32 Mathematical Concepts

CHAPTER

CONTENTS

LOGARITHMIC FUNCTIONS

We know that $\log_2 8$ is the number to which 2 must be raised to get 8.

$$\therefore \qquad \log_2 8 = 3$$

In general, if $a^x = y$, $(a > 1)$, then we say that $\log_a y = x$. If $e^x = y$, then we say that the natural logarithm of y is x and we write $\log y = x$. In other words, if the base of a logarithm is not mentioned, then it is understood that the base is e. In fact, we cannot think of logarithm of a number without any base.

Two Important Results

Prove that (*i*) $\log_a 1 = 0$

(*ii*) $\log_a a = 1$

Proof: (*i*) $\because a^\circ = 1; \qquad \therefore \log_a 1 = 0$

(*ii*) $\because a^1 = a \qquad \therefore \log_a a = 1$

SOLVED PROBLEM 1. Find value of $\log_{.5} 256$.

SOLUTION. Let $\log_{.5} 256 = x$

$$\therefore \qquad \left(\frac{1}{2}\right)^x = 2^8$$

$\therefore \quad 2^{-x} = 2^{8}$

$\Rightarrow \quad -x = 8, \text{ or } x = -8$

$\therefore \quad \log_{.5} 256 = -8$

SOLVED PROBLEM 2. If $x = 2^{\frac{-1}{3}\log_2 64}$, find x

SOLUTION. Let $\log_2 64 = y$

$\therefore \quad 2^y = 64$

$\Rightarrow \quad 2^y = 2^6$

$\Rightarrow \quad y = 6$

$\Rightarrow \quad \log_2 64 = 6$

$\therefore \quad x = 2^{\frac{-1}{3}\log_2 64}$

$\Rightarrow \quad x = 2^{\frac{-1}{3}(6)} = 2^{-2} = \frac{1}{4}$

Fundamental Properties of Logarithms

(1) **Product Formula.** Logarithm of the product of two numbers to any base is equal to the sum of logarithms of the number to the same base.

i.e. $\log_a (mn) = \log_a m + \log_a n$

(2) **Quotient Formula.** Logarithm of the quotient of two numbers to any base is equal to the difference of logarithms of the numerator and denominator to the same base.

i.e. $\log_a \left(\frac{m}{n}\right) = \log_a m - \log_a n$

(3) **Power Formula.** Logarithm of a number raised to a power is equal to the index of the power multiplied by the logarithm of the number to the same base.

i.e. $\log_a m^n = n \log_a m$

(4) **Base Changing Formula**

$$\log_n m = \frac{\log_a m}{\log_a n}$$

Two Systems of Logarithms

(1) **Natural Logarithms.** Logarithms to the base e (= 2.7183 approximately) are called natural logarithms. They are used in all theoretical calculations.

(2) **Common Logarithms.** Logarithms to base 10 are called common logarithms. They are used in arithmetical calculations.

CHARACTERISTIC AND MANTISSA

The integral part of the logarithm of a number, after expressing the decimal part as positive, if not already so, is called the **characteristic** and the positive decimal part is called **mantissa.** The mantissa is always positive.

Two Rules to Find the Characteristics

Rule 1. The characteristic of the logarithm of a number greater than 1 is positive and 1 less than the number of digits in the integral part of the number.

Example. (*i*) If the number is 732, then the characteristic of the logarithm = 2 (= 3 – 1)

(*ii*) If the number is 7.8256, then the characteristic of the logarithm = 0 (= 1 –1)

Rule 2. The characteristic of the logarithm of a number of a positive number less than 1 is negative and numerically 1 more than the number of zeros immediately after the decimal point.

Example. (*i*) Consider the number .1205, which is positive but less than 1. There is no zero immediately after decimal point.

$\therefore$ Characteristic of logarithm $= -(0+1)$

$= -1 = \bar{1}$

(*ii*) Consider the number ·002007. This number is positive and less than 1. Also there are two zeros immediately after the decimal.

$\therefore$ Characteristic of logarithm $= -(2+1)$

$= -3 = \bar{3}$

Rule to Find Mantissa

We can explain this rule by an example. Suppose we wish to find mantissa of log 57.6932.

(*i*) Remove the decimal point from 57.6932 we get 576932. We take its first four significant figures only. Therefore, number is 5769. The first two figures from the left form 57, the third figure is 6 and the fourth is 9.

(*ii*) In the table of logarithms, we find 57 in the first column.

(*iii*) In the horizontal row beginning with 57 and under the column headed by 6, we find the number 7604 at the intersection. We note it down.

(*iv*) In continuation of this row and under the small column on the right headed by 9, we find the number 7 at the intersection.

(*v*) Adding 7 to 7604, prefixing the decimal point, the mantissa = ·7611.

ANTILOGARITHM

The number whose logarithm is x, is called the antilogarithm of x and is written as antilog x.

Example. $\because$ log 3 = .4771

$\therefore$ antilog (.4771) = 3

Rule to Find Antilog of a Number

We can find the number whose logarithm is 2.6078.

(*i*) The characteristic of the logarithm = 2. This is less than the number of digits in the integral part of the required number.

$\therefore$ Number of digits in the integral part of the required number = 2 + 1 = 3

(*ii*) Removing the integral part 2 from the given logarithm 2.6078, we get, .6078 from the table of antilogarithms.

The number corresponding to .607 = 4046

Mean difference for 8 = 7

$\therefore$ Number corresponding to ·6078 = 4053

$\therefore$ Required number = 405.3

SOLVED PROBLEM 1. Given log 2 = .30103, find the number of digits in 2^{64}.

SOLUTION. Let $x = 2^{64}$

$\therefore \log x = \log 2^{64} = 64 \log 2 = 64\,(0.30103)$

$\therefore \log x = 19.26592$

$\therefore$ Characteristic = 19

$\therefore$ Number of digits in x or $2^{64} = 19 + 1 = 20$

SOLVED PROBLEM 2. Find the fifth root of 8.012.

SOLUTION. Let $x = (8.012)^{1/5}$

$\therefore \quad \log x = \log\ (8.012)^{1/5}$

$= \frac{1}{5} \log\ (8.012) = \frac{1}{5}\ (0.9037)$

$\therefore \quad \log x = 0.1807$

$\therefore \quad x = 1.516$

SOLVED PROBLEM 3. Evaluate $\sqrt{\frac{.0075 \times .014}{80.35}}$ using log tables.

SOLUTION. Let $x = \sqrt{\frac{.0075 \times .014}{80.35}}$

$\therefore \quad \log x = \log \left[\frac{.0075 \times .014}{80.35}\right]^{\frac{1}{2}}$

$= \frac{1}{2} \log \left[\frac{.0075 \times .014}{80.35}\right]$

$= \frac{1}{2} \left[\log\ (.0075 \times .014) - \log 80.35\right]$

$= \frac{1}{2} \left[\log .0075 + \log .014 - \log 80.35\right]$

$= \frac{1}{2}\ [\bar{3} - .8751 + \bar{2} - .1461 - 1.9050]$

$= \frac{1}{2}\ [-3 + .8751 - 2 + .1461 - 1.9050]$

$= \frac{1}{2}\ [-6.9050 + 1.0212]$

$= \frac{1}{2}\ (-5.8838)$

$= -2.9419 = -2 - .9419$

$= (-2 - 1) + (1 - .9419) = -3 + .0581$

$\therefore \quad \log x = \bar{3}.0581$

$\therefore \quad x = .001143$

EXPONENTIAL FUNCTIONS

Let $f(x) = e^x$ where $e = 2.7182818$......... The function e^x is also written as exponential (x) or, in short, as exp(x). For Instance, the exponential function $e^{-E/RT}$ which we shall frequently came across in the text, is written as exp ($-E/RT$). The algebraic operations with respect to exponential functions are

$$e^x \cdot e^y = e^{x+y}$$
$$e^x / e^y = e^{x-y}$$

The exponential and logarithmic functions are related as $\frac{1}{e^x} = \ln x$ and $\frac{1}{10^x} = \log x$

Polynomial

A polynomial is a function such as

$$f(x) = a_0 + a_1 x + a_2 x^2 + ... + a_n x^n$$

where $a_1, a_2, a_3, ... a_n$ are constants and exponent n, which is a positive integer, is called degree of polynomial.

Curve Sketching

The relationship between the x and y co-ordinates of points lying on a straight line is represented by a straight line graph.

DISPLACEMENT–TIME GRAPHS

The shortest distance between the initial and final positions of an object is called its displacement. When the displacement of a moving object is plotted against time, we obtain displacement– time graph. For plotting this graph, time is represented on x-axis and displacement on y-axis as shown in Fig. 32.1 :

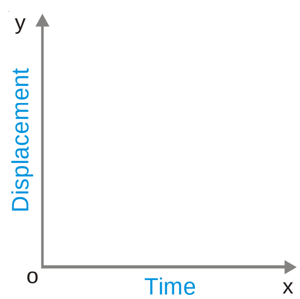


Figure 32.1

Types of Diaplacement–Time Graphs

(1) This graph shows uniform positive velocity *i.e.* displacement increases with time in this type as shown in Fig. 32.2 :

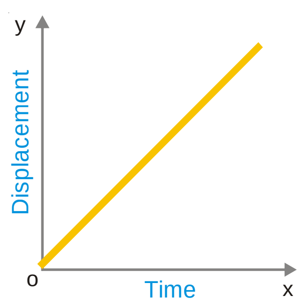


Figure 32.2

(2) This graph shows uniform negative velocity *i.e.* displacement decreases with time in this type as shown in Fig. 32.3 :

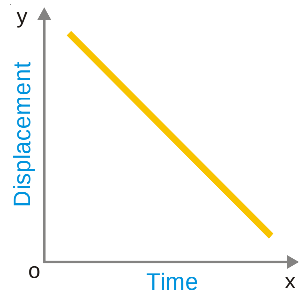


Figure 32.3

(3) This graph shows variable positive velocity *i.e.* displacement increases but variably as shown in Fig. 32.4 :

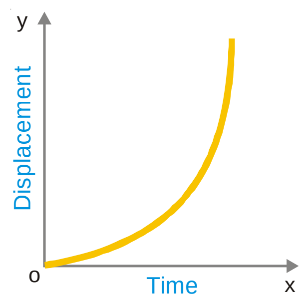


Figure 32.4

(4) This graph shows variable negative velocity *i.e.* displacement decreases with time in a variable manner *i.e.* not constantly as shown in Fig. 32.5

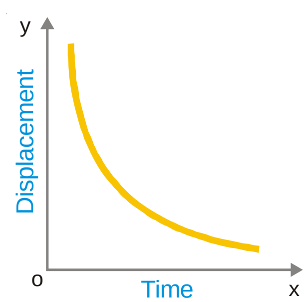


Figure 32.5

Notes :

(*i*) In displacement–time graph velocity $= \dfrac{\text{Change in displacement}}{\text{Change in time}}$

$$= \frac{\text{Final displacement} - \text{Initial displacement}}{\text{Final time} - \text{Initial Time}}$$

(*ii*) The graph as shown in Fig. 32.6 is impossible as displacement is changing without any change in time.

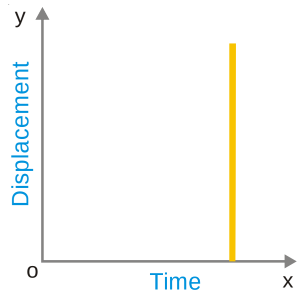


Figure 32.6

VELOCITY–TIME GRAPHS

When the velocity of an object is plotted against time, the graph so obtained is called velocity–time graph. For plotting this graph, the time is represented along *x*-axis and velocity along *y*-axis (Fig. 32.7).

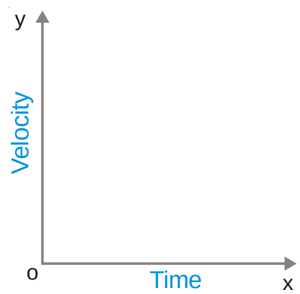


Figure 32.7

Types of Velocity–Time Graphs

(1) This graph shows uniform positive acceleration *i.e.* velocity increases with time in this type as shown in Fig. 32.8 :

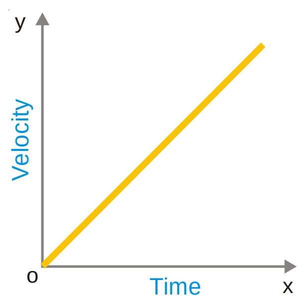


Figure 32.8

(2) This graph shows uniform negative acceleration *i.e.* velocity decreases with time in this type as shown below in Fig. 32.9 :

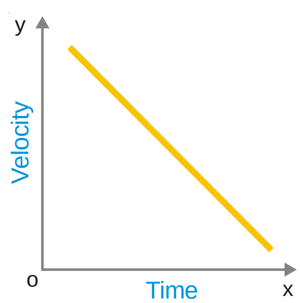


Figure 32.9

(3) This graph shows variable positive acceleration as shown below in Fig. 32.10 :

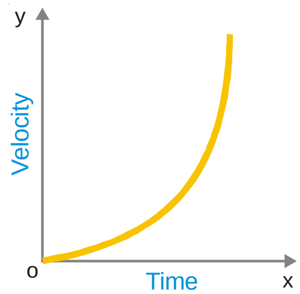


Figure 32.10

(4) This graph shows variable negative acceleration as shown below in Fig. 32.11 :

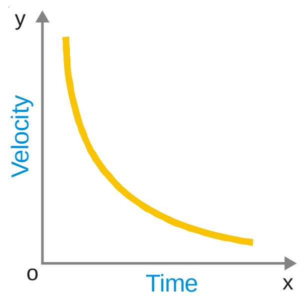


Figure 32.11

Notes :

(*i*) In velocity–time graphs,

$$\text{Acceleration} = \text{Rate of change of velocity}$$

$$= \frac{\text{Change in velocity}}{\text{Change in time}}$$

$$= \frac{\text{Final velocity} - \text{Initial velocity}}{\text{Final time} - \text{Initial time}}$$

(*ii*) For any time–interval, the area enclosed between the velocity–time graph and *x*-axis is equal to the distance travelled in that interval.

(*iii*) The graph as shown below in Fig. 32.12 is impossible state. It shows that velocity is changing without any change in time, which is not possible.

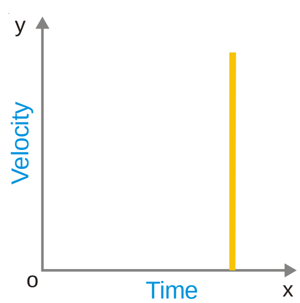


Figure 32.12

Graphs of Linear Equations

The graph of linear equation of the form $y = mx + c$ will always be a straight line.

It is very easy to draw the graph of linear equations as illustrated in following example.

SOLVED PROBLEM 1. Draw the graph of the equation $y = x + 3$

SOLUTION. We draw the following value table :

x	0	1	2
y	3	4	5

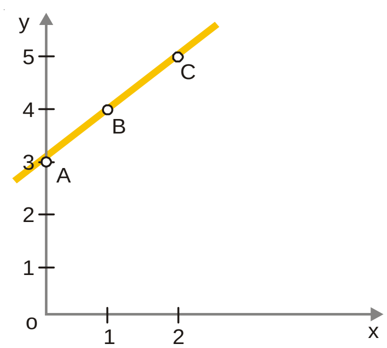


Figure 32.13
Graph of equation y = x + 3

We now plot the points *A* (0, 3), *B* (1, 4) and *C* (2, 5) on graph paper and join the points *A*, *B* and *C* and produce on either side. The line ABC is required graph of given equation.

SOLVED PROBLEM 2. Solve the given equations graphically,

$$2x - 3y = 1 \text{ and } 3x - 4y = 1$$

SOLUTION. The given equations can be written as :

$$y = \frac{2}{3}x - \frac{1}{3} \quad \text{and} \quad y = \frac{3}{4}x - \frac{1}{4}$$

Table of values for $2x - 3y = 1$

x	2	5	8
y	1	3	5

Table of values for $3x - 4y = 1$

x	3	7	11
y	2	5	8

On the same graph paper draw the graph of each given equation. Both lines drawn meet at point *P* as is clear from the graph. The co-ordinates of common point *P* are (–1, –1)

Hence, solution of given equation is $x = -1$ and $y = -1$

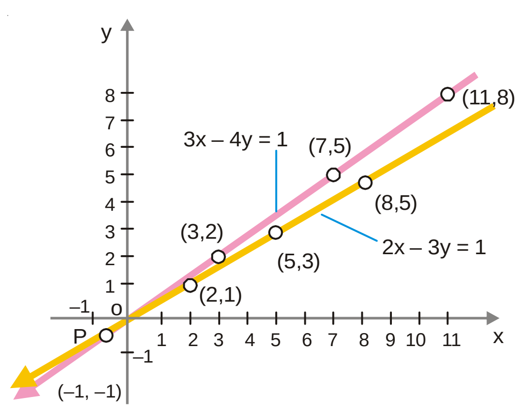


Figure 32.14

SLOPE OF A LINE

(1) Inclination of a straight line

The angle which the line makes with positive direction of x-axis measured in anti-clockwise direction is called the inclination of the line. It is denoted by θ (Fig. 32.15).

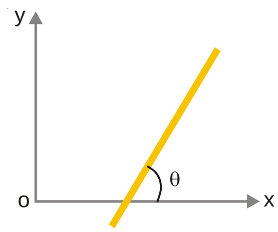


Figure 32.15

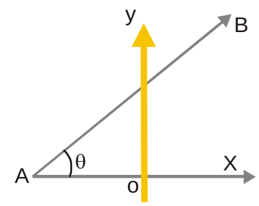


Figure 32.16

It should be noted that :

(*i*) inclination of line parallel to x-axis or x-axis itself is 0°.

(*ii*) inclination of line parallel to y-axis or y-axis itself is 90°.

(2) Slope (or Gradient of a line)

Slope of a line is the tangent of the angle which the part of the line above the x-axis makes with positive direction of x-axis.

The slope of a line is generally denoted by m.

∴ If θ is the angle which the line AB makes with x-axis, then slope is $\tan\theta$ (Fig. 32.16).

$$\therefore \quad m = \tan\theta$$

Note : If $\theta = \frac{\pi}{2}$, then $m = \tan\frac{\pi}{2}$, which is not defined.

∴ Slope of a vertical line is not defined.

(3) Slope of a line passing through two fixed points

The slope of a line passing through two fixed points $A\,(x_1, y_1)$ and $B\,(x_2, y_2)$ is given by

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} \quad i.e. \quad m = \frac{y_2 - y_1}{x_2 - x_1}$$

(4) Slope–Intercept form

$y = mx + b$ where m is the slope of the line and b is the intercept on the y–axis.

(5) Slope–Point form

The equation of a line with slope m and passing through a point (x_1, y_1) is

$$y - y_1 = m\,(x - x_1)$$

(6) Two–Point form

The equation of a line passing through two points (x_1, y_1) and (x_2, y_2) is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}\,(x - x_1)\,; \quad x_2 \neq x_1$$

(7) The slope of the line $ax + by + c = 0$

The slope of line $ax + by + c = 0$ is

$$\text{is } \frac{-a}{b} = -\frac{\text{coeff. of } x}{\text{coeff. of } y}$$

(8) Slope of parallel lines

Two lines are parallel if and only if their slopes are equal.

(9) Slope of perpendicular lines

Two lines are perpendicular if and only if the product of their slope is –1.

Note. In cases, 8 and 9 the lines taken should be non-vertical.

(10) Intercept form $\frac{x}{a} + \frac{y}{b} = 1$

where x-intercept of the line is a and y-intercept is b.

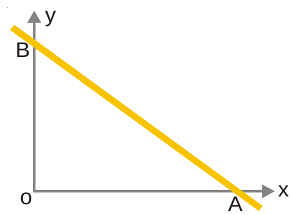


Figure 32.17

Notes :

Let a straight line AB, meet x-axis in A and y-axis in B. Then

(*i*) OA is called intercept of the line on x-axis.

(*ii*) OB is called the intercept of the line on y-axis and

(*iii*) The two together, taken in this order, are called the intercepts of the line on the axes.

SOLVED PROBLEM 1. Find the slope of the line whose inclination is 45°.

SOLUTION. Let m be the slope of line,

Then, $m = \tan 45° = 1$

SOLVED PROBLEM 2. The equation of a line is $2x - 2y - 5 = 0$. Find inclination and gradient of the line.

SOLUTION. The equation of line is $2x - 2y - 5 = 0$.

$$\therefore \quad \text{gradient} = \frac{2}{-2} = 1$$

Let θ be inclination of the line, then

$$\tan\theta = m = 1$$

$$\Rightarrow \quad \theta = 45°$$

SOLVED PROBLEM 3. A straight line passes through the points $P(4, -5)$ and $Q(6, 7)$. Find the slope of the line PQ.

SOLUTION. Given points are $P(4, -5)$ and $Q(6, 7)$

$$\therefore \quad \text{The slope of the line } PQ = \frac{7-(-5)}{6-4} = \frac{12}{2} = 6$$

TRIGONOMETRIC FUNCTIONS

The trigonometric functions such as $f(x) = \sin x, \cos x, \tan x, \cot x, \sec x$ and cosec x are periodic. For example, $\sin x$ and $\cos x$ have the period $= 2\pi$ radian. Some fundamental relations among trigonometric functions are mentioned below :

(*a*) $\sin^2 x + \cos^2 x = 1$; $\sec^2 x = 1 + \tan^2 x$, $\text{cosec}^2 x = 1 + \cot^2 x$

(*b*) $\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$

(*c*) $\cos(x \pm y) = \cos x \cos y \pm \sin x \sin y$

(*d*) $\tan(x \pm y) = (\tan x \pm \tan y) / (1 \pm \tan x \tan y)$

(*e*) $\cos 2x = \cos^2 x - \sin^2 x$, $\sin 2x = 2 \sin x \cos x = \dfrac{2\tan x}{1+\tan^2 x}$

(*f*) $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$; $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

(*g*) $\sin 3x = 3\sin x - 4\sin^3 x$; $\cos 3x = 4\cos^3 x - 3\cos x$

(*h*) $\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$; $\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$

(*i*) $\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$

(*j*) $\cos x - \cos y = 2\sin\frac{x+y}{2}\sin\frac{x-y}{2} = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$

Inverse Trigonometric Functions

Inverse trigonometric functions such as $f(x) = \sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$ etc. are the inverse of corresponding trigonometric functions.

For example, if $x = \sin y$

Then $y = \sin^{-1} x$

DIFFERENTIATION

Derivative of a function

The derivative of a function $y = f(x)$ at a point x is defined as

$$f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$$

$$= \lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = \frac{dy}{dx}$$

where $\Delta x = h$ and $\Delta y = f'(x+h) - f(x)$ are the increments in the variables x and y, respectively. The derivative of $f(x)$ is denoted by $\frac{dy}{dx}$ provided the limit exists, *i.e.*

$$\frac{dy}{dx} = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$$

and may be interpreted as the rate of change of y w.r.t. x. The process of finding the derivative is called differentiation. The derivative of a function $f(x)$ at a given point represents a slope of the tangent drawn to the curve of $y = f(x)$ at a point where the function is defined. The derivative is also called differential coefficient.

SOLVED PROBLEM 1. If $y = x^2 + 6x + 8$, find $\frac{dy}{dx}$

SOLUTION

$$y + \Delta y = (x+h)^2 + 6(x+h) + 8$$

$$= x^2 + 2xh + h^2 + 6x + 6h + 8$$

$$\therefore \quad \Delta y = (2x+6)\,h + h^2$$

$$\therefore \quad \frac{\Delta y}{\Delta x} = \frac{(2x+6)\,h + h^2}{h} = 2x + 6 + h$$

$$\therefore \quad \frac{dy}{dx} = \lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = \lim_{h\to 0}(2x + 6 + h) = 2x + 6$$

SOLVED PROBLEM 2. Differentiate sin *x* with respect to *x* from first principles.

SOLUTION

$$\frac{d}{dx}\sin x = \lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h}$$

$$= \lim_{h\to 0}\frac{2\cos\left(x+\frac{h}{2}\right)\sin\frac{h}{2}}{h}$$

$$= \lim_{h\to 0}\cos\left(x+\frac{h}{2}\right)\frac{\sin\frac{h}{2}}{\frac{h}{2}} = \cos x \qquad \left[\because \lim_{h\to 0}\frac{\sin\frac{h}{2}}{\frac{h}{2}} = 1\right]$$

Derivatives of Some simple functions

The following differentiation formulas should be memorized by the reader. We have assumed in these formulas that *u* and *v* are differentiable functions of *x* and *c*, *n* are arbitrary constants :

(1) $$\frac{d}{dx}(c) = 0$$

(2) $$\frac{d}{dx}(u+v) = \frac{du}{dx}+\frac{dv}{dx}$$

(3) The Constant Multiple Rule

$$\frac{d}{dx}(cu) = \frac{cdu}{dx}$$

(4) The Product Rule

$$\frac{d(uv)}{dx} = u\frac{dv}{dx}+v\frac{du}{dx}$$

(5) The Quotient Rule

$$\frac{d}{dx}(u/v) = \frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2},\quad v\neq 0$$

(6) The Power Rule for Positive and Negative Integers

$$\frac{d}{dx}x^{n} = x^{n-1}$$

(7) $$\frac{d}{dx}e^{x} = e^{x}$$

(8) $$\frac{d}{dx}a^{x} = a^{x}\ \log a$$

(9) $$\frac{d}{dx}(\log x) = \frac{1}{x}$$

(10) $$\frac{d}{dx}(\sin x) = \cos x$$

(11) $$\frac{d}{dx}(\cos x) = -\sin x$$

(12) $$\frac{d}{dx}(\tan x) = \sec^2 x$$

(13) $$\frac{d}{dx}(\cot x) = -\text{cosec}^2 x$$

(14) $$\frac{d}{dx}(\sec x) = \sec x \,.\, \tan x$$

(15) $$\frac{d}{dx}(\text{cosec } x) = -\text{cosec } x \cot x$$

(16) $$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

(17) $$\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$$

(18) $$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

(19) $$\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$$

(20) $$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$$

(21) $$\frac{d}{dx}(\text{cosec}^{-1} x) = \frac{-1}{x\sqrt{x^2-1}}$$

Chain Rule. According to this rule the derivative of the composite of two differentiable functions is the product of their derivatives evaluated at appropriate points.

Thus, $$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

SOLVED PROBLEM 1. Find the differential coefficients of

(*i*) $x^{4/3}$ (*ii*) $\log(1+x)$ (*iii*) a^{mx}

SOLUTION. (*i*) Let $y = x^{4/3}$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{4}{3} x^{4/3-1}$$

$$= \frac{4}{3} x^{1/3}$$

(*ii*) Let $y = \log(1+x)$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{1+x}$$

(*iii*) Let $y = a^{mx}$

$$\Rightarrow \quad \frac{dy}{dx} = a^{mx} m \log a$$

SOLVED PROBLEM 2. Find the differential coefficients of

(*i*) 4^x (*ii*) e^x (*iii*) $\log_{10} x$

SOLUTION. (*i*) Let $y = 4^x$

$$\Rightarrow \quad \frac{dy}{dx} = 4^x \log 4$$

(*ii*) Let $y = e^x$

$\Rightarrow$ $\frac{dy}{dx} = e^x$

(*iii*) Let $y = \log_{10} x$

$\Rightarrow$ $\frac{dy}{dx} = \frac{1}{x}\log_{10} e$

PARTIAL DIFFERENTIATION

Partial Derivatives

Consider $z = f(x, y)$

Then $\underset{h\to 0}{\text{Lt}} \frac{f(a+h, b) - f(a, b)}{h}$ if it exists, is said to be partial derivatives of f w.r.t. x at (a, b) and is denoted by $\left(\frac{\partial z}{\partial x}\right)_{(a,\, b)}$ or $fx\,(a, b)$.

Again $\underset{k\to 0}{\text{Lt}} \frac{f(a, b+k) - f(a, b)}{k}$ if it exists, is said to be partial derivatives of f w.r.t. x at (a, b) and is denoted by $\left(\frac{\partial z}{\partial y}\right)_{(a,\, b)}$ or $fy\,(a, b)$.

Partial Differentiation of Higher Orders

Partial derivatives of first order can be formed as we formed those of f.

Therefore, $$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial^2 z}{\partial x^2} \quad \text{...(1)}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial^2 z}{\partial y\,\partial x} \quad \text{...(2)}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial^2 z}{\partial y^2} \quad \text{...(3)}$$

(1), (2) and (3) can be denoted as f_{xx}, f_{yx} and f_{yy} respectively.

SOLVED PROBLEM 1. Differentiate $(3x^2 + 1)^2$ with respect to x.

SOLUTION. Let $y = (3x^2 + 1)^2 = u^2$

where $u = 3x^2 + 1$

Hence $$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} = \frac{d(u^2)}{du}\,\frac{d}{dx}\,(3x^2+1)$$
$$= (2u)\,(6x)$$
$$= 2\,(3x^2 + 1)\,6x$$
$$= 36x^3 + 12x$$

SOLVED PROBLEM 2. Find the first order partial derivatives of

(*i*) $u = y^x$ (*ii*) $u = \log\,(x^2 + y^2)$

SOLUTION. (*i*) $u = y^x$

$$\log u = x \log y$$

$$\Rightarrow \quad \frac{1}{u}\frac{\partial u}{\partial x} = \log y$$

$$\Rightarrow \quad \frac{\partial u}{\partial x} = u \log y = y^x \log y$$

Also

$$\log u = x \log y$$

$$\Rightarrow \quad \frac{1}{u}\frac{\partial u}{\partial y} = x\frac{1}{y}$$

$$\Rightarrow \quad \frac{\partial u}{\partial y} = u\frac{x}{y} = y^x\frac{x}{y} = xy^{x-1}$$

(*ii*)

$$u = \log(x^2 + y^2)$$

$$\Rightarrow \quad \frac{\partial u}{\partial x} = \frac{1}{x^2 + y^2}(2x) = \frac{2x}{x^2 + y^2}$$

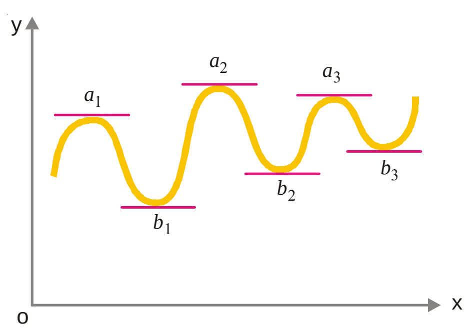
Also

$$\frac{\partial u}{\partial y} = \frac{1}{x^2 + y^2}(2y) = \frac{2y}{x^2 + y^2}$$

MAXIMA AND MINIMA

Graphs of functions show maxima and/or minima and in some cases the functions are merely increasing or decreasing.

In Fig. 32.18, the function $y = f(x)$ has maximum values at the points a_1, a_2, a_3 .... and minimum values at the points b_1, b_2, b_3, ...



■ Figure 32.18

Working rules for finding maximum and minimum values of a function

Step 1. Put the given function = y

Step 2. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$

Step 3. Put $\frac{dy}{dx} = 0$ and solve this equation.

Step 4. Put $x = a$ in $\frac{d^2y}{dx^2}$. If the result is –ve, the function is maximum at $x = a$ and maximum $y = f(a)$.

If by putting $x = a$ in $\frac{d^2y}{dx^2}$, result is +ve, the function has minimum value at $x = a$ and minimum $y = f(a)$.

Similarly, test for other values of b, c, ... of x found in Step 4.

SOLVED PROBLEM 1. Find maximum and minimum values of $x^3 - 12x + 10$

SOLUTION. Let $$y = x^3 - 12x + 10 \quad ...(1)$$

$$\Rightarrow \quad \frac{dy}{dx} = 3x^2 - 12$$

For maximum and minimum values, $\frac{dy}{dx} = 0$

$$\Rightarrow \quad 3x^2 - 12 = 0$$

$$\Rightarrow \quad x^2 - 4 = 0$$

$$\Rightarrow \quad (x-2)(x+2) = 0$$

$$\Rightarrow \quad x = 2, -2$$

Also $$\frac{d^2y}{dx^2} = 6x \quad ...(2)$$

putting $x = 2$ in (2), $$\frac{d^2y}{dx^2} = 6 \times 2 = 12 > 0$$

$\therefore$ at $x = 2$, the function has a minimum value and this minimum value is obtained by putting $x = 2$ in (1)

$\therefore$ Minimum $$y = (2)^3 - 12(2) + 10$$
$$= 8 - 24 + 10 = -6$$

Again putting $x = -2$ in (2), $\frac{d^2y}{dx^2} = 6 \times (-2) = -12 < 0$

$\therefore$ at $x = -2$, the function has a maximum value and this maximum value is obtained by putting $x = -2$ in (1)

$\therefore$ Maximum $$y = (-2)^3 - 12(-2) + 10$$
$$= -8 + 24 + 10$$
$$= 26$$

SOLVED PROBLEM 2. Find the maximum and minimum values of $2x^3 - 9x^2 + 12x + 6$

SOLUTION. Let $$y = 2x^3 - 9x^2 + 12x + 6 \quad ...(1)$$

$$\Rightarrow \quad \frac{dy}{dx} = 6x^2 - 18x + 12$$

For maximum and minimum values, $\frac{dy}{dx} = 0$

$$\Rightarrow \quad 6x^2 - 18x + 12 = 0$$

$$\Rightarrow \quad x^2 - 3x + 2 = 0$$

$$\Rightarrow \quad (x-1)(x-2) = 0$$

$$\Rightarrow \quad x = 1, 2$$

Also, $$\frac{d^2y}{dx^2} = 12x - 18 \quad ...(2)$$

Putting $x = 1$ in (2), $$\frac{d^2y}{dx^2} = 12 - 18 = -6 < 0$$

$\therefore$ at $x = 1$, the function has a maximum value and this maximum value is obtained by putting $x = 1$ in (1).

$\therefore$ Maximum $$y = 2.1^3 - 9.1^2 + 12.1 + 6 = 11$$

Again putting $x = 2$ in (2)

$$\frac{d^2y}{dx^2} = 24 - 18 = 6 \text{ which is +ve}$$

$\therefore$ at $x = 2$, the function has a minimum value and this minimum value is obtained by putting $x = 2$ in (1).

Minimum $$y = 2.2^3 - 9.2^2 + 12.2 + 6$$
$$= 10$$

INTEGRATION

Integration is the process which is inverse of differentiation. In differentiation, a function is given and it is required to find its differential coefficient. But integration is its reverse process *i.e.*, given the differential coefficient of a function, it is required to find the function.

Thus if $\frac{d}{dx}[f(x)] = g(x)$, then $g(x)$ is called an integral of $g(x)$ w.r.t. x and is written as

$$\int g(x)\, dx = f(x)$$

The function $g(x)$ to be integrated is called **Integrand** and function sought is called **integral** or **primitive,** here $f(x)$. The symbol dx after the integrand $g(x)$ denotes that x is the independent variable and integration is done w.r.t. x. The process of finding the integral of a function is called **Integration.** For example,

Since $$\frac{d}{dx}(x^4) = 4x^3$$

$\therefore$ $$\int 4x^3\, dx = x^4$$

Constant of Integration

If $\frac{d}{dx}[f(x)] = g(x)$, then we know that $\frac{d}{dx}[f(x) + c]$ is also equal to $g(x)$, c being an arbitrary constant. It, therefore, follows that $\int g(x)\, dx = f(x) + c$, also and the arbitary c is called the constant of integration.

Some Important Results

(1) $$\int x^n = \frac{x^{n+1}}{n+1} \quad [\text{where } n \neq 1]$$

(2) (*i*) $$\int 1\, dx = x$$

(*ii*) $$\int 0\, dx = c \text{ (constant)}$$

(3) $$\int \frac{1}{x}\, dx = \log x$$

(4) $$\int e^x \, dx = e^x$$

(5) $$\int a^x \, dx = \frac{a^x}{\log a}$$

(6) $$\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{(n+1)\, a}$$

(7) $$\int \frac{1}{ax+b} \, dx = \log (ax+b)$$

(8) $$\int \sin x \, dx = -\cos x$$

(9) $$\int \cos x \, dx = \sin x$$

(10) $$\int \sec^2 x \, dx = \tan x$$

(11) $$\int (\sec x \times \tan x) \, dx = \sec x$$

(12) $$\int \operatorname{cosec}^2 x \, dx = -\cot x$$

(13) $$\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x$$

SOLVED PROBLEM 1. Integrate the following functions w.r.t. x :

(*i*) x^7 (*ii*) $e^{-nx}\, dx$ (*iii*) a^{7x+8}

SOLUTION. (*i*) $$\int x^7 \, dx = \frac{x^{7+1}}{7+1} = \frac{x^8}{8}$$

(*ii*) $$\int e^{-nx} \, dx = \frac{e^{-nx}}{-n}$$

(*iii*) $$\int a^{7x+8} \, dx = \frac{a^{7x+8}}{7 \log a}$$

SOLVED PROBLEM 2. Evaluate the following integrals :

(*i*) $\displaystyle\int \frac{\sin^2 x}{1+\cos x} \, dx$ (*ii*) $\displaystyle\int \left[\frac{2\cos x}{3\sin^2 x} + 5x^2 - 6\right] dx$

SOLUTION. (*i*) $$\int \frac{\sin^2 x}{1+\cos x} \, dx = \int \frac{1-\cos^2 x}{1+\cos x} \, dx$$

$$= \int \frac{(1-\cos x)(1+\cos x)}{(1+\cos x)} \, dx$$

$$= \int (1-\cos x) \, dx$$

$$= x - \sin x + c$$

(*ii*) $$\int \left[\frac{2\cos x}{3\sin^2 x} + 5x^2 - 6\right] dx$$

$$= \int \left[\frac{2\cos x}{3\sin x}\operatorname{cosec} x + 5x^2 - 6\right] dx$$

$$= \int \left[\frac{2}{3}\cot x \operatorname{cosec} x + 5x^2 - 6\right] dx$$

$$= \frac{-2}{3}\operatorname{cosec} x + \frac{5}{3}x^3 - 6x + c$$

PERMUTATIONS AND COMBINATIONS

Factorial of an Integer

Let $n \in$ N. The combined product of first n natural number is called the factorial n. It is denoted by $\lfloor n$.

$$\therefore \qquad \lfloor n = n(n-1)(n-2)\ldots\ldots\ldots\ldots 4.3.2.1$$

$$= 1.2.3.\ldots\ldots\ldots\ldots n$$

$$\lfloor 1 = 1$$

$$\lfloor 3 = 3 \times 2 \times 1 = 6$$

Factorial Zero

$$\lfloor 0 = 1$$

or $\qquad 0! = 1$

Permutations

It is an arrangement that can be made by taking some or all of a number of given things.

Meaning of ${}^nP_r$

It means the number of permutations of n different things taken r at a time.

Illustrations

Consider three letters a, b, c. The permutations of three letters taken two at a time are :

$$ab,\ bc,\ ca$$

$$ba,\ cb,\ ac$$

$\therefore$ The number of arrangements of three letters taken two at a time is 6 *i.e.* ${}^3P_2 = 6$

Note. ${}^nP_r$ is also written as $P(n, r)$

Combination

It is a group (or selection) that can be made by taking some or all of a number of a given things at a time.

Meaning of ${}^nC_r$

${}^nC_r$ means the number of combinations of n different things taken r at a time.

Illustration

Consider three letters a, b, c the group of these 3 letters taken two at a time are ab, bc, ca.

As far as group is concerned ac or ca is the same group, as in a group we are concerned with the number of things contained, whereas in the case of arrangement we have to take into consideration the order of things.

Note. ${}^nC_r$ is also written as $C(n, r)$

Theorem

Let the number of distinct objects be n and $1 \leqslant r \leqslant n$. Then the number of all permutations of n objects taken r at a time is given by $\dfrac{\lfloor n}{\lfloor n-r}$

SOLVED PROBLEM 1. Evaluate :

(*i*) $P(9, 5)$ (*ii*) $P(12, 0)$

SOLUTION. (*i*) We have $P(9, 5) = \dfrac{\lfloor 9}{\lfloor 9-5}$

$$= \frac{\lfloor 9}{\lfloor 4} = \frac{(9)(8)(7)(6)(5)\lfloor 4}{\lfloor 4}$$

$$= 15120$$

(*ii*) We have $P(12, 0) = \dfrac{\lfloor 12}{\lfloor 12-0}$

$$= \frac{\lfloor 12}{\lfloor 12}$$

$$= 1$$

SOLVED PROBLEM 2. Determine the number of different 5-letter words formed from the letters of the word 'EQUATION'.

SOLUTION. The given word is 'EQUATION'.

$\therefore$ number of letters $= 8$

Number of letters to be taken at a time = 5

$\therefore$ Total number of words formed $= P(8, 5)$

$$= 8 \times 7 \times 6 \times 5 \times 4$$

$$= 6720$$

Notes. (1) The number of permutations of n dissimilar things taken r at a time is given by

$$^{n}P_{r} = \frac{n!}{(n-r)!}$$

(2) Number of permutations of n dissilmilar things, taken all at a time is $^{n}P_{n} = n!$

(3) Number of circular permutations of n different things taken all at a time is $(n-1)!$

(4) Number of permutations of n things, taken all at a time, when P_1, are alike of one kind, P_2 are alike of second kind, ... P_r are alike of r^{th} kind is given by

$$^{n}C_{r} = \frac{n!}{(P_1!)(P_2!)\ldots\ldots\ldots(P_r!)}$$

Notes. (1) Number of combinations of n different things taken r at a time is given by

$$\frac{n!}{(n-r)!(r!)} = \frac{^{n}P_{r}}{r!}$$

(2) $^{n}C_{0} = 1, \ ^{n}C_{n} = 1$

(3) $^{n}C_{p} = \ ^{n}C_{q}$

$\Rightarrow \qquad p + q = n \text{ or } p = q$

(4) $\quad {}^{n}C_{r} = {}^{n}C_{n-r}$

(5) $\quad {}^{n}C_{r-1} + {}^{n}C_{r} = {}^{n+1}C_{r}$

(6) Number of combinations of n different things, taken r at a time when p particular things always occur

$$= {}^{n-p}C_{r-p}$$

(7) Number of combinations of n different things taken r at a time when p particular things never occur is

$$= {}^{n-p}C_{r}$$

SOLVED PROBLEM 3. Verify that $C(8, 4) + C(8, 3) = C(9, 4)$

SOLUTION.

$$\text{L.H.S.} = C(8, 4) + C(8, 3)$$

$$= {}^{8}C_{4} + {}^{8}C_{3}$$

$$= \frac{8 \times 7 \times 6 \times 5}{1 \times 2 \times 3 \times 4} + \frac{8 \times 7 \times 6}{1 \times 2 \times 3} = 70 + 56 = 126$$

$$\text{R.H.S.} = C(9, 4) = {}^{9}C_{4} = \frac{9 \times 8 \times 7 \times 6}{1 \times 2 \times 3 \times 4} = 126$$

$\therefore \qquad$ R.H.S. = L.H.S.

SOLVED PROBLEM 4. Show that total number of ways in which six '+' and four '–' signs can be arranged in a line such that no two '–' signs occur together is 35.

Solution. $x + x + x + x + x + x + x$

Since no two '–' signs occur together 4 '–' signs can be arranged in $7x$ marked places in ${}^{6}C_{6}$ ways.

$\therefore$ Required number of ways

$$= {}^{7}C_{4} \times {}^{6}C_{6} = \frac{7 \times 6 \times 5 \times 4}{1 \times 2 \times 3 \times 4} \times 1$$

$$= 35$$

PROBABILITY

Introduction

When we perform an operation again and again under the same conditions, than

(*i*) either the result is certain,

(*ii*) or the result is not unique but may be one of the several possibilities.

Suppose we toss a coin. Then we are not certain of head or tail. In this case, we talk of chance or probability which is taken to be quantitative measure of certainity.

Now we define certain terms which are used frequently:

Trial and Event. An experiment repeated under essentially identical conditions may not give unique result but may result in any one of the several possible outcomes. The experiment is called a Trial **(or random experiment)** and the outcomes are known as events or cases. For example, Tossing of a coin is **trial** and getting head or tail is an **event**.

Exhaustive Events. The total number of possible outcomes in any trial is known as exhaustive events. For example, in tossing a coin, there are two exhaustive cases, head and tail.

Favourable events or cases. The number of cases favourable to an event in a trial is the number of outcomes which ensure the happening of the event. For example, in tossing a die, the total number of cases favourable to the appearance of a multiple of 3 are two viz., 3 and 6.

Mutually Exclusive Events. Events are said to be mutually exclusive or incompatiable if the happening of any one of them rules out the happening of all others. For example, in tossing a coin the events head and tail are mutually exclusive.

Independent and Dependent Events. Two or more events are said to be independent if the happening or non-happening of any one does not depend (or is not affected) by the happening or non-happening of any other. Otherwise they are said to be dependent. For example, if a card is drawn from a pack of well shuffled cards and replaced before drawing. The second card, the result of second draw is independent of first draw. However, if the first card is not replaced, then the second draw is dependent on the first draw.

Probability. If a trial results in n exhaustive, mutually exclusive and equally likely cases and m of them are favourable to the happening of an event E, then probability of happening of E is given by :

$$P \text{ or } P(E) = \frac{\text{Favourable Number of cases}}{\text{Exhaustive Number of cases}}$$

SOLVED PROBLEM 1. A coin is tossed once. What are all possible outcomes? What is the probability of the coin coming up "Heads"?

SOLUTION. The coin can come up either "Heads" (H) or "tails" (T).

$\therefore \quad S = \{H, T\}$

$\therefore$ Total number of possible ways = 2

Number of favourable ways = 1

$\therefore \quad \text{Required probability} = \frac{1}{2}$

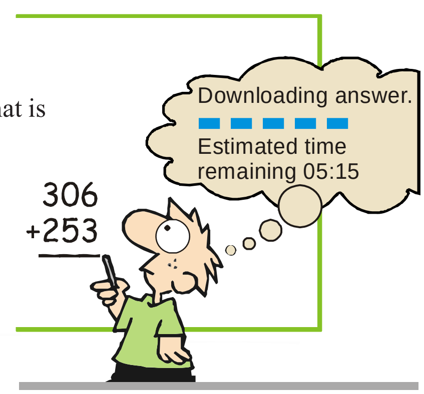
SOLVED PROBLEM 2. In a single throw of two dice, what is the probability of obtaining a total greater than 10?

SOLUTION. Total number of outcomes = 6 × 6 = 36

Favourable outcomes are (5, 6), (6, 5), (6, 6)

$\therefore$ Number of favourable outcomes = 3

$\therefore \quad \text{Required probability} = \frac{3}{36} = \frac{1}{12}$



EXAMINATION QUESTIONS

1. Find value of

(*a*) $\log_{81} 243$ (*b*) $\log_{\frac{1}{3}} 243$

(*c*) $\log_{.5} .04$ (*d*) $\log_{\sqrt{2}} 324$

Answer. (*a*) $\frac{5}{4}$; (*b*) –5 ; (*c*) –2 ; (*d*) 4

2. If $\log_x 243 = 5$; find x

Answer. 3

3. Given log 3 = .4771, find

(*a*) the number of digits in 3^{62}.

(*b*) the number of digits in 27^5.

(*c*) the position of first significant figure in 3^{-65}.

(*d*) the position of first significant figure in 3^{-32}.

Answer. (*a*) 30; (*b*) 8; (*c*) 32^{nd}; (*d*) 16^{th}

4. (*a*) Find the seventh root of .03457.

(*b*) Find the fifth root of .096.

(*c*) Find the seventh root of .00001427.

(*d*) Find the seventh root of .001.

Answer. (*a*) .6183; (*b*) .6259; (*c*) .2035; (*d*) .3727

5. Find the values of :

(*a*) $\dfrac{0.0518 \times 4.68}{.0054 \times 25.5 \times 0.9}$ (*b*) $\dfrac{368.4361 \times .006143}{4384.612 \times 0.8391}$

(*c*) $\dfrac{(435)^3 \sqrt{.056}}{(380)^4}$

Answer. (*a*) 1.956; (*b*) .0006149; (*c*) .0009342

6. Evaluate :

(*a*) $\log_2 3$ (*b*) $\log_{43} 57$

Answer. (*a*) 1.59 approximately; (*b*) 1.07 approximately

7. Draw graph for the equation : $2x + 3y = 6$

8. Solve graphically the following equations :

$$x + y = 3, \quad 2x + 5y = 12$$

Answer. $x = 1$, $y = 2$

9. Find the slope of a line whose inclination is :

(*a*) 30° (*b*) 45°

Answer. (*a*) $\dfrac{1}{\sqrt{3}}$; (*b*) 1

10. Find the inclination of a line whose gradient is

(*a*) $\dfrac{1}{\sqrt{3}}$ (*b*) 1

(*c*) $\sqrt{3}$

Answer. (*a*) 30°; (*b*) 45°; (*c*) 60°

11. Find the slope of line $-5y + 1 = 0$.

Answer. 0

12. Find the differential coefficient of following functions :

(*a*) $x^{5/3}$ (*b*) $(3x + 5)^{7/3}$

(*c*) $\log_5 x$ (*d*) e^{3x}

(*e*) $\frac{3^x}{2^x}$

Answer. (*a*) $\frac{5}{3}x^{2/3}$; (*b*) $7(3x+5)^{4/3}$; (*c*) $\frac{1}{x}\log_5 e$; (*d*) $3e^{3x}$; (*e*) $\frac{3^x}{2^x}\log\frac{3}{2}$

13. Find the maximum and minimum values of

(*a*) $x^4 - 14x^2 + 21x + 9$ (*b*) $x^3 + 2x^2 - 4x + 8$

Answer. (*a*) Maximum Value = 20 at $x = 1$

Minimum Value = 17 at $x = 2$

(*b*) Minimum Value = $\frac{-256}{27}$ at $x = \frac{2}{3}$

Maximum Value = 0 at $x = -2$

14. Integrate the following functions w.r.t. *x*.

(*a*) x^{10} (*b*) a^{5x+7}

(*c*) $\int \frac{1}{1+\sin x}dx$ (*d*) $\int (x^a + a^x + e^x a^x + \sin a)\, dx$

(*e*) $\int \sin x \sec^2 x\; dx$

Answer. (*a*) $\frac{x^{11}}{11}$; (*b*) $\frac{a^{5x+7}}{5\log a}$; (*c*) $\tan x - \sec x$;(*d*) $\frac{x^{a+1}}{a+1} + \frac{a^x}{\log a} + \frac{(ae)^x}{\log(ae)} + \sin a\, x$; (*e*) $\sec x$

15. Evaluate :

(*a*) $P(15,3)$ (*b*) $P(30,2)$

Answer. (*a*) 2730; (*b*) 870

16. How many different words containing all the letters of word 'TRIANGLE' can be formed?

Answer. 40320

17. Evaluate : $C(19,17) + C(19,18)$

Answer. 190

18. In a single throw of a pair of two dice, write the probability of getting a doublet of even numbers.

Answer. $\frac{1}{12}$

MULTIPLE CHOICE QUESTIONS

1. The value of log1 is

(*a*) 1 (*b*) 0

(*c*) 2 (*d*) 3

Answer. (*b*)

2. The value of log (*xy*) is

(*a*) 0 (*b*) 1

(*c*) $\log x + \log y$ (*d*) 2

Answer. (*c*)

3. $\log\left(\frac{x}{y}\right)$ is equal to

(*a*) $\log x - \log y$ (*b*) 0

(*c*) 1 (*d*) 3

Answer. (*a*)

4. What is the value of $\log x^y$?

(*a*) $\log x + \log y$ (*b*) $\log x - \log y$

(*c*) $y \log x$ (*d*) zero

Answer.(*c*)

5. The value of $\log_{81} 243$ is

(*a*) $\frac{6}{4}$ (*b*) $\frac{5}{4}$

(*c*) $\frac{7}{4}$ (*d*) $\frac{9}{4}$

Answer. (*b*)

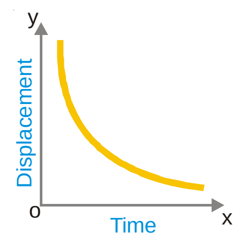
6. The fifth root of .096 is

(*a*) .6253 (*b*) .6257

(*c*) .6259 (*d*) .6371

Answer. (*c*)

7. The following graph shows



(*a*) uniform positive velocity

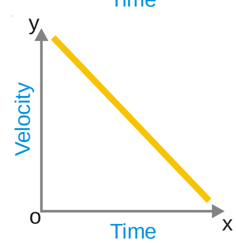
(*b*) uniform negative velocity

(*c*) variable positve velocity

(*d*) variable negative velocity

Answer. (*d*)

8. The following graph shows



(*a*) uniform positive acceleration

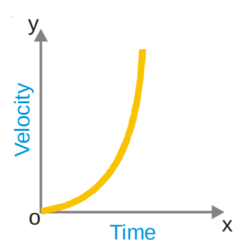
(*b*) uniform negative acceleration

(*c*) variable positive acceleration

(*d*) variable negative acceleration

Answer. (*b*)

9. The following graph shows



(*a*) uniform positive acceleration

(*b*) uniform negative acceleration

(*c*) variable positive acceleration

(*d*) variable negative acceleration

Answer. (*c*)

10. The graph of linear equation is always in the form of

(*a*) circle (*b*) sphere

(*c*) straight line (*d*) curve

Answer. (*c*)

11. Slope of a line is not defined if the line is

(*a*) parallel to x-axis
(*b*) parallel to the line $x - y = 0$
(*c*) parallel to the line $x + y = 0$
(*d*) parallel to y-axis

Answer. (*d*)

12. Slope of any line parallel to x-axis is

(*a*) 1
(*b*) –1
(*c*) 0
(*d*) not defined

Answer. (*d*)

13. The equation $y - y_1 = m(x - x_1)$, $m \in R$ represents the line

(*a*) parallel to x-axis
(*b*) parallel to y-axis
(*c*) parallel to the line $x - y = 0$
(*d*) parallel to the line $x + y = 0$

Answer. (*b*)

14. The equation $y - y_1 = m(x - x_1)$ for different values of m and (x_1, y_1) fixed, represents

(*a*) a family of parallel lines
(*b*) a straight line
(*c*) a family of lines which are concurrent
(*d*) a family of concurrent lines

Answer. (*d*)

15. The straight lines $y = m_1x$ and $y = m_2x$ are perpendicular to each other if

(*a*) $m_1 = 1/m_2$
(*b*) $m_1m_2 = -1$
(*c*) $m_1 = m_2$
(*d*) $m_1 = -m_2$

Answer. (*b*)

16. Two straight lines, whose gradients are m_1, m_2 respectively are parallel if

(*a*) $m_1 = 0$
(*b*) $m_2 = 0$
(*c*) $m_1m_2 = -1$
(*d*) $m_1 = m_2$

Answer. (*d*)

17. The intercept form of line is given by

(*a*) $y = mx + b$
(*b*) $\frac{x}{a} + \frac{y}{b} = 1$
(*c*) $2x + 3y = 1$
(*d*) $x + 2y = 2$

Answer. (*b*)

18. The relation between $\sin^2 x$ and $\cos^2 x$ is

(*a*) $\sin^2 x - \cos^2 x = 1$
(*b*) $\sin^2 x + \cos^2 x = 1$
(*c*) $\frac{\sin^2 x}{\cos^2 x} = 1$
(*d*) $\sin^2 x \times \cos^2 x = 1$

Answer. (*b*)

19. The relation between $\operatorname{cosec}^2 x$ and $\cot^2 x$ is

(*a*) $\frac{\operatorname{cosec}^2 x}{\cot^2 x} = 1$
(*b*) $\operatorname{cosec}^2 x + \cot^2 x = 1$
(*c*) $\operatorname{cosec}^2 x - \cot^2 x = 1$
(*d*) $\operatorname{cosec}^2 x \times \cot^2 x = 1$

Answer. (*c*)

20. The value of $\sin(x + y)$ is

(*a*) $\sin x \cos y - \cos x \sin y$
(*b*) $\sin x \cos y + \cos x \sin y$
(*c*) $\sin x \sin y + \cos x \cos y$
(*d*) $\sin x \sin y - \cos x \cos y$

Answer. (*b*)

21. The largest value of sin θ.cos θ is

(*a*) 1 (*b*) $\frac{1}{2}$

(*c*) $\frac{1}{\sqrt{2}}$ (*d*) $\frac{\sqrt{3}}{2}$

Answer. (*b*)

22. If $\sqrt{x}+\sqrt{y}=1,$ then $\frac{dy}{dx}$ at $\left(\frac{1}{4}, \frac{1}{4}\right)$ is

(*a*) $\frac{1}{2}$ (*b*) 1

(*c*) –1 (*d*) 2

Answer. (*c*)

23. If $x = a\,(t - \sin t), y = a\,(1 + \cos t)$, then $\frac{dy}{dx}$ equals

(*a*) $-\tan\frac{t}{2}$ (*b*) $\cot\frac{t}{2}$

(*c*) $-\cot\frac{t}{2}$ (*d*) $\tan\frac{t}{2}$

Answer. (*c*)

24. $\frac{d}{dx}\left(\cos^{-1}x+\sin^{-1}x\right)$ is

(*a*) $\frac{\pi}{2}$ (*b*) 0

(*c*) $\frac{2}{\sqrt{1-x^2}}$ (*d*) none of these

Answer. (*b*)

25. If $x = a\cos^3 t,\ y = a\sin^3 t$, then $\frac{dy}{dx}$ is

(*a*) cos *t* (*b*) cot *t*

(*c*) cosec *t* (*d*) – tan *t*

Answer. (*d*)

26. If $y = t^2 + t - 1$, then $\frac{dy}{dx}$ is equal to

(*a*) $2t + 1$ (*b*) $t^2 + t - 1$

(*c*) 0 (*d*) not defined

Answer. (*d*)

27. If *x* be real, the minimum value of $x^2 - 8x + 17$ is

(*a*) –1 (*b*) 0

(*c*) 1 (*d*) 2

Answer. (*c*)

28. The maximum value of $\frac{\log x}{x}$ is

(*a*) 1 (*b*) $\frac{2}{e}$

(*c*) *e* (*d*) $\frac{1}{e}$

Answer. (*d*)

29. The function $f(x) = 2x^3 - 3x^2 - 12x + 4$ has

(*a*) two maxima
(*b*) two minima
(*c*) one maxima and one minima
(*d*) no maxima and minima

Answer. (*c*)

30. At $x = \frac{5\pi}{6}$, $f(x) = 2 \sin 3x + 3 \cos 3x$ is

(*a*) maximum
(*b*) minimum
(*c*) zero
(*d*) none of these

Answer. (*d*)

31. $\frac{d}{dx}\left(\int f(x)\, dx\right)$ is equal to

(*a*) $f'(x)$
(*b*) $\frac{(f(x))^2}{2}$
(*c*) $f(x)$
(*d*) none of these

Answer. (*c*)

32. $\int f(x)\, dx = f(x)$, then

(*a*) $f(x) = x$
(*b*) $f(x)$ = constant
(*c*) $f(x) = 0$
(*d*) $f(x) = e^x$

Answer. (*d*)

33. $\int \frac{dx}{\sqrt{4x^2 - 1}}$ is

(*a*) $\frac{1}{2}\tan^{-1} 2x$
(*b*) $\log\left(\sqrt{4x^2 - 1}\right)$
(*c*) $\sqrt{4x^2 - 1}$
(*d*) none of these

Answer. (*d*)

34. $\int x^2 e^{x^2}\, dx$ is equal to

(*a*) $x^2 (e^{x^2} - 1)$
(*b*) $\frac{1}{2} x^2 (e^{x^2} - 1)$
(*c*) $\frac{1}{2} e^{x^2} (x^2 - 1)$
(*d*) $\frac{1}{2} (e^{x^2} - 1)$

Answer. (*c*)

35. $\int (\tan x + \cot x)\, dx$ is equal to

(*a*) $\log (C \tan x)$
(*b*) $\log (\sin x + \cos x) + C$
(*c*) $\log x + C$
(*d*) none of these

Answer. (*a*)

36. $\int \frac{1}{x \log x}\, dx$ equals

(*a*) $\log x$
(*b*) $\log (\log x)$
(*c*) $\log (\log (\log x)$
(*d*) none of these

Answer. (*b*)

37. $\int \frac{\sin (\log x)}{x}\, dx$ equals

(*a*) $\cos (\log x)$
(*b*) $\sin (\log x)$

(*c*) $-\cos(\log x)$ (*d*) $-\sin(\log x)$

Answer. (*a*)

38. $\int \log x \, dx$ will be equal to

(*a*) $x \log\left(\frac{x}{e}\right)$ (*b*) $x \log x$

(*c*) $\frac{\log x}{x}$ (*d*) $x \log\left(\frac{e}{x}\right)$

Answer. (*a*)

39. If $C(n, 10) = C(n, 12)$ then n is equal to

(*a*) 2 (*b*) 10×12

(*c*) 22 (*d*) none of these

Answer. (*c*)

40. The number of diagonals of a hexagon is

(*a*) 3 (*b*) 6

(*c*) 9 (*d*) 12

Answer. (*c*)

41. The probability of a sure event is

(*a*) 1 (*b*) 2

(*c*) $\frac{1}{2}$ (*d*) unlimited

Answer. (*a*)

42. The probability of an impossible event is

(*a*) 1 (*b*) 2

(*c*) $\frac{1}{2}$ (*d*) 0

Answer. (*d*)

43. The probability of having at least one tail in 4 throws with a coin is

(*a*) $\frac{15}{16}$ (*b*) $\frac{1}{16}$

(*c*) $\frac{1}{4}$ (*d*) 1

Answer. (*a*)

44. A dice is thrown once. Then the probability of getting a number greater than three is

(*a*) $\frac{1}{2}$ (*b*) $\frac{2}{3}$

(*c*) 6 (*d*) 0

Answer. (*a*)

45. The probability that a leap year selected at random will contain 53 Sundays is

(*a*) $\frac{1}{7}$ (*b*) $\frac{2}{7}$

(*c*) $\frac{6}{7}$ (*d*) $\frac{6}{14}$

Answer. (*b*)

Top