DEVELOPMENT OF MATHEMATICAL MODEL FOR SIMPLE CONICAL FLUME IN TRAPEZOIDAL OPEN CHANNEL

Pritam J. Gadge , **Surajkumar S. Sonkusale**
Dept. of Civil Engineering, Karmaveer Dadasaheb Kannamwar College of Enineering, Nagpur, 440009
Under The Guidance Of Prof. A. M. Badar
Dept. of Civil Engineering, Karmaveer Dadasaheb Kannamwar College of Enineering, Nagpur, 440009

ABSTRACT

This paper presents the development of simple conical flume for measuring flow discharge in trapezoidal open channel. The purpose of this paper is to develop the necessary equation for a simple flume that consist of a cone installed axially in trapezoidal channel with a side slope of 1:1. The flume is obtained by inserting a cone in a trapezoidal cross section perpendicular to the flow. Using dimensional analysis, the stage discharge relationship for the flume is theoretically deduced. For determining the two coefficients of power stage discharge equation, some experimental runs are carried out. The presence of conical flume constricts the flow area, creating a critical flow condition. The apparatus may be used for both permanent and mobile discharge evaluation in open channel. The flow resulting from a circular cone immersed in a trapezoidal channel is investigated under critical flow condition.

Keywords: Flume, Discharge Measurement, Open Channel Flow, Dimensional Analysis

INTRODUCTION

Water is the greatest natural source of humanity. It not only helps in survival but also helps in making life comfortable and luxurious. Besides various other uses of water, the largest use of water is made for irrigation system and municipal supply, industrial water use, power generation, navigation, recreation etc. Hence Water is an important constraint. At the same time, environmental water needs are increasingly being recognized, limiting the water sources of new water and further increasing the Water measurement in open channel is one of the most important and basic element of water conservation with the increasing demand of water measurement techniques,

there is a serious need for low cost and accurate water measuring devices. The usual discharge measurement structures in open channels are based on critical flow concept. For constant discharge and steady flow condition, either width is constricted or bottom is elevated to achieve critical flow condition (Hager 1985b). The first configuration is often termed as venturi flume, while the second is called as weir. Most existing flow measuring devices have been relatively expensive to maintain and difficult to install or costly to construct.

Hence the concept of conical flume would be convenient to use as a discharge measuring device due to its low weight, high precision of finishing and its reasonable price when compared to other conventional structures. It can be used both as permanent and mobile discharge measuring device. This paper is based on previous works presented by Hager(1985,1986,1988) and Samani and Magallanez(1993). Hager(1985,1988) showed that a cylinder or a circular cone installed axially in a prismatic channel can be used to measure the discharge.

Dimensionless Analysis by Buckingham's π −Theorem

1. For a simple conical flume, the relationship between the upstream energy head 'H'& the discharge 'Q' for a trapezoidal flume having horizontal bottom, formed by contracting the flow by placing a cylinder of diameter 'D' centrally at the canal bottom with width 'B', can be expressed as –

$$f(Q, H, B, D, g) = 0 \quad \text{..........................(1)}$$

2. Total number of variables is 5.
3. There are only two fundamental dimensions involved i.e. L & T.
4. Therefore, there are 3 dimensionless π terms.
5. According to the Π theorem of dimensional analysis, the functional relationship given by Equation (6) can be expressed by using only three dimensionless groups.

$$\phi(\Pi_1, \Pi_2, \Pi_3) = 0 \quad \text{...(2)}$$

6. Where Π_1, Π_2 & Π_3 are dimensionless groups whose expressions have to be determined.
7. Let B and g be the repeating variables.
8. Π terms have been grouped as follows:

$$\Pi_1 = B^a g^b Q \quad \text{...(3)}$$

$$\Pi_2 = B^c g^d H \quad \text{...(4)}$$

$$\Pi_3 = B^e g^l D \quad \text{...(5)}$$

9. Where,
   a, b, c, d, e and l are numerical constants.
10. Substituting the measurement units of each variable in Equations (3), (4) and (5)

$$\pi_1 = \frac{Q}{B^{\frac{5}{2}} * g^{\frac{1}{2}}} \quad \text{.............................. (6)}$$

$$\Pi_2 = \frac{H}{B} \quad \text{..................................(7)}$$

$$\Pi_3 = \frac{D}{B} \quad \text{..................................(8)}$$

Substituting Equations (6), (7), (8) into Equation (2),

$$\phi\left(\frac{Q}{B^{\frac{5}{2}},g^{\frac{1}{2}}},\frac{H}{B},\frac{D}{B}\right)=0$$

……………………………………………..(9)

Similarly, from the permutations and combinations, following equations can be obtained

$$\phi_1\left(\frac{Q}{B^{\frac{5}{2}},g^{\frac{1}{2}}},\frac{H}{D},\frac{B}{D}\right)=0$$

……………………………………………(10)

$$\phi_2\left(\frac{Q}{H^{\frac{5}{2}}g^{\frac{1}{2}}},\frac{B}{H},\frac{D}{H}\right)=0$$

……………………………………………(11)

Thus the stage discharge relationship for simple cylindrical flume in rectangular channel can be expressed in the form:

$$\frac{Q}{B^{\frac{5}{2}} * g^{\frac{1}{2}}} = f\left(\frac{H}{D}\right) = k\left(\frac{H}{D}\right)^n$$

Experimental Setup And Methodology

Fig.1 shows the experimental setup of trapezoidal open channel with side slope as 1:1. The experimental runs were carried out in trapezoidal open channe in the lanoratory of fluid mechanics in Department of Civil Engineering, KDKCE. The channel has the dimensions as bottom width of 20cm,side length of 60cm and side slope of 1:1 keeping bottom slope horizontal.

Figure:1. Experimental setup

Fig.2 shows a conical flume of diameter 151mm and axial height of 201mm.The flume is placed in the channel axially with its diameter at a distance of 1.32 m from the outlet of channel. The purpose of the flume is to constrict the flow area so as to assure critical flow condition.

Figure:2. Conical Flume

Regression Analysis

A discharge model can be developed using the observed head readings. The dimensionless discharge can be expressed as the exponential function of dimensionless head. The regression analysis has been carried out for the best fit correlation between dimensionless discharge and dimensionless head for establishment of dimensionless relationship for the data given. Nine combinations of dimensionless head have been considered. A total of eighteen combinations have been considered for assessing the variation of dimensionless discharge with dimensionless head. Regression analysis has been carried out and power model for fitness of trend line, in the form of $y = k \cdot x^n$ was found to be approximate to yield high value of R^2. Figure shows the variation of $(Qm/B^{5/2} * g^{1/2})$ with H/D using measured flow rates.

Correlation coefficient (R^2) is a measure of well predicted value from a set of observed values. Numerically it cannot exceed unity. It always lies between 0 and 1. Correlation between the variables of equation, $(Qm/(B^{5/2} * g^{1/2})$ and H/D gave value of coefficient of determination, $R^2 = 0.9988$. This regression coefficient suggest that the model is very good and hence can be adopted

The developed mathematical model is given by,

$$Qp = 0.6843 * B^{5/2} * g^{1/2} * (H/D)^{2.3138};$$

$$R^2 = 0.9988 \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

Table 1: OBSERVATIONS AND CALCULATION

H (m)	Volume (lit)	Time (sec)	Qm (m^3/sec)	B	H/D	Qm/(B^(5/2)*9.81^(1/2)	K	N	Qp	% errors
0.13	332.5	22.645	0.014683153	0.2	0.65	0.262065488	0.684	2.313	0.014149	-3.63599
0.1255	332.5	25.595	0.012990819	0.2	0.6275	0.231860636	0.684	2.313	0.013042	0.394434
0.1195	332.5	28.485	0.01167281	0.2	0.5975	0.208336773	0.684	2.313	0.011645	-0.23924
0.115	332.5	30.955	0.010741399	0.2	0.575	0.191712905	0.684	2.313	0.010656	-0.7989
0.1115	332.5	33.78	0.009843102	0.2	0.5575	0.175680076	0.684	2.313	0.00992	0.785482
0.1075	332.5	36.78	0.009040239	0.2	0.5375	0.161350543	0.684	2.313	0.009117	0.844233

0.103	332.5	40.81 5	0.008146 515	0. 2	0.51 5	0.145399313	0.684	2.31 3	0.0082 58	1.3686 98
0.097	332.5	47.68 5	0.006972 843	0. 2	0.48 5	0.124451567	0.684	2.31 3	0.0071 88	3.0804 66
0.091 5	332.5	53.57 5	0.006206 253	0. 2	0.45 75	0.110769444	0.684	2.31 3	0.0062 8	1.1860 72
0.081 5	332.5	69.73 5	0.004768 05	0. 2	0.40 75	0.085100351	0.684	2.31 3	0.0048 05	0.7743 3
0.073	332.5	88.04 5	0.003776 478	0. 2	0.36 5	0.067402726	0.684	2.31 3	0.0037 24	- 1.3806 7
0.066 5	332.5	106.9 5	0.003108 929	0. 2	0.33 25	0.055488293	0.684	2.31 3	0.0030 02	- 3.4485 5
0.061	332.5	137.2 65	0.002422 32	0. 2	0.30 5	0.043233694	0.684	2.31 3	0.0024 58	1.4890 73

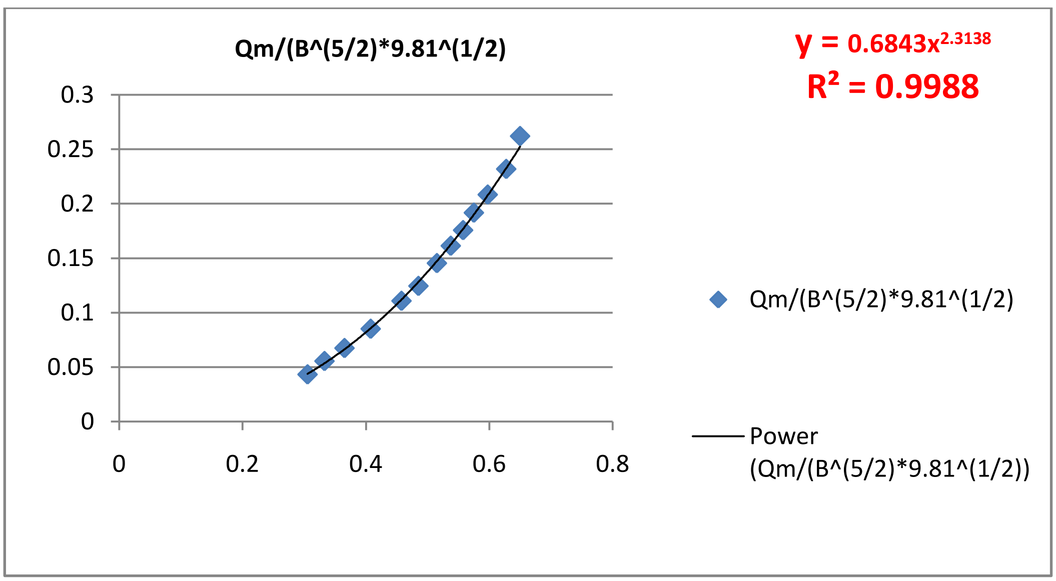

Figure:3. Plot of ($Q_m/(D^{3/2}*B*g^{1/2})$) with H/D

RESULT

The values of dimensionless constants k and n for the power equation y= k x ^{n} are found to be 0.6843 and 2.3138 resp. with the coefficient of determination R^2= 0.9988.

Also the maximum percentage error is found to be 3.08% which is within the permissible limit as per the specification, so the developed mathematical model for conical flume can be efficiently used for the measurement of discharge in trapezoidal open channel for side slope of 1:1.

CONCLUSION

Simple conical flume can be used as a mobile discharge measurement device for small sized trapezoidal canals on irrigation fields. The discharge can be estimated using the proposed mathematical model {Equation (12)}, which incorporates the channel bed width, diameter of cone and the water head reading. The results of the experimental

show that the proposed mathematical model {Equation (12)} predicts more accurate

discharge with maximum error upto 3.08%. The proposed mathematical model is valid for only the trapezoidal section with side slopes of 1:1. There is need to collect more experimental data for different side slopes (m:1), so that a more generalized mathematical model can be developed.

NOTATIONS

The following symbols are used in this paper:

B - Canal bottom width;
D - Diameter of column pipe;
f - Function;
g - Acceleration due to gravity;
K - Coefficient in head-discharge relation;
m - Side slope;
n - Exponent in head-discharge relation;
N - Number of observations;
Q - Flow rate;
Qm - Measured Discharge;
Qp - Predicted Discharge by proposed model;
R^2 - Coefficient of determination;

REFERENCES

[1]Hager (1986). "Modified,Trapezoidal venture channel," Journal of Irrigation and Drainage Engineering, ASCE, Vol –112, No -3,Pg.-225–241.

[2]Samani, Z., and Magallanez, H. (1993). ''Measuring water in trapezoidal Canals,'' Journal of Irrigation and Drainage Engineering, ASCE, Vol –119, No -1,Pg.-181–186.

[3]Skogerboe, G. V., and Hyatt, M.L., (1967), "Rectangular cut throat flumes", Journal of Irrigation and Drainage Engineering Div., 98(4), 569-583.

[4]Hager, W. H. (1985). "Modified Venturi Channel," Journal of Irrigation and Drainage Engineering., ASCE, Vol– 111, No - 1, Pg. -19-35

[5]Samani, Z., and Magallanez, H. (2000)."Simple flume for flow measurement in open channel."Journal of Irrigation and Drainage Engineering, ASCE, Vol – 126, No. 2, Pg. No.127–129.

[6]Baiamonte, G., and Ferro, V., (2007). "Simple flume for measurement in sloping open channel", Journal of Irrigation and Drainage Engineering, ASCE, V01-113, No.-1, Pg -71-78