follows:

Type of Stress	Flexural Analogy	Differential Equation
Compression/tension stress = $f_b + f_{b,w}$ = 59.1* + 77.2	136.30 MPa	108.2 MPa
Web shear stress = $f_{st} + f_{sb}$ = 17.8 + 7.7*	25.50 MPa	21.96 MPa
Flange shear stress = $f_{st} + f_{sb} + f_{sw}$ = 29.1 + 1.66* + 1.67	32.43 MPa	25.30 MPa

*Values taken from Example 18.2

It is observed that the stresses obtained by flexure analogy are very conservative, especially the value of lateral bending stress f_{bw} (77.2 N/mm^2 as against 49.1 N/mm^2 using differential equations). Hence, Salmon and Johnson (1996) suggest the use of a reduction factor β, using which the value of f_{bw} may be calculated more precisely. β values are given by them in the form of tables for various end conditions in terms of λL.

Example 18.4 *Design a cantilever beam of length 4 m, subjected to a load at the tip of the cantilever of 50 kN, which is applied at an eccentricity of 30 mm.*

Solution

Assuming an ISMB 450 section

The horizontal force acting on each flange

$$P_H = 50 \times 1000 \times 30/(450 - 17.4) = 3467 \text{ N.}$$

Properties of ISMB 450

$$D = 450 \text{ mm}, t_w = 9.4 \text{ mm}, t_f = 17.4 \text{ mm}, B = 150 \text{ mm}$$

$$M_f = 3467 \times 4000 = 13.868 \times 10^6 \text{ N mm}$$

$$M_z = 50 \times 1000 \times 4000 = 0.2 \times 10^9 \text{ N mm}$$

$$Z_{p,z} = 2Bt_f(D - t_f)/2 + td^2/4$$

$$= 2 \times 150 \times 17.4(450 - 17.4)/2 + 9.4 \times (450 - 2 \times 17.4)^2/4$$

$$= 1534.2 \times 10^3 \text{ mm}^3$$

$$Z_{p,y} = 2t_f B^2/4 + dt^2/4$$

$$= 2 \times 17.4 \times 150^2/4 + (450 - 2 \times 17.4) \times 9.4^2/4$$

$$= 204.9 \times 10^3 \text{ mm}^3$$

$$C_n = (Z_{p,z}/Z_{p,y}) = 1534.2 \times 10^3/204.9 \times 10^3 = 7.486$$

Required $Z_{p,z} \geq (M_z + 2C_n M_f)/(f_y/\gamma_m)$

Thus $Z_{p,z} \geq (0.2 \times 10^9 + 2 \times 7.486 \times 13.868 \times 10^6)/(250/1.1)$

$$= 1793.58 \times 10^3 \text{ mm}^3 > 1534.2 \times 10^3 \text{ mm}^3$$

Hence, adopt ISMB 500 with $Z_{p,z} = 2074.67 \times 10^3 \text{ mm}^3$

$Z_{p,y}$ of ISMB 500 $= 2t_f B^2/4 + dt^2/4$

$$= 2 \times 17.2 \times 180^2/4 + (500 - 2 \times 17.2) \times 10.2^2/4$$

$$= 290.74 \times 10^3 \text{ mm}^3$$

Check

$$[M_{uz}/(Z_{pz} \times f_y/\gamma_m)] + [M_{uy}/(Z_{py} \times f_y/\gamma_m)]$$

$$= [(0.2 \times 10^9)/(2074.67 \times 10^3 \times 250/1.1)] + [(13.868 \times 10^6)/(290.74 \times 10^3 \times 250/1.1)]$$

$$= 0.424 + 0.210 = 0.634 < 1.0$$

Hence the beam is safe.

Example 18.5 *Design a suitable wide flange section to carry a 6 kN/m dead load, in addition to the weight of the beam, and a live load of 20 kN/m. The load is applied eccentrically 150 mm from the centre of the web on the simply supported beam of 8 m span as shown in Fig. 18.18. Assume the ends of the beam have torsional simple supports.*

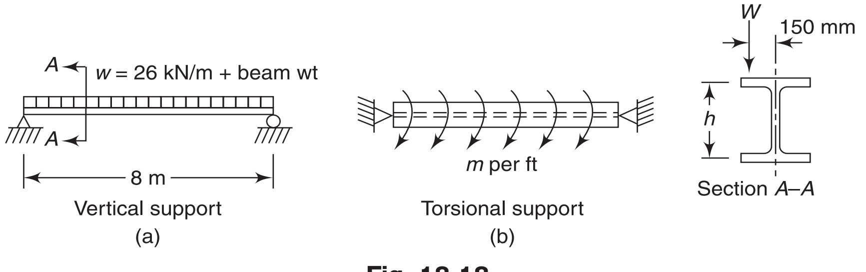


Fig. 18.18

Solution

(a) Compute factored loads, assuming the weight of beam as 1.96 kN/m (assuming W 360 × 370 × 196 section, $h = 372 - 26.2 = 345.8$ mm).

$$w_u = 1.5(20 + 6 + 1.96) = 41.94 \text{ kN/m}$$

(b) The bending moment is

$$M_{uz} = wL^2/8 = 41.94 \times 8^2/8 = 335.52 \text{ kN/m}$$

(c) Bending moment due to torsion

The factored uniformly distributed torsional moment

$$m_u = 1.5(20 + 6) \times 0.15 = 5.85 \text{ kN m/m}$$

Using the flexure analogy, the lateral bending moment acting on one flange is

$$M_f = m_u L^2/(8 \times h)$$
$$= 5.85 \times 8^2/(8 \times 0.3458) = 135.34 \text{ kN m}$$

Thus, the required $Z_{p,z}$ (assuming $Z_{p,z}/Z_{py}$ as 3.0)

$$Z_{p,z} \geq M_{uz}/(f_y/1.1) + [M_{uy}/(f_y/1.1)]\,[Z_{p,z}/Z_{py}]$$
$$= 335.52 \times 10^6/(250/1.1) + 3 \times 135.34 \times 10^6/(250/1.1)$$
$$= 3262.7 \times 10^3 \text{ mm}^3$$

$Z_{p,z}$ of W 360 × 370 × 196 (B = 374 mm, D = 372 mm, t_f = 26.2 mm, t_w = 16.4 mm)

$$= 2Bt_f(D - t_f)/2 + t_w d^2/4$$
$$= 2 \times 374 \times 26.2(372 - 26.2)/2 + 16.4 \times (372 - 2 \times 26.2)^2/4$$
$$= 3807.2 \times 10^3 \text{ mm}^3 > 3262.7 \times 10^3 \text{ mm}^3$$

Hence the section is safe.

Note that wide flange beams are most suitable where high torsional strength is required. For the same weight per metre, ISMB sections give a reduced stress from in-plane (of web) flexure but an increased stress from restraint of torsional warping. Also note that the section can be reduced by using the tables of modification factors given in Salmon and Johnson (1996), pp. 448-449. We must also perform the buckling check, in addition to capacity check {see Eqns (18.20)–(18.22)}.

Example 18.6 *Design a rectangular hollow section member of a dome which is subjected to the following factored forces and moments. Axial compression = 50.0 kN, moments M_z = 7.5 kN m, M_y = 0.75 kN m, and torsion = 0.80 kN m. Assume the length of the member as 4527 mm and f_y = 250 MPa.*

Solution

Try a cold rolled section 172 × 92 × 5.4 RHS

A = 2659 mm², $Z_{yy} = 82.99 \times 10^3$ mm³, r_z = 61.7 mm,

$M_{p,z} = 146.5 \times 10^3$ mm³

$Z_{zz} = 117.73 \times 10^3$ mm³, r_y = 37.9 mm,

$d/t = (172 - 3 \times 5.4)/5.4 = 28.85 > 27.3\varepsilon$ but $< 29.3\varepsilon$ (see Table 8.5)

Hence, this section is a compact section. Therefore, local buckling will not be a problem and the design may be based on the whole section. Since the member is not supported laterally.

$$L/r_y = 0.85 \times 4527/37.9 = 101.53$$

From Table 7.4(b) of the code (for L/r = 101.53, f_y = 250 MPa),

$$f_{cd} = 115.86 \text{ N/mm}^2$$

and

$$P_d = 115.86 \times 2659 \times 10^{-3} = 308.07 \text{ kN}$$

For $L/r = 4527/37.9 = 119.4$, $D/t = 31.8$, from Tables 8.1 and 8.2 of the code,

$$f_{bd} = 64.72 \text{ MPa}$$

$$M_{dz} = 64.72 \times 146.5 \times 10^{-3} = 9.48 \text{ kN m}$$

$$f_y Z_y = 250 \times 82.99 \times 10^{-3} = 20.747 \text{ kN m}$$

Combined Axial and Bending

$$50.0/308.07 + 7.5/9.48 + 0.75/20.747 = 0.989 < 1.0$$

Torsional shear stress = $T/(2tbd)$

$2tbd = \{(172 - 5.4) \times (92 - 5.4) \times 5.4 \times 2\} = 155{,}816 \text{ mm}^3$

Shear stress due to torsion = $0.8 \times 10^6/(155{,}816) = 5.13$ MPa

Bending stress *x-x* axis = $f_{bc} = 7.5 \times 10^6/(117.73 \times 10^3) = 63.71$ MPa

Bending stress *y-y* axis = $f_{bc} = 0.75 \times 10^6/(82.99 \times 10^3) = 9.04$ MPa

Combined Shear and Bending

$$\sqrt{[(63.71 + 9.04)^2 + 3(5.13)^2]} = 73.29 < 250/\sqrt{3} = 144 \text{ MPa}$$

Hence, the provided section is safe.

Example 18.7 *Compute the maximum load a column with a cruciform cross section as shown in Fig. 18.19 can carry. The column is made of Fe 410 steel, 4.5 m long and supported in such a way that warping at the ends is prevented.*

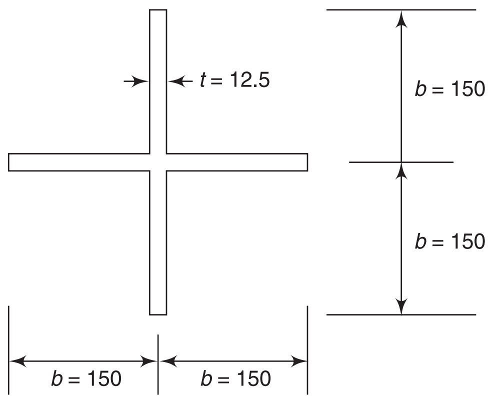


Fig. 18.19

Solution

$$I_z = I_y = 2tb^3/3 = (2 \times 12.5 \times 150^3)/3 = 28.125 \times 10^6 \text{ mm}^4$$

$$I_P = I_z + I_y = 56.25 \times 10^6 \text{ mm}^4$$

$$I_t = 4(bt^3)/3 = 4 \times 150 \times 12.5^3/3 = 39.0625 \times 10^4 \text{ mm}^4$$

$$I_w = b^3t^3/9 = (12.5 \times 150)^3/9 = 732.422 \times 10^6 \text{ mm}^6$$

$$A = 4 \times 12.5 \times 150 = 7500 \text{ mm}^2$$

$$r_z = r_y = \sqrt{I/A} = \sqrt{(28.125 \times 10^6/7500)} = 61.24 \text{ mm}$$

$r_e^2 = [I_w + 0.04 I_t (KL)^2]/I_p$

$= (732.422 \times 10^6 + 0.04 \times 39.0625 \times 10^4 \times (0.5 \times 4500)^2)/(56.25 \times 10^6)$

$= (732.422 \times 10^6 + 791.016 \times 10^8)/56.25 \times 10^6 = 1419 \text{ mm}^2$

Therefore,

$r_e = 37.67$ mm

Since $r_e < r_z < r_y$, the column fails by torsional buckling

$KL/r_e = 0.5 \times 4500/37.67 = 59.73$

From Table 7.4(b) of the code, $f_{cd} = 181.35 \text{ N/mm}^2$

Hence $P_{cd} = 181.35 \times 7500/1000 = 1360.125$ kN

Note that the contribution of I_w to the torsional resistance of the cruciform section is small and hence can be neglected. Similarly the warping resistance of other cross sections such as angles and tees will also be small and hence can be neglected.

Example 18.8 *Compute the maximum load the column with T cross section shown in Fig. 18.20 can carry. The column is made of Fe 410 steel, 3 m long and supported so that warping, z-axis bending and y-axis bending are all prevented at each end.*

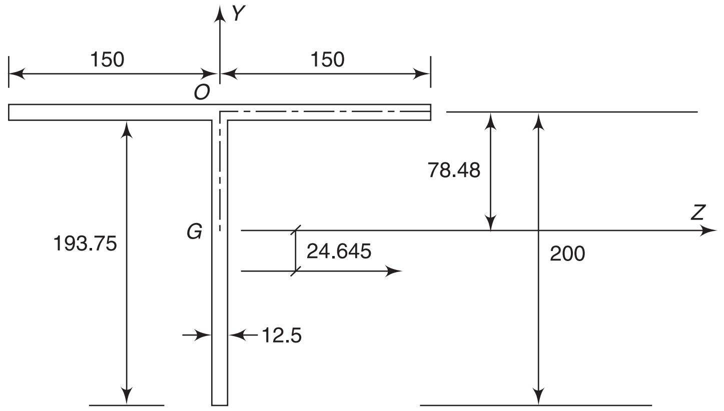


Fig. 18.20

Solution

$A = 12.5\ (300 + 193.75) = 6171.875 \text{ mm}^2$

$I_z = 300 \times 12.5 \times 78.48^2 + 12.5^3 \times 300/12 + 12.5 \times 193.75^3/12 + 193.75 \times 12.5 \times 24.645^2$

$= 32.193 \times 10^6 \text{ mm}^4$

$I_y = 12.5 \times 300^3/12 + 193.75 \times 12.5^3/12$

$= 28.156 \times 10^6 \text{ mm}^4$

$I_p = I_z + I_y + Ay_o^2 = 32.193 \times 10^6 + 28.156 \times 10^6 + 1671.875 \times 24.645^2$

$= 64.098 \times 10^6 \text{ mm}^4$

$I_t = (1/3)(300 \times 12.5^3 + 193.75 \times 12.5^3) = 321.45 \times 10^3 \text{ mm}^4$

$I_w \approx 0.0$

$r_z^2 = I_z/A = 32.193 \times 10^6/6171.875 = 5216 \text{ mm}^2$

$r_y^2 = I_y/A = 28.156 \times 10^6/6171.875 = 4562\ \text{mm}^2$

$r_p^2 = I_p/A = 64.098 \times 10^6/6171.875 = 10385\ \text{mm}^2$

$r_t^2 = [I_w + 0.04 I_t (KL)^2]/I_p = [0 + 0.04 \times 321.45 \times 10^3\ (0.5 \times 3000)^2]/(64.098 \times 10^6)$

$= 451.3\ \text{mm}^2$

As per Eqn (9.57) of Chapter 9,

$$(1 - z_0^2/r_p^2) r_{tb}^4 - (r_y^2 + r_t^2) r_{tb}^2 + r_y^2 r_t^2 = 0$$

where z_0 is the distance between the centroid and shear centre.
Thus,

$$(1 - 78.48^2/10385) r_{tb}^4 - (4562 + 451.3)\, r_{tb}^2 + 4562 \times 451.3 = 0$$

$$0.4069\ r_{tb}^4 - 5013.3\, r_{tb}^2 + 2058831 = 0$$

Hence,

$$r_{tb}^2 = (5013.3 \pm \sqrt{25{,}133{,}176} - 3{,}350{,}952)/0.8138 = 425.4\ \text{mm}^2$$

$$r_{tb} = \sqrt{425.4} = 20.63\ \text{mm}$$

$$r_y = \sqrt{4562} = 67.54\ \text{mm}$$

Since $r_{tb} < r_y$, the column will fail by flexural torsional buckling.

$KL/r_{tb} = 0.5 \times 3000/20.63 = 72.71$

Hence from Tables 7.2 and 7.4(c) of the code,

$f_{cd} = 147.66\ \text{N/mm}^2$

Capacity of the column = 147.66 × 6171.875/1000 = 911.34 kN

Summary

In actual practice, torsion occurs rarely and hence the code does not contain any provision for design of members subjected to torsional moments. However, on many occasions, torsion may occur as a secondary moment in combination with bending moments. Hence, few design guidelines are provided in this chapter. It is also of interest to note that the sections which are effective in resisting bending (such as I-sections) are not equally efficient in resisting torsional moments.

Torsion mainly occurs when transverse loads are applied at a plane not passing through the shear centre. Torsion may be classified as primary and secondary torsion. Primary torsion, whose effect must be considered, is again classified as free (uniform), restrained (warping or non-uniform), and destabilizing. Destabilizing torsion should not be allowed in structures. Free or uniform torsion results when plane surfaces remain plane after deformation. Closed tubular and hollow sections resist torsion better than open cross sections. The torsional shear stress of open cross sections such as I- or channel section depends on the torsional constant I_t, which is approximately given by $\Sigma bt^3/3$, where t is the thickness and b is the breadth of the individual rectangular sections.

The end conditions of most of the structural members will prevent uniform torsion and hence the cross sections of such members undergo warping. Elastic analysis of members subjected to non-uniform torsion is complex and involves the use of exponential and hyperbolic functions. The deformations due to non-uniform torsion are also dependent upon both torsional constant I_t and warping constant I_w. The value of the coefficient k_t may be used to predict whether pure torsion or warping effects are predominant (high values of k_t indicate uniform torsion and low values indicate that warping torsion is predominant).

Some equations are given for calculating the distance of the shear centre form the centroid. An approximate design procedure, which is conservative, is suggested for the design of beams subjected to bending and torsion. In most of the practical situations, it may be easier to detail the beam ends to minimize the effects of torsion, rather than allowing torsion and calculating the stresses due to the combined effect of bending and torsion. Such details are discussed.

The flexural torsional buckling of cruciform sections is discussed and the approximate design method of these sections based on an equivalent radius of gyration has been provided. All the concepts discussed are explained with examples.

Exercises

1. A twisting moment of 20 kN m is applied at the end of a 2-m long shaft. Determine (a) the maximum shear stress, and (b) the maximum angle of rotation for
   (i) a solid circular section of 250 mm diameter
   (ii) a circular hollow section of inner radius 225 mm and outer radius 250 mm
   (iii) a square box section of outer side 250 mm and thickness 25 mm
2. A wide flange section W 310 × 165 × 62 beam used for a 6 m simply supported span is applied with a torsional moment of 3.0 kN m at the mid-span. The ends of the beam are simply supported with respect to torsional restraint (i.e., $\phi = 0$). Compute the combined bending and torsional stresses using the differential equation solution.
3. Compute the stresses on a wide flange beam of Exercise 2 using flexural analogy rather than the differential equation solution and compare the results.
4. Design a cantilever beam of length 5 m, subjected to a load at the tip of the cantilever of 20 kN, which is applied at an eccentricity of 25 mm. Assume $f_y = 250$ MPa.
5. Design a suitable wide flange section to carry a factored $L.L + D.L$ of 20 kN m. The load is applied eccentrically 30 mm from the centre of the web on a simply supported beam of 6 m span. Assume that the ends of the beam have torsional simple supports and $f_y = 250$ MPa.
6. Design a suitable ISMB section for the simply supported beam given in Exercise 5 and compare the results.
7. Design a rectangular hollow section member to carry the following factored forces and moments. Axial compression = 30 kN, moments $M_z = 6$ kN m, $M_y = 0.5$ kN m

and torsion = 0.50 kN m. Assume the length of the member as 3.5 m and f_y = 250 MPa.

8. Compute the maximum load of a 4-m long column which has warping at the ends prevented with:
   (a) a cruciform section, with each leg 100 mm long and 10 mm thick (see Fig. 18.19)
   (b) a T-section having flange size 200 × 10 mm and stem 150 × 10 mm.

Review Questions

1. Which of these sections are stronger in resisting torsion?
   (a) I-section (b) channels (c) tubes and hollow sections
2. What are the three classifications of primary torsion?
3. How is the magnitude of secondary torsions predicted?
4. Cite an example where a destabilizing torsion may occur.
5. When can St Venant's torsion occur?
6. State the equation for calculating the St Venant's torsional shear stress of
   (a) solid circular section, with radius R,
   (b) thin walled boxed section,
   (c) hollow circular section with internal radius R_i and outside radius R_o,
   (d) solid rectangular section, and
   (e) thin walled open sections.
7. State the equation for calculating the St Venant torsional constant for
   (a) solid rectangular sections,
   (b) thin walled open sections, and
   (c) single cell closed sections.
8. Draw the approximate shear stress distribution in an I-section subjected to uniform torsion, non-uniform torsion, and shear force.
9. What does a high value of the coefficient $k_t = L\sqrt{\dfrac{GI_t}{EI_w}}$ signify?
10. Write short notes on the following:
    (a) shear centre
    (b) approximate design procedure for torsion
11. What are the methods adopted in practice to minimize the effects of torsion?
12. State the equation for predicting the critical stress of a column with a cruciform cross section, due to torsional buckling.
13. How one can calculate the load carrying capacity of a cruciform or Z-section?
14. State the equation for calculating the angle of rotation of a cross section subjected to torsion.